Interactive Fuzzy Goal Programming Approach for Optimization of Extended Hub-and-Spoke Regional Port Transportation Networks

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Abstract. Based on interactive fuzzy goal programming, a model is introduced for extended hub-and-spoke regional port transportation network optimization problem where one of the objective functions is considered as a non-linear function. The proposed model considers the imprecise nature of the input data and assumes that each objective function has a fuzzy goal, and aims at jointly minimizing the total costs including the sailing cost and handling cost as well as the total transit times consisting of sailing time and handling time. Meanwhile it optimizes the following factors: transportation quantities via a hub port from an original port to a destination port, transportation quantities directly from an original port to a destination port. The solution procedure to the proposed model is then presented. At last, a numerical example is given to demonstrate the effectiveness of the proposed model and evaluate the performance of the solution procedure.

Keywords: Fuzzy goal programming, Hub-and-spoke, Optimization, Port transportation network.

1 Introduction

In regional port transportation networks, the sizes and types of ships calling at every port are different because individual port's natural condition and capacity are different. The route selection is important in regional port transportation network optimization. The port condition and economy of scale in ship transportation should be considered to select the routes from original ports to destination ports. The ports are connected by direct transports or via a transshipment port. This has led to the formation of hub-and-spoke networks in regional port transportation industry. Cargoes from original ports are usually consolidated at hub port and shipped to different destination ports. Hub-and-spoke transportation can be classified into two types: pure and extended [1]. The pure hub-and-spoke transportation network is characterized by transshipment in which direct transport between ports is excluded and all cargoes have to be transported via a hub port. The extended hub-and–spoke transportation network consists of direct transportation between ports and transshipment via the hub port.

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In this paper, based on extended hub-and-spoke port transportation system, a decision model is developed to determine the following factors: shipment volume via a hub port from an original port to a destination port, shipment volume directly from an original port to a destination port.

The decision problem of hub-and-spoke systems has received many attentions in academic literature. Some researchers solved it as the location –allocation problem that determines locations of hubs and assigns shipments to each route[2][3][4]. Another researchers examined it as the pure allocation problem that finds the optimal assignment of shipments under predetermined hub locations[5][6][7]. But the models and methods in these studies are applied in air or truck transportation networks and have been less focused on linking hub-and-spoke system to ports. Most of them only consider transportation (or travel) cost and don't include the handling costs occurring at nodes. Furthermore, these studies only consider one objective function. This paper introduces a model to examine the pure allocation problem in extended hub-and-spoke regional port transportation network. The model aims at jointly minimizing the total costs consisting of sailing cost and handling cost occurring at the ports as well as total time consisting of sailing time and handling time occurring at the ports, in which the time objective function is a non-linear function. In addition, the existing decision problem of hub-and-spoke system is a deterministic mathematical programming problem. However, in practice, the input data or decision parameters, such as transportation capacity, cost, time and objective function are often imprecise because some information is uncertain. Therefore, the values of these parameters are rarely constant. To deal with ambiguous parameters in the above decision problem, this paper uses an interactive fuzzy goal programming model to formulate the problem of extended hub-and-spoke regional port transportation network optimization, and propose a solution procedure for this model.

2 The Decision Problem of Extended Hub-and-Spoke Port Transportation Network

2.1 Notations

M **,** the number of ports excluding the hub port 0;

D_{ij}, transportation demand quantity from port i to port j;

C_{ij}, the transportation cost per unit cargo from port i to port j;

S_{ij}, the transportation capacity in the route from port i to port j;

T_{ij}^{k}, the sailing time from port i to port j;

C_i, the handling cost per unit cargo occurred at port i;

T_i the handling time per unit cargo occurred at port i;

x_{ij}**,** transportation quantity from port i to port j ($i \neq j$); (Decision variables)

2.2 Mathematical Model

The decision problem of extended hub-and-spoke port transportation system can be formulated as follows:

$$\text{(P1)}\ \min\ Z_1 = \sum_{i=0}^{M} \sum_{\substack{j=0 \\ i \neq j}}^{M} C_{ij} x_{ij} + \sum_{i=0}^{M} C_i \sum_{\substack{j=0 \\ j \neq i}}^{M} x_{ij} + \sum_{i=0}^{M} C_i \sum_{\substack{j=0 \\ j \neq i}}^{M} x_{ji} \tag{1}$$

$$\min\ Z_2 = \sum_{i=0}^{M} \sum_{\substack{j=0 \\ i \neq j}}^{M} T_{ij} x_{ij} + \sum_{i=0}^{M} \sum_{\substack{j=0 \\ i \neq j}}^{M} (T_i \sum_{\substack{j=0 \\ j \neq i}}^{M} x_{ij} + T_j \sum_{\substack{i=0 \\ i \neq j}}^{M} x_{ij}) x_{ij} \tag{2}$$

$$\text{s.t.}\ x_{i0} + \sum_{\substack{j=1 \\ j \neq i}}^{M} x_{ij} = \sum_{\substack{j=1 \\ j \neq i}}^{M} D_{ij}\,,\ i = 1,2,\ldots M, i \neq 0 \tag{3}$$

$$x_{0i} + \sum_{\substack{j=1 \\ j \neq i}}^{M} x_{ji} = \sum_{\substack{j=1 \\ j \neq i}}^{M} D_{ji}\ \ i = 1,2,\ldots M, i \neq 0 \tag{4}$$

$$x_{ij} \leq S_{ij}\ \ i = 0,1,\ldots M, j = 0,1,\ldots M, i \neq j \tag{5}$$

$$x_{ij} \geq 0\ \ \ i = 0,1,\ldots M, j = 0,1,\ldots M, i \neq j \tag{6}$$

In real world, the input data or parameters of (P1) problem, such as transportation capacity and objective function are often imprecise because some information is unobtainable. Conventional mathematical programming can't capture such vagueness in the critical information. In this case, a fuzzy programming approach is commonly used to treat the information in a fuzzy environment. However, some researchers have presented the shortcomings of using fuzzy programming in solving some multi-objective decision problems. Abd El-Wahed (2001) proved that using fuzzy programming in solving such multi-objective transportation problem changes the standard form of the transportation problem [8]. In addition, Li and Lai (2000) proved that using the min-operator does not guarantee an efficient solution[9]. In this paper, we introduce an interactive fuzzy goal programming model for regional port transportation networks optimization, which is the combination of interactive programming, fuzzy programming and goal programming, and leverages the advantages of the three approaches as well as reduces some or all of the shortcomings of each individual approach [10].

3 Interactive Fuzzy Goal Programming Approach

3.1 IFGP Model

In our model, we capture the ambiguity about the fuzzy information related to the total transportation cost, total transportation time and transportation capacities by transforming the (P1) model into the following (P2) model.

(P2) $$\sum_{i=0}^{M}\sum_{\substack{j=0\\ i\neq j}}^{M} C_{ij}x_{ij} + \sum_{i=0}^{M} C_i \sum_{\substack{j=0\\ j\neq i}}^{M} x_{ij} + \sum_{i=0}^{M} C_i \sum_{\substack{j=0\\ j\neq i}}^{M} x_{ji} \lesssim \tilde{Z}_1 \tag{7}$$

$$\sum_{i=0}^{M}\sum_{\substack{j=0\\ i\neq j}}^{M} T_{ij}x_{ij} + \sum_{i=0}^{M}\sum_{\substack{j=0\\ i\neq j}}^{M} (T_i \sum_{\substack{j=0\\ j\neq i}}^{M} x_{ij} + T_j \sum_{\substack{i=0\\ i\neq j}}^{M} x_{ij}) x_{ij} \lesssim \tilde{Z}_2 \tag{8}$$

s.t. $$x_{ij} \lesssim \tilde{S}_{ij} \quad i = 0,1,...M,\ j = 0,1,...M, i \neq j, \tag{9}$$

as well as constraints given in (3), (4) and (6)

where the symbol " $\lesssim$ " indicates "essentially smaller than or equal to " and allows one reach some aspiration level, $\tilde{Z}_1, \tilde{Z}_2$, and $\tilde{S}_{ij}$ denote fuzzy values.

In this paper, a linear membership function defined in Bellman and Zadeh(1970) has been considered for all fuzzy parameters in (P2) problem.

To formulate model (P2) as a goal programming model, we introducing the following positive and negative deviation variables:

$$Z_k(x) - d_k^+ + d_k^- = G_k, \qquad k = 1,2 \tag{10}$$

where G_i is the aspiration level of the objective function i.

By using the given membership functions and introducing an auxiliary variable L the fuzzy programming model (P2) can be formulated as the following equivalent programming model (P3) (Zimmermann, 1978):

(P3) Max L

s.t. $$L(Z_k^{\max} - Z_k^{\min}) + Z_k(x) \leq Z_k^{\max} \qquad k = 1,2 \tag{11}$$

$$L(S_{ij}^U - S_{ij}^L) + x_{ij} \leq S_{ij}^U \quad i = 0,1,...M,\ j = 0,1,...M, i \neq j \tag{12}$$

$$Z_k(x) - d_k^+ + d_k^- = G_k \qquad k = 1,2 \tag{13}$$

$$0 \leq L \leq 1, \tag{14}$$

$$d_k^+, d_k^- \geq 0, \qquad k = 1,2 \tag{15}$$

as well as constraints given in (3), (4) and (6).

3.2 The Solution Procedure

In order to obtain the solution to Model (P3), the following procedure can be applied.

Step1: Solve the model (P1) as a single objective problem to obtain the initial solutions for each objective function, i.e. $X^1 = \{x_{ij}^1\}$ and $X^2 = \{x_{ij}^2\}$, $i = 0,1,..M, j = 0,1,...,M,.\ i \neq j$. If $X^1 = X^2$, select one of them as an optimal solution and go to Step 6. Otherwise, go to Step 2.

Step 2: Determine the best lower bound ($Z_k^{\min}$) and the worst upper bound ($Z_k^{\max}$) for each objective function: $Z_1^{\min}=Z_1(X^1)$, $Z_1^{\max}=Z_1(X^2)$, $Z_2^{\min}=Z_2(X^2)$, $Z_2^{\max}=Z_2(X^1)$, $i=0,1,..M$, $j=0,1,...,M$, $i\neq j$.

Step 3: Define the membership functions of each objective function.

Step 4: Solve problem (P4) as a goal programming and obtain the solution to it. Compare the upper bound of each objective function with the new value of the objective function. If the new value of each objective function is equal to the upper bound, go to Step 6. Otherwise, go to Step 5.

Step 5: Update the upper bound of each objective function. If the new value of the objective function is lower than the upper bound, consider this as a new upper bound. Otherwise, keep the old one as is. Go to Step 3.

Step 6: Stop.

4 Numerical Example

We consider the following numerical example to demonstrate the application of IFGP. Assume that there are four ports and one hub in regional port transportation network. There are two types of ships employed in the route between two ports. The transportation capacity of ship for type 1 is 5000 Ton, whereas that for type 2 is 8000 ton. The handling cost per ton occurring at Port 1, Port 2, Port 3, Port 4 and Port 0 are 0.40,0.55,0.60,0.65 and 0.40 Yuan, respectively. The handling time per ton occurring at Port 1, Port 2, Port 3, Port 4 and Port 0 are 0.5,1.0,1.0,1.5 and 0.5 hour, respectively. The other data of the problem is given in Table1, Table2 and Table3.

Table 1. Transportation demand between ports (ton)

Original Ports	Destination ports				
	Port1	Port 2	Port 3	Port 4	Hub 0
Port 1	----	1790	1960	2680	750
Port 2	2500	----	3030	6830	1000
Port 3	2320	2670	----	9490	2100
Port 4	890	2330	3690	----	650
Hub 0	450	1500	2700	560	----

Table 2. Transportation costs (Yuan per Ton) and sailing time(Hours) in different routes

Original ports	Destination ports									
	Port 1		Port 2		Port 3		Port 4		Hub 0	
	cost	time	cost	time	cost	time	cost	time	cost	time
Port 1	----	----	3.0	10.0	4.0	8.0	4.0	8.0	1.5	5.0
Port 2	2.5	11.0	----	----	2.5	11.0	4.5	8.0	1.1	4.0
Port 3	3.0	10.0	2.8	9.0	----	----	4.0	8.0	1.5	5.0
Port 4	1.6	5.5	3.0	10.0	4.0	8.0	----	----	2.0	11.0
Hub 0	1.5	5.0	1.1	4.0	1.5	5.0	1.8	11.0	----	----

Table 3. Transportation capacity provided by carriers for different routes (Ton)

Original ports	Destination ports									
	Port 1		Port 2		Port 3		Port 4		Hub 0	
	Lower bound	Upper bound	Lower bound	Upper bound	Lower bound	Upper bound	Lower bound	Upper bound	Lower bound	Upper bound
Port 1	----	----	1850	2000	2350	2500	2850	3000	1570	1600
Port 2	2880	3000	----	----	2950	3000	6850	7000	1460	1500
Port 3	2450	2500	2850	3000	----	----	9350	9500	2350	2500
Port 4	850	900	2450	2500	3850	4000	----	----	1150	1200
Hub 0	560	600	1580	1600	2860	3000	560	600	----	----

4.1 Solution

The initial model (P1) of the numerical example can be obtained as

$$\text{Min } Z_1 = 2.3x_{01} + 2.05x_{02} + 2.5x_{03} + 2.85x_{04} + 3.95x_{12} + 5.0x_{13} + 5.05x_{14} + 2.3x_{10} + 3.45x_{21} + 3.65x_{23}$$
$$+ 5.70x_{24} + 2.05x_{20} + 4.0x_{31} + 3.95x_{32} + 5.25x_{34} + 2.5x_{30} + 2.65x_{41} + 4.2x_{42} + 5.25x_{43} + 3.05x_{40}$$
$$Z_2 = 5.0x_{01} + 4.0x_{02} + 5.0x_{03} + 11.0x_{04} + 10.0x_{12} + 8.0x_{13} + 8.0x_{14} + 5.0x_{10} + 11.0x_{21} + 11.0x_{23}$$
$$+ 8.0x_{24} + 4.0x_{20} + 10.0x_{31} + 9.0x_{32} + 8.0x_{34} + 5.0x_{30} + 5.5x_{41} + 10.0x_{42} + 8.0x_{43} + 11.0x_{40}$$
$$+ x_{01}^2 + x_{01}x_{02} + x_{01}x_{03} + x_{01}x_{04} + x_{01}x_{21} + x_{01}x_{31} + x_{01}x_{41} + 1.5x_{02}^2$$
$$+ x_{02}x_{03} + x_{02}x_{04} + 2.0x_{02}x_{12} + 2.0x_{02}x_{32} + 2.0x_{02}x_{42} + 1.5x_{03}^2 + 1.0x_{03}x_{04} + 2.0x_{03}x_{13}$$
$$+ 2.0x_{03}x_{23} + 2.0x_{03}x_{43} + 2.0x_{04}^2 + 3.0x_{04}x_{14} + 3.0x_{04}x_{24} + 3.0x_{04}x_{34} + x_{10}^2 + x_{10}x_{12}$$
$$+ x_{10}x_{13} + x_{10}x_{14} + x_{10}x_{20} + x_{10}x_{30} + x_{10}x_{40} + 1.5x_{12}^2 + x_{12}x_{13} + x_{12}x_{14} + 2.0x_{12}x_{32}$$
$$+ 2.0x_{12}x_{42} + 1.5x_{13}^2 + x_{13}x_{14} + 2.0x_{13}x_{23} + 2.0x_{13}x_{43} + 2.0x_{14}^2 + 3.0x_{14}x_{24} + 3.0x_{14}x_{34}$$
$$1.5x_{20}^2 + 2.0x_{20}x_{21} + 2.0x_{20}x_{23} + 2.0x_{20}x_{24} + x_{20}x_{30} + x_{20}x_{40} + 1.5x_{21}^2 + 2.0x_{21}x_{23}$$
$$+ 2.0x_{21}x_{24} + x_{21}x_{31} + x_{21}x_{41} + 2.0x_{23}^2 + 2.0x_{23}x_{24} + 2.0x_{23}x_{43} + 2.5x_{24}^2 + 3.0x_{24}x_{34}$$
$$+ 2.0x_{30}x_{34} + 2.0x_{31}x_{34} + 2.0x_{32}x_{34} + 2.5x_{34}^2 + 2.0x_{30}x_{32} + 2.0x_{31}x_{32} + 2.0x_{32}^2 + 2.0x_{32}x_{42}$$
$$+ 2.0x_{30}x_{31} + 1.5x_{31}^2 + 1.5x_{30}^2 + 1.0x_{31}x_{41} + 1.0x_{30}x_{40} + 2.0x_{40}^2 + 3.0x_{40}x_{41} + 3.0x_{40}x_{42} + 3.0x_{40}x_{43}$$
$$+ 2.0x_{41}^2 + 3.0x_{41}x_{42} + 3.0x_{41}x_{43} + 2.5x_{42}^2 + 3.0x_{42}x_{43} + 2.5x_{43}^2$$

$$\text{s.t. } x_{10} + x_{12} + x_{13} + x_{14} = 7180,\quad x_{20} + x_{21} + x_{23} + x_{24} = 13360$$
$$x_{30} + x_{31} + x_{32} + x_{34} = 16580,\quad x_{40} + x_{41} + x_{42} + x_{43} = 7560$$
$$x_{01} + x_{21} + x_{31} + x_{41} = 6160,\quad x_{02} + x_{12} + x_{32} + x_{42} = 8290$$
$$x_{03} + x_{13} + x_{23} + x_{43} = 11380,\quad x_{04} + x_{14} + x_{24} + x_{34} = 19560$$
$$x_{10} \le 1600,\ x_{12} \le 2000,\ x_{13} \le 2500,\ x_{14} \le 3000,\ x_{20} \le 1500,\ x_{21} \le 3000$$
$$x_{23} \le 3000, x_{24} \le 7000,\ x_{30} \le 2500,\ x_{31} \le 2500, x_{32} \le 3000,\ x_{34} \le 9500$$
$$x_{40} \le 1200, x_{41} \le 900,\ x_{42} \le 2500,\ x_{43} \le 4000, x_{01} \le 600,\ x_{02} \le 1600$$
$$x_{03} \le 3000,\ x_{04} \le 600,\ x_{ij} \ge 0 \quad i = 0,1,\ldots 4, j = 0,1,\ldots 4, i \ne j$$

By using the above-mentioned solution procedure, the set of solutions (Z^1, Z^2) can be obtained after 17 iterations. The values of objective function are converged at (212509.0, 1366308000), the corresponding optimal solution is as follows:

$$x_{01}=0, x_{02}=790.1, x_{03}=2200.0, x_{04}=507.0, x_{12}=2000.0, x_{13}=2180.1, x_{14}=3000.0, x_{10}=0.0,$$
$$x_{21}=3000.0, x_{23}=3000.0, x_{24}=6553.1, x_{20}=807.0, x_{31}=2260.0, x_{32}=3000.0, x_{34}=9500.0, x_{30}=1820.0$$
$$x_{41}=900.0, x_{42}=2500.0, x_{43}=4000.0, x_{40}=160.0, d_1^+=234.0, d_1^-=0.0, d_2^+=2614425.0, d_2^-=0.0$$

4.2 Performance Evaluation

To evaluate the performance of the proposed model, the solution to the illustrative example by using different methods is considered. The fuzzy programming approach gives the following results: $Z^1=212729$, $Z^2=1368880000$. The interactive fuzzy goal programming approach provides the following results: $Z^1=$ 212505, $Z^2=1366640000$. To determine the degree of closeness of the IFGP model results to the ideal solution, the family of distance functions presented in [10][11] is considered.

$$D_{\mathrm{p}}(\lambda, K)=[\sum_{k=1}^{K}\lambda_k^p(1-d_k)^p]^{1/p}, \tag{16}$$

where d_k represents the degree of closeness of the solution to the ideal optimal solution with respect to the k th objective function, $d_k=$ the ideal optimal value of Z^k / the solution of Z^k. λ_k is the weight of the k th objective and $\sum_{k=1}^{K}\lambda_k=1$. The power p represents a distance parameter $1\le p\le\infty$.

Thus, we can state the approach is better than others if $Min\ D_p(\lambda, K)$ is achieved by its solution with respect to some p. Assuming $\lambda_1=\lambda_2=0.5$, $p=1,2$ and ∞, the results of the different approaches are given in Table 4.

Table 4. Comparison of results by different approaches

	Fuzzy programming	Interactive fuzzy programming	Interactive fuzzy goal programming	Optimal solution
(Z^1,Z^2)	(212729, 1368880000)	(212505, 1366640000)	(212509.0, 1366308000)	(212274.5, 1363694000)
D_1	0.0030	0.0016	0.0015	--
D_2	0.0022	0.0012	0.0011	--
D_∞	0.0019	0.0011	0.0010	--

It is clear from Table 2 that the solution to the proposed approach is better than the solution by the other approaches for all the distance functions.

5 Conclusions

This paper proposes an interactive fuzzy goal programming model of the extended hub-and-spoke regional port transportation network optimization problem, where transportation capacity and objective function are considered as o imprecise/fuzzy, and aims at jointly minimizing the total costs including the sailing cost between ports and handling cost occurring at all ports as well as the total transit times consisting of sailing time between ports and handling time in all ports. To obtain the solution of the above-mentioned problem, the solution procedure for the model is illustrated by updating both the membership values and the aspiration levels. At last, a numerical example is given to demonstrate the effectiveness of the proposed model and evaluate the performance of the solution procedure by comparing its results with those of the fuzzy programming approach and interactive fuzzy programming approach.

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References

1. Zapfel, G., Wasner, M., Planning and optimization of hub-and-spoke transportation networks of cooperative third party logistics providers, *International Journal of Production Economics,* (2002), 78 :207-220.
2. Aykin,T.,Networking policies for hub-and-spoke systems with applications to the air transportation system, *Transportation Science*, (1995),26:201-221.
3. O'Kelly, M.E., A quadratic integer program for location of interacting hub facilities, *European Journal of Operational Research*, 1987, 32: 393-404.
4. Pirkul,H.,Schilling,D.A.,An efficient procedure for designing single allocation hub and spoke systems, *Management Science*, (1998), 44: S235-242.
5. Abdinnourhelm,S.,Venkataramanan,M.A., Solution approaches to hub location problems, *Annals of Operations Research*, (1998), 78:31-50.
6. Bertazzi,L.,Speranza,M.G.,Ukovich,W.,Minimization of logistics costs with given frequencies, *Transportation Research* B, (1997), 31: 327-340.
7. Liu, J.,Li,C.L.,Chan,C.Y.,Mixed truck delivery systems with both hub-and-spoke and direct shipment. *Transportation Research* E, (2003), 39:325-339.
8. Abd El-Wahed WF, A multi-objective transportation problem under fuzziness. *Fuzzy Sets and Systems*, (2001), 117:27-33
9. Li,L.,Lai KK., A fuzzy approach to the multi-objective transportation problem, *Computers & Operational Research*, (2000), 27: 43-57
10. Abd El-Wahed WF, Lee, S.M., Interactive fuzzy goal programming for multi-objective transportation problems, *Omega*, (2006), 34: 158-166
11. Steuer, R., Multiple criteria optimization: theory, computation, and application. New York: Wiley, (1986).