Quantum-Inspired Genetic Algorithm Based Time-Frequency Atom Decomposition*

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Abstract. The main problem of time-frequency atom decomposition (TFAD) lies in an extremely high computational load. This paper presents a fast implementation method based on quantum-inspired genetic algorithm (QGA). Instead of finding the optimal atom in greedy implementation algorithm, this method is to search a satisfactory atom in every iteration of TFAD. Making full use of QGA's advantages such as good global search capability, rapid convergence and short computing time, the method reduces greatly the computational load of TFAD. Experiments conducted on radar emitter signals verify the effectiveness and practicality of the introduced method.

Keywords: Quantum-inspired genetic algorithm, time-frequency atom decomposition, fast implementation algorithm.

1 Introduction

Time-frequency atom decomposition (TFAD), also known as matching pursuit or adaptive Gabor representation [1,2], was introduced independently in [3] and [4]. TFAD is an approach that decomposes any signal into a linear expansion of waveforms selected from a redundant dictionary of time-frequency atoms that well localized both in time and frequency [3]. Unlike Wigner and Cohen class distributions, the energy distribution obtained by TFAD does not include interference terms. Different from Fourier and Wavelet orthogonal transforms, the information in TFAD is not diluted across the whole basis [3]. Hence, TFAD has become an attractive analysis technique in signal processing and harmonic analysis [1, 3-5]. However, the the necessary dictionary of time-frequency atoms being very large, the computational load turns out to be the main problem of TFAD [2]. After a greedy algorithm was presented in [3], several fast implementation algorithms were introduced in [6-9]. Although genetic algorithm was used to reduce the computational load in [1,2,6-8], this problem need be studied further by using some unconventional computation methods with some advantages over conventional genetic algorithms.

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This paper introduces quantum-inspired genetic algorithm (QGA) into TFAD to lower the computational complexity. Based on the concepts of quantum computing, QGA falls into the latest category of unconventional computation. Due to some outstanding advantages such as good global search capability, rapid convergence and short computing time [10-12], QGA is able to accelerate greatly the process of searching the most satisfactory time-frequency atom in each iteration of TFAD. In Section 2, the detailed algorithm of QGA based TFAD is described. Next, some experiments are conducted on radar emitter signals in Section 3. Finally, some conclusions are drawn.

2 QGA Based TFAD

The structure of QGA Based TFAD is presented in Algorithm 1 and the detailed description is as follows.

Algorithm 1. *Algorithm of QGA Based TFAD*

```
        Begin
(i)       Initialization of TFAD; % Initial iteration T=1
(ii)      While (not termination condition of TFAD) do
            % Searching the most satisfactory atom using QGA
(iii)       Setting initial values of parameters in QGA; % generation g=0
(iv)        Initializing P(g);
(v)         Generate R(g) by observing P(g); %
(vi)        Fitness evaluation; %
(vii)       Store the best solution among P(g) into B(g);
            While (not termination condition of QGA) do
              g=g+1;
(viii)        Generate R(g) by observing P(g-1); %
(ix)          Fitness evaluation; %
(x)           Update P(g) using quantum rotation gate; %
(xi)          Store the best solution among P(g) and B(g-1) into B(g); %
              If (migration condition)
(xii)           Migration operation;
              End if
              If (catastrophe condition)
(xiii)          Catastrophe operation;
              End if
            End while
            T=T+1;
(xiv)       Computing residual signal %
          End while
        End
```

(i) In the first step, the initial iteration T is set to 1. The original signal, represented linearly by time-frequency atoms, is loaded from the database or generated in simulation way. Let f be the original signal, $f \in \mathbf{H}$, where

H is a Hilbert space. By using TFAD, the signal f can be represented with a linear expansion of time-frequency atoms that are dilations, translations, and modulations of a single window function g_γ ($\|g_\gamma\| = 1$) [3]. Let $\mathscr{D} = \{g_\gamma\}_{\gamma \in \Gamma}$, where Γ is the set of the index γ that is composed of y parameters, i.e. $\gamma = (\gamma_1, \gamma_2, \cdots, \gamma_y)$. In this paper, time-frequency atom uses Gabor function:

$$g_\gamma(t) = \frac{1}{\sqrt{s}} g(\frac{t-u}{s}) \cos(\nu t + \omega) \ . \tag{1}$$

where the index γ=(s, u, ν, ω) is a set of parameters and s, u, ν, ω are scale, translation, frequency and phase, respectively. $g(\cdot)$ is a Gauss-modulated window function as

$$g(t) = e^{-\pi t^2} \ . \tag{2}$$

In the index γ=(s, u, ν, ω), there are four parameters to optimize. The method for choosing the four parameters is $\gamma = (c^x, bc^x \Delta u, dc^{-x} \Delta\nu, z\Delta\omega)$, $c = 1.5$, $\Delta u = 1/2$, $\Delta\nu = \pi$, $\Delta\omega = \pi/6$, $0 < x \le \log_2 L$, $0 < b \le L/2^{x-1}$, $0 \le z \le 12$, $0 \le d \le 2^{x+1}$, where L is the length of the signal f [3].

(ii) In TFAD, the maximal number of iteration is usually used as the termination condition.

(iii) The initial values of some parameters in QGA are set. The parameters include population size N, the number y=4 of variables, the number m of binary bits of each variable, the maximal evolutionary generation g of QGA for searching the sub-optimal time-frequency atom in the over-complete dictionary (i.e. in each iteration), immigration generation M_g and catastrophe generation C_g.

(iv) In this step, population $\boldsymbol{P}(g)$=$\{\boldsymbol{p}_1^g, \boldsymbol{p}_2^g, \cdots, \boldsymbol{p}_N^g\}$, where $\boldsymbol{p}_i^g (i = 1, 2, \cdots, N)$ is an arbitrary individual in population and $\boldsymbol{p}_i^g$ is represented as

$$\boldsymbol{p}_i^g = \begin{bmatrix} \alpha_{i1}^g | \alpha_{i2}^g | \cdots | \alpha_{i(my)}^g \\ \beta_{i1}^g | \beta_{i2}^g | \cdots | \beta_{i(my)}^g \end{bmatrix} \ . \tag{3}$$

where $\alpha_{ij}^g = \beta_{ij}^g = 1/\sqrt{2}$ $(j$=$1, 2, \cdots, my)$, which means that all states are superposed with the same probability. Here, α_{ij}^g and β_{ij}^g $(j = 1, 2, \cdots, my)$ are the probability amplitudes of one qubit and satisfy the normalization equality: $|\alpha|^2 + |\beta|^2 = 1$, where $|\alpha|^2$ and $|\beta|^2$ are the probabilities that the qubit will be observed in '0' state and in '1' state in the act of observing the quantum state.

(v) According to probability amplitudes of all individuals in $\boldsymbol{P}(g)$, observation states $\boldsymbol{R}(g)$ is constructed by observing $\boldsymbol{P}(g)$. Here $\boldsymbol{R}(g)$=$\{\boldsymbol{a}_1^g, \boldsymbol{a}_2^g, \cdots, \boldsymbol{a}_N^g\}$, where $\boldsymbol{a}_i^g (i = 1, 2, \cdots, N)$ is an observation state of an individual. $\boldsymbol{a}_i^g$ is a binary string with the length of my, that is $\boldsymbol{a}_i^g = b_1 b_2 \cdots b_{my}$, where b_j $(j = 1, 2, \cdots, my)$ is one binary number '0' or '1'. Observation states $\boldsymbol{R}(g)$ is generated in probabilistic way. For the probability amplitude $[\alpha\ \beta]^T$ of a qubit, a random number r in the range [0, 1] is generated. If $r < |\alpha|^2$, the corresponding observation value is set to '0', otherwise, the value is set to ‘1’. In the process of constructing observation state $\boldsymbol{R}(g)$ using $\boldsymbol{P}(g)$, the decoding operation in conventional genetic algorithm is included. After decoding, the parameter values of all optimization parameters can be obtained.

(vi) The value of fitness function of each individual in population is computed by using the values of y variables and each individual in population is evaluated. Here, the fitness is chosen as $|\langle f, g_\gamma \rangle|$. The reason is as follows. The signal f will be decomposed into $f = \langle f, g_\gamma \rangle g_\gamma + Rf$, where Rf is the residual signal after approximating f in the direction of g_γ. Obviously, g_γ is orthogonal to Rf. So the following equality can be obtained [3].

$$\|f\|^2 = |\langle f, g_\gamma \rangle|^2 + \|Rf\|^2 \; . \tag{4}$$

In (4), only if $|\langle f, g_\gamma \rangle|$ $(g_\gamma \in \mathscr{D})$ is maximum, $\|Rf\|$ will be minimum.

(vii) The best solution in $\boldsymbol{P}(g)$ at generation g is stored into $B(g)$.

(viii) According to probability amplitudes of all individuals in $\boldsymbol{P}(g-1)$, observation states $\boldsymbol{R}(g)$ is constructed by observing $\boldsymbol{P}(g-1)$. This step is similar to step (v). The detailed procedures have been described.

(ix) This step is similar to step (vi).

(x) The probability amplitudes of all qubits in population are updated by using quantum rotation gates given in (5).

$$\boldsymbol{G} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \; . \tag{5}$$

where θ is the rotation angle of quantum rotation gate and θ is defined as

$$\theta = k \cdot f(\alpha, \beta) \; . \tag{6}$$

where k is a coefficient whose value has a direct effect on convergent speed of QGA. In this paper, k is defined as a variable, which is relative to the evolutionary generation g. A kind of definition of k is given in (7).

$$k = \frac{\pi}{2} e^{-\frac{mod(g,100)}{10}} \; . \tag{7}$$

where mod(g,100) is a function for computing modulus after g is divided by 100. k varies from $\pi/2$ to 0, which indicates QGA searches the optimal solution at large grid at the beginning of the algorithm and then the search grid declines gradually to 0 as the evolutionary generation g increases to 100. When the evolutionary generation g amounts to 100, the value of k comes back to $\pi/2$. $f(\alpha, \beta)$ is a function for determining the search direction of QGA to a global optimum. The look-up table of $f(\alpha, \beta)$ is shown in Table 1, in which $\xi_1 = \arctan(\beta_1/\alpha_1)$ and $\xi_2 = \arctan(\beta_2/\alpha_2)$, where α_1, β_1 are the probability amplitudes of the best solution stored in $B(g)$ and α_2, β_2 are the probability amplitudes of the current solution [12]. Thus, the formula for updating an individual $\boldsymbol{p}_i^g$ in $\boldsymbol{P}(g)$ using quantum rotation gate $\boldsymbol{G}$ can be described as

$$\boldsymbol{p}_i^{g+1} = \boldsymbol{G}(g) \cdot \boldsymbol{p}_i^g \; . \tag{8}$$

Table 1. Look-up table of function $f(\alpha, \beta)$ (Sign is a symbolic function)

$\xi_1 > 0$	$\xi_2 > 0$	$f(\alpha, \beta)$	
		$\xi_1 \geq \xi_2$	$\xi_1 < \xi_2$
True	True	+1	-1
True	False	sign$(\alpha_1 \cdot \alpha_2)$	
False	True	-sign$(\alpha_1 \cdot \alpha_2)$	
False	False	sign$(\alpha_1 \cdot \alpha_2)$	-sign$(\alpha_1 \cdot \alpha_2)$
$\xi_1, \xi_2 = 0$ or $\pi/2$		± 1	

where g is evolutionary generation, $\boldsymbol{G}(g)$ stands for the gth generation quantum rotation gate, $\boldsymbol{p}_i^g$ and $\boldsymbol{p}_i^{g+1}$ are the probability amplitudes at gth generation and at $(g+1)$th generation, respectively. Once the probability amplitudes of all individuals in $\boldsymbol{P}(g)$ are updated using quantum rotation gate, the individuals in the next generation are generated.

(xi) The best solution among $\boldsymbol{P}(g)$ and $B(g-1)$ is stored into $B(g)$.

(xii) When immigration operation is made every M_g generation, the 10% of probability amplitudes in the stored best individual are replaced by the new probability amplitudes generated using the similar method in step (ii). That is, each α is a random value from 0 to 1 and corresponding β equals $\pm\sqrt{1-|\alpha|^2}$. How to choose M_g is discussed in the next section.

(xiii) If the best solution stored in $B(g)$ is not changed in many generations, such as C_g generations, then population catastrophe operation should be performed.

(xiv) The residual signal $R^{(T+1)}f$ is $R^{(T+1)}f = R^T f - \langle R^T f, g_{\gamma_T} \rangle g_{\gamma_T}$. The residual signal is used as the original signal in the next iteration.

After the signal f is decomposed up to the order T, f can be represented with the concatenated sum

$$f = \sum_{i=0}^{T} \langle f, g_{\gamma_i} \rangle g_{\gamma_i} + R^{T+1} f \ . \tag{9}$$

where g_{γ_i} satisfies

$$|\langle R^T f, g_{\gamma_i} \rangle| = \alpha \sup_{\gamma \in \Gamma} |\langle R^T f, g_\gamma \rangle| \ . \tag{10}$$

According to the conclusion [3]: $\lim_{T \to \infty} \|R^T f\| = 0$, the signal f can be represented as

$$f = \sum_{i=0}^{T} \langle f, g_{\gamma_i} \rangle g_{\gamma_i} \ . \tag{11}$$

To compute the correlation between the original signal f and the restored signal f_r with parts of decomposed time-frequency atoms, resemblance coefficient method [13] is used to the correlation ratio C_r of f and f_r.

$$C_r = \frac{\langle f, f_r \rangle}{\|f\| \cdot \|f_r\|} \ . \tag{12}$$

The decaying value d_c is a function of the number of iteration T. d_c is computed using the following formula.

$$d_c = \log_{10} \frac{\|R^T f\|}{\|f\|} . \qquad (13)$$

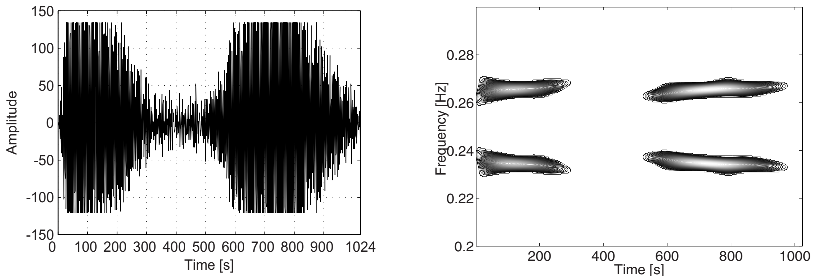
where $R^T f$ is the residual signal after approximating the original signal f using the first T time-frequency atoms [3]. The variable d_c is directly related to convergent speed of TFAD implementation algorithm.

3 Experiments

To test the validity of the introduced method, a real radar emitter signal is used to conduct the experiment, in which greedy implementation algorithm (GIA) [3] is brought into comparison with QGA. Experimental environment is chosen as: the maximal number of iteration is set to 500 as the termination condition of TFAD; From our prior tests, c=1.5 is much better than c=2 in [3]; In QGA, population size N, the number m of binary bits of each variable, the maximal evolutionary generation g, immigration generation M_g and catastrophe generation C_g are set to 20, 10, 200, 15 and 25, respectively. These experiments are carried out on the personal computer with 3.06GHz CPU,1GHz EMS memory and 160GB hard disk. The performances of the two algorithms are evaluated by using computing efficiency and optimization result. Computing efficiency includes computing time and the decay performance given in (13). Optimization result is evaluated by using the correlation ratio of the original signal and approximation signal given in (12). Figure 1 shows the noised radar emitter signal and its time-frequency distribution. Experimental results are given in Fig.2, Table 2 and Fig.3. The decaying curves of GIA and QGA in Fig.2 show the difference between the two algorithms. Figure 3 illustrates time-frequency distributions of restored radar emitter signals from the decomposed time-frequency atoms. As can be seen from Fig.2 and Table 2, the introduction of QGA into TFAD shortens greatly the computing time. In Table 2, the computing time of QGA is about 30 times as small as that of GIA. Figure 3 brings us some important hints that time-frequency distributions constructed by decomposed time-frequency atoms are nearly identical with that of the original radar emitter signals and TFAD is a good decomposition approach of a signal. The experimental results manifest that QGA is a better method than GIA for solving the problem of computing load in time-frequency atom decomposition.

Table 2. Performance comparisons of GIA and QGA

Methods	QGA	GIA
Correlation ratio	0.9903	0.9904
Computing time (Second)	4130.8750	123850.5781



(a) Time-domain signal (b) Time-frequency distribution

Fig. 1. A radar emitter signal with noise

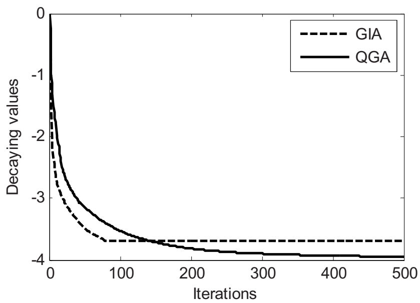
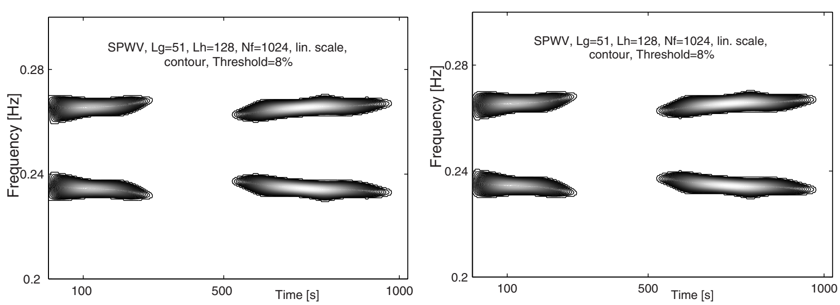


Fig. 2. The decaying curves of GIA and QGA



(a) Obtained by QGA (b) Obtained by GIA

Fig. 3. Time-frequency distributions of restored signals

4 Concluding Remarks

This paper presents a fast and efficient implementation method for TFAD based on QGA. The steps of the method are described in detail. The experiment on a real radar emitter signal has been carried out to verify the validity and efficiency of the introduced method. Based on the concepts and principles of quantum computing and taking advantage of the strong parallelism of quantum computing, QGA uses its good characteristics of good global search capability, rapid convergence and short computing time to decrease greatly the computational load of TFAD. Our further study will aim at feature extraction and analysis of radar emitter signals by using the introduced method.

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