An Algorithm for Improving Hilbert-Huang Transform

Song Guo[1], Guochang Gu[1], and Changyou Li[2]

[1] School of Computer Science and Technology, Harbin Engineering University, Harbin 150001, China
guosong@hrbeu.edu.cn
[2] Department of General Mechanics and Mechanics Foundation, School of Aerospace, Harbin Institute of Technology, Harbin 150001, China
lichangyou_1980@163.com

Abstract. The Hilbert-Huang transform is viewed as a promising method to process the nonlinear and non-stationary signal. However, it hasn't been a general method in signal processing due to its some deficiencies. In this paper, an algorithm is proposed to resolve one of its deficiencies that the EMD will generate redundant IMFs at the low-frequency region. This deficiency may cause misinterpretation to the result of processing signal. The experimental results show that the proposed algorithm successfully resolves this deficiency.

Keywords: eliminating redundant IMFs; Hilbert-Huang transform; EMD.

1 Introduction

The wavelet transform has become one of the fast-evolving tools for analyzing nonlinear and non-stationary signals in the past decade. However, with the wide application of wavelet transform, some crucial deficiencies are reported. Hence, a new type of time-frequency analysis called Hilbert-Huang transform (HHT) has been proposed by N.E. Huang in 1998[1].

Compared with the wavelet transform, HHT has many advantages which were described in literature [2]. Nevertheless, in practical application, HHT has also some deficiencies, which will be described in detail in section 2. In this paper, an algorithm is proposed to resolve one of them.

2 The Deficiencies of Hilbert-Huang Transform

HHT consists of two main steps, the first step is data "sifting" to generate the intrinsic mode functions (IMFs) and the second step is to apply the Hilbert transform to the IMFs. The Empirical Mode Decomposition (EMD) which is the key part of this method is employed to "sift" data, by means of which any complicated data can be decomposed into a finite number of IMFs. The instantaneous frequency, determined by Hilbert Transform (HT) of IMF, provided much sharper identification of embedded events beyond the limitation of the Uncertainty Principle. HHT is described in detail in literature [1].

Y. Shi et al. (Eds.): ICCS 2007, Part III, LNCS 4489, pp. 137–140, 2007.
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HHT seems to be a perfect method to process the nonlinear and non-stationary signal. However, in practice, HHT has some unsolved problems. First, the EMD will generate some undesired low amplitude IMFs at the low-frequency region and raise some undesired frequency components. Second, the first IMF may cover a wide frequency range at the high-frequency region, therefore cannot satisfy the mono-component definition very well. Third, the EMD operation often cannot separate some low-energy components from the analysis signal; therefore those components may not be able to appear in the frequency-time plane. In this paper, all effort is dedicated to resolve the first deficiency because the second and third have been solved in [3] [4].

Now, let's observe the first deficiency according to one example. Let the signal is equal to: $x(t) = \sin(2*\pi*10*t) + \sin(2*\pi*15*t) + \sin(2*\pi*20*t) + \sin(2*\pi*30*t)$ and the results of EMD of $x(t)$ and the FFT of IMFs are described in fig. 1.

There are four mono-components in the signal $x(t)$ so that four IMFs should be acquired by EMD. However, ten IMFs are seen according to the left section of fig. 1 and we can observe that all the redundant IMFs have lower frequency than the lowest frequency of the signal $x(t)$ from the right section.

3 The Proposed Algorithm

In [3], [4], the correlation coefficients of IMFs and the signal is used as a criterion to decide which IMFs should be retained and which IMFs should be eliminated because the author thinks that the real IMF components will have relative good correlation with the original signal and on the other hand, the pseudo-components will only have poor correlation with the signal. However, the analysis is not logical. If the real IMFs have relative good correlation with the original signal, they will be also relative good correlation each other. This is inconsistent with the characteristic of EMD which the IMFs are almost an orthogonal representation for the analyzed signal. In [5], [6], Kolmogorov-Smirnov test is employed to resolve the same problem. Nevertheless, Kolmogorov-Smirnov test is mainly used to check if two independent distributions are similar or different.

According to the theory of EMD, the following equation is obtained

$$v_i(t) = x(t) - h_i(t) = \sum_{j=1}^{i-1} h_i(t) + \sum_{j=i+1}^{n} h_j(t) + r_n(t) \cdot \tag{1}$$

where, $x(t)$, $h_i(t)$, $r_n(t)$ are the processed signal, IMF and the residual signal respectively.

The redundant IMFs have not only lower frequency than the lowest frequency of the signal $x(t)$ but also less amplitude. This can be explained that the redundant IMFs are generated due to leaking in the decomposing process of EMD. Therefore, if the energy of $v_i(t)$ is almost equal to that of $x(t)$, $h_i(t)$ is viewed as the redundant IMF. The ratio of the energy of $v_i(t)$ to $x(t)$ will be constructed to eliminate the redundant IMFs . It is defined as

$$L_i = \frac{\int v_i^2(t)dt}{\int x^2(t)dt} = \frac{\int (x(t) - h_i(t))^2 dt}{\int x^2(t)dt} \cdot \tag{2}$$

All L_i make a vector L,

$$L = [L_1, L_2, \cdots, L_n], \tag{3}$$

and D_L is defined as

$$D_L = \{D_i | D_i = |L_{i+1} - L_i|, i = 1,2,\cdots, n-1\}. \tag{4}$$

If D_m is equal to a local maximum of D_L and L_{m+1} is more than T_0 (in general, $0.95 \leqslant T_0 < 1$), $h_i(t)$ $(1 \leqslant i \leqslant m)$ are viewed as the real IMFs.

There are two reasons why the proposed method is logical to eliminate the redundant IMFs. Firstly, it is based on the fact that the redundant IMFs have lower frequency and less amplitude than the lowest frequency of the signal $x(t)$. Secondly, the frequency of $h_k(t)$ is higher than that of $h_n(t)$ if k is lesser than n. In a word, there is a local maximum of D_L at the boundary between the real IMFs and the redundant IMFs.

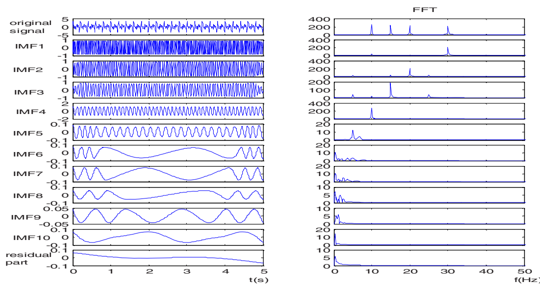

Fig. 1. The results of EMD of $x(t)$ and the FFT of IMFs

4 Experiment and Analysis

The example of the section 2 is discussed in succession. Above all, all L_i of IMFs are calculated by equation (2). The results are described in table 1. The results of D_L are described in table 2.

Table 1. All L_i of IMFs $(1 \leqslant i \leqslant 10)$

IMF*i*	1	2	3	4	5
L_i	0.6495	0.5345	0.6406	0.5539	0.9908
IMF*i*	6	7	8	9	10
L_i	0.9971	0.9987	0.9989	0.9992	0.9989

Table 2. The result of D_L

L_i	1	2	3	4	5
D_i	0.1149	0.1061	0.0867	0.4369	0.0063
L_i	6	7	8	9	--
D_i	0.0016	0.0003	0.0003	0.0003	--

From the table 2, it can be seen that m is equal to four. So, the first four IMFs are viewed as the real IMFs and other are the redundant IMFs. This is consistent with the fact.

The instantaneous frequency estimated by Hilbert-Huang transform without the proposed algorithm is presented in fig. 2 and with the proposed algorithm in Fig. 3. From the two figures, those can be seen that the first deficiency of HHT will cause misinterpretation to the result and the proposed algorithm successfully eliminate the redundant IMFs so that obtain the real instantaneous frequency of the signal $x(t)$.

5 Conclusion

HHT provides a new method for processing the nonlinear and non-stationary signal and has received great attention in various areas, but it is not a perfect tool in practical application due to some deficiencies. In this paper, only one of them, that is, the EMD will generate redundant IMFs at the low-frequency, is discussed. To resolve the problem, an algorithm is proposed which is based on the cause of generating the redundant IMFs and the characteristics of EMD. In the algorithm, a threshold m is calculated which must satisfy some proper conditions, then $h_i(t)$ $(1 \leqslant i \leqslant m)$ are viewed as the real IMFs and other the redundant IMFs to be eliminated finally. The results of experiment show that the proposed algorithm is effective on improving HHT.

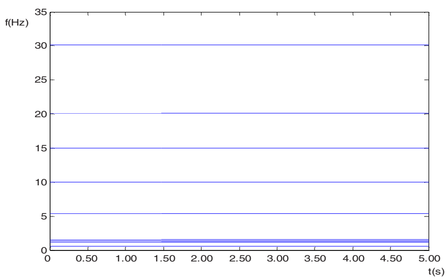

Fig. 1. The instantaneous frequency estimated by HHT without using the proposed algorithm

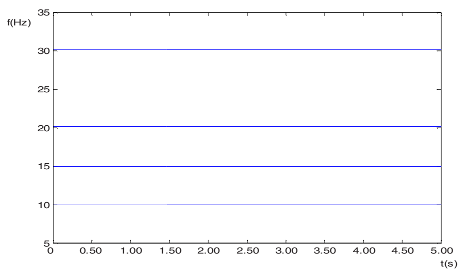

Fig. 2. The instantaneous frequency estimated by HHT with using the proposed algorithm

References

1. N.E. Huang, Z. Shen, S.R. Long, M. Wu, H. Shih, N. Zheng, C. Yen, C.C. Tung, H.H. Liu. The empirical mode decomposition and the Hilbert spectrum for non-linear and non-stationary time series analysis. Proceedings of the Royal Society of London Series A—Mathematical Physical and Engineering Sciences 454 (1998) 903–995.
2. Olivier Adam. The use of the Hilbert-Huang transform to analyze transient signals emitted by sperm whales. Applied Acoustics 67 (2006) 1134–1143
3. Z.K. Peng, Peter W. Tse, F.L. Chu. An improved Hilbert–Huang transform and its application in vibration signal analysis. Journal of Sound and Vibration 286 (2005) 187–205
4. Z.K. Peng, Peter W. Tse, F.L. Chu. A comparison study of improved Hilbert–Huang transform and wavelet transform: Application to fault diagnosis for rolling bearing. Mechanical Systems and Signal Processing 19 (2005) 974–988
5. Liu YB, Wu Q, Ma ZY, Yan KG. An improved Hilbert-Huang transform and its application in faults signal analysis. IEEE ICMA 2006: PROCEEDING OF THE 2006 IEEE INTERNATIONAL CONFERENCE ON MECHATRONICS AND AUTOMATION 1-3(2006) 2426-2431
6. Qi Wu, Yi-bing Liu, Ke-guo Yan. A new approach to improved Hilbert-Huang transform. Sixth World Congress on Intelligent Control and Automation Dalian, China, 21-23 (2006)