A Model for Representing Topological Relations Between Simple Concave Regions

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Abstract. At present, qualitative spatial reasoning has become the hot issues in many research fields. The most popular models of spatial topological relations are Region Connection Calculus (RCC) and 9-intersection model. However, there are few contributions on topological relations of concave regions in which the representative model is Cohn's RCC23. There are some limitations of RCC23 especially in practical applications due to its less expressiveness. In order to construct a more expressive model of topological relations between concave regions, this paper completed the following works: 9-intersection matrix is extended to 16-intersection matrix, and RCC23 is refined to RCC62 based on 16-intersection matrix. More relations can be distinguished in RCC62, which is more expressive than RCC23. In order to further reason about relations in RCC62, the Conceptual Neighborhood Graph (CNG) and the Closest Topological Relation Graph (CTRG) of RCC62 are given.

Keywords: topological relation, simple concave region, 9-intersection model, 16-intersection matrix, RCC23.

1 Introduction

The development of formal models on spatial relations is an essential topic in spatial reasoning, geographic information system (GIS) and computer vision which got much attention from researchers in relative fields during recent years [1], [2], [3]. Particularly, significant progresses have been made in models on topological spatial relations of regions. Topology is perhaps the most basic aspect of space. It is clear that topology must form a fundamental aspect of qualitative spatial reasoning since topology certainly can only make qualitative distinctions [3].

At present, most of the models on topological spatial relations adopt either logical or algebraic methods, among which the most typical models are Region Connection Calculus (RCC) [4] and intersection models [5]. Moreover, there are many improved methods proposed by other researchers. Most of the existing models mainly concentrate on topological relations between convex regions [4], [5], but less works have been made on topological relations between concave

Y. Shi et al. (Eds.): ICCS 2007, Part I, LNCS 4487, pp. 160–167, 2007.

regions, in which one of the typical models is Cohn's RCC23 [6], which twenty-three topological relations between two simple concave regions are defined based on two primitive of connection and convex hull. However, there are limitations of RCC23 especially in spatial query on account of its less expressiveness. Sometimes, the querying results would be inexact or the querying request may be failed. Thus, it is essential to establish more expressive models for topological relations between concave regions.

This paper make an extension of 9-intersection to 16-intersection matrix, and based on 16-intersection matrix, RCC23 [6] is refined to a set of sixty-two relations, thus derived RCC62 which is more expressive than RCC23. In order to further research the reasoning of RCC62, the Conceptual Neighborhood Graph (CNG) [7] and the Closest Topological Relation Graph (CTRG) [8] of RCC62 are given.

2 Background

2.1 RCC8 and RCC23

Randell et al. [4] proposed the RCC theory based on a calculus of individuals developed by Clarke. The basic part of RCC theory assumes a primitive dyadic relation: $C(x, y)$ read as 'x connects with y' which is defined on regions. Using the relation $C(x, y)$, a basic set of dyadic relations are defined: DC, EC, PO, TPP, NTPP, TPPi, NTPPi, EQ. To reason about spatial change, the Conceptual Neighborhood Graph (CNG) [7] is used. Fig. 1(left) shows the basic relations and also the CNG of RCC8 which indicates the continuous transitions among the RCC8 relations.

Cohn et al. [6] defined three predicates (INSIDE, P-INSIDE, and OUTSIDE) based on the primitive of convex hull to test for a region being inside, partly inside and outside another. Each of these relations is asymmetric so they have inverses, denoted by INSIDEi, P-INSIDEi and OUTSIDEi. The definitions naturally give rise to a set of twenty-three JEPD relations, which we call RCC23, as illustrated in Fig. 1 (right).

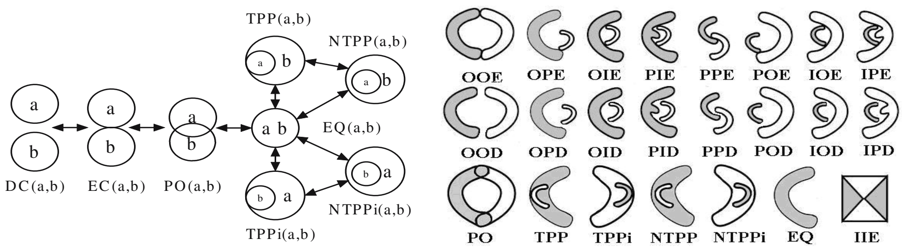

Fig. 1. (left) 2D illustrations of the relations of RCC 8 and their continuous transitions (CNG) (right) The pictorial representation of the relations of RCC23

2.2 Intersection Models

Egenhofer et al. [5] presented the 9-intersection model for binary topological relations between crisp regions by comparing the interiors (A°), boundaries (∂A) and exteriors (A^-) of the two regions. Considering the binary values empty and non-empty for these intersections, one can distinguish 512 binary topological relationships. Eight topological relations can be realized between two regions embedded in $\mathbf{IR}^2$. The 9-intersection matrix is shown in Fig. 2 (left). In order to represent the continuous change of spatial relations, Egenhofer presented the Closest Topological Relation Graph (CTRG) [7] based on the definition of topological distance. Fig. 2 (right) shows the CTRG of eight binary relations.

$$\begin{pmatrix} A^\circ \cap B^\circ & A^\circ \cap \partial B & A^\circ \cap B^- \\ \partial A \cap B^\circ & \partial A \cap \partial B & \partial A \cap B^- \\ A^- \cap B^\circ & A^- \cap \partial B & A^- \cap B^- \end{pmatrix}$$

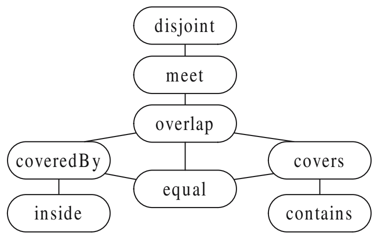

Fig. 2. (left) The 9-intersection matrix (right) The CTRG of eight binary relations

3 RCC62

An extension of 9-intersection matrix to 16-intersection matrix is made, then RCC23 is refined by adding 39 new relations based on 16-intersection matrix, thus sixty-two topological relations between two simple concave regions are derived, which we call RCC62. The CNG and the CTRG of RCC62 are also given.

3.1 16-Intersection Matrix

Some basic definitions used in this paper are given below.

Definition 1. *The convex hull of region x, denoted by* $\texttt{conv}(x)$*, is the smallest convex region of which x is a part.* [4]

Definition 2. *The inside of concave region x, denoted by* $\texttt{inside}(x)$*, is the difference of the convex hull of region x and the region itself.*[4]

$$\texttt{inside}(x) \equiv_{def} \iota y[\forall z[C(z,y) \leftrightarrow \exists w[\texttt{INSIDE}(w,x) \wedge C(z,w)]]] \quad (1)$$

Definition 3. *The outside of concave region x, denoted by* $\texttt{outside}(x)$*, is the complement part of the convex hull of region x.*[4]

$$\texttt{outside}(x) \equiv_{def} \iota y[\forall z[C(z,y) \leftrightarrow \exists w[\texttt{OUTSIDE}(w,x) \wedge C(z,w)]]] \quad (2)$$

Definition 4. *x is a concavity of region y, denoted by* $\texttt{Concavity}(x, y)$*, when predicate* $\texttt{Concavity}(x, y)$ *is true.*[9]

$$\texttt{Concavity}(x, y) \equiv_{def} \texttt{MAX\_P}(x, \texttt{inside}(y)) \tag{3}$$

Definition 5. *Region x is a simple concave region, if it is concave and have only one concavity.*

Definition 6. *The topological relation between two concave regions a and b, is characterized by comparing a's outside (a_0), boundary (a_1), interior (a_2), inside (a_3) with b's outside (b_0), boundary (b_1), interior (b_2), inside (b_3), thus concisely represented as a 4×4 matrix, called the 16-intersection matrix which denoted by*

$$R(a, b) = \begin{pmatrix} a_0 \cap b_0 & a_1 \cap b_0 & a_2 \cap b_0 & a_3 \cap b_0 \\ a_0 \cap b_1 & a_1 \cap b_1 & a_2 \cap b_1 & a_3 \cap b_1 \\ a_0 \cap b_2 & a_1 \cap b_2 & a_2 \cap b_2 & a_3 \cap b_2 \\ a_0 \cap b_3 & a_1 \cap b_3 & a_2 \cap b_3 & a_3 \cap b_3 \end{pmatrix} \tag{4}$$

If the intersection of a_i and b_j is non-empty, then the value of $a_i \cap b_j$ is 1, otherwise the value of $a_i \cap b_j$ is 0. Different combinations in the intersection matrix can represent different topological relations. Note that since a_3 is a virtual area component, no boundary is defined between a_3 and a_0. Thus if a_3 has a non-empty intersection with b_1, then it has also to intersect with b_2. Similarly, if b_3 has a non-empty intersection with a_1, then it has also to intersect with a_2.

As illustrated in Fig. 3 (left), the topological relation between two simple concave regions a and b is represented by their 16-intersection matrix.

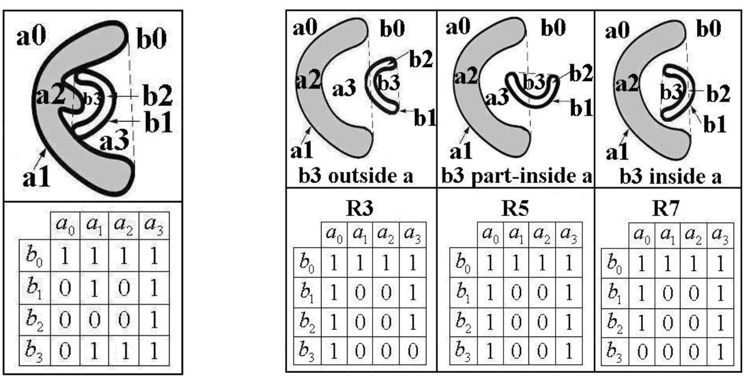

Fig. 3. (left) The topological relation between two simple concave region a, b and their 16-intersection matrix. (right) 16-intersection matrix for distinguishing b_3 (the bay of the island) outsides, partially-insides or insides island a.

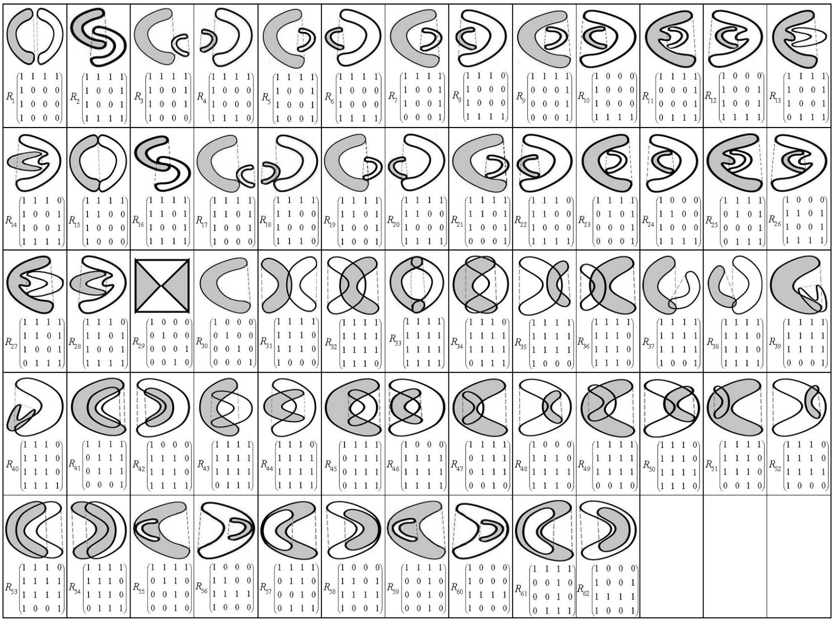

Fig. 4. A pictorial representation of the basic relations of RCC62 and their corresponding 16-intersection matrixes

3.2 Basic Relations of RCC62

The definition of 16-intersection matrix is useful for distinguish topological spatial relations of simple concave regions such as the bay of a small island b (i.e.b_3) is outside, partially-inside, or inside a large island a, as shown in Fig. 3 (right). These distinctions cannot be made using RCC23 since they represent the same relation OPD.

In RCC23, the concave regions are considered as a whole part, while in RCC62 it is decomposed into four sub-parts (i.e. a_0, a_1, a_2, a_3), thus based on 16-intersection matrix more topological relations can be derived between two concave regions.

Here relation OPD in RCC23 is refined to three new relations (i.e. R_3, R_5, R_7) by considering that b_3 intersects only with a_0, both with a_0, a_3, and only with a_3. In the same way, RCC23 is refined based on 16-intersection matrix, and totally sixty-two topological relations between two simple concave regions are defined, which we call RCC62. The pictorial representation of the base relations of RCC62 and their corresponding 16-intersection matrixes are shown in Fig. 4.

3.3 Relationship Between RCC23 and RCC62

Since RCC62 is derived from RCC23, there is a natural connection between them. The relationship between RCC23 and RCC62 is given in Fig. 5. Basic

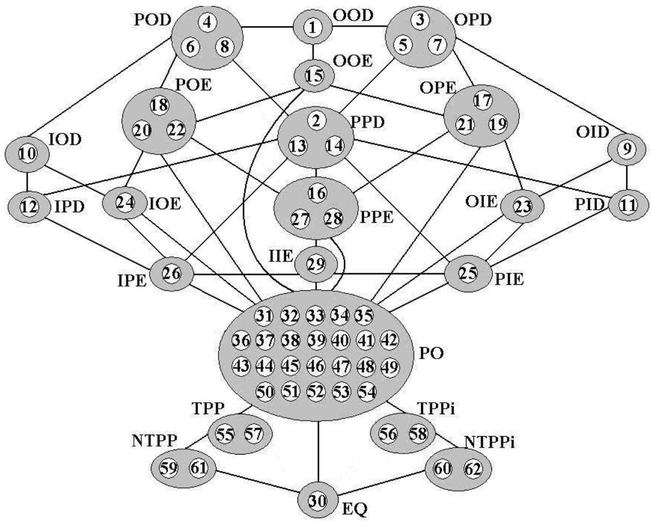

Fig. 5. Basic relations of RCC23 (denoted by the solid ovals labeled with letters), and their corresponding refinements to RCC62 (denoted by the hollow circles labeled with numbers), e.g. relation POD in RCC23 are refined to three new relations (i.e. R_4, R_6, R_8) in RCC62

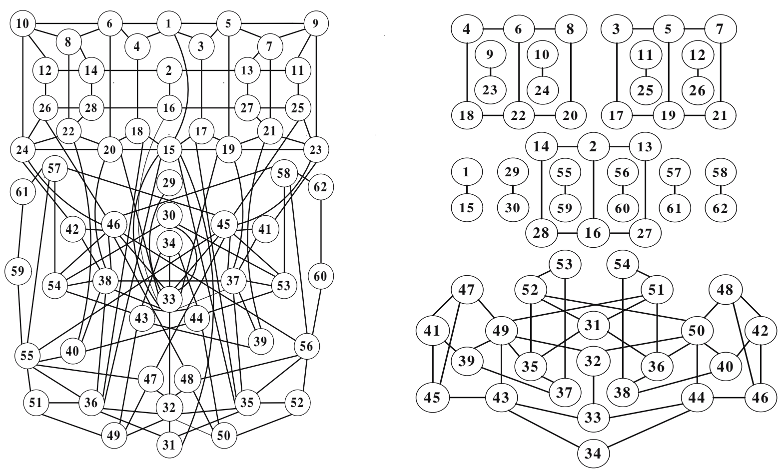

Fig. 6. (left) The CNG of RCC62, in which a node for each relation and an arc between two relations represents a smooth transition that can transform a relation to the other and vice versa. (right) The CTRG of RCC62, in which a node for each relation and an arc for each pair of matrices at minimum topological distance that measured in terms of the number of different values in the corresponding matrices.

relations of RCC23 are represented by the solid ovals labeled with letters, while relations in RCC62 are described by the hollow circles labeled with numbers. Take an example, relation POD in RCC23 are refined to three new relations (i.e. R_4, R_6, R_8) in RCC62. The arc between two basic relations of RCC23 denotes a smooth transition between them. As shown in Fig. 5, there are more relations can be distinguished in RCC62, thus it is more expressive than RCC23.

3.4 CNG and CTRG of RCC62

It is necessary to represent the change of spatial relations in spatial reasoning. In qualitative reasoning we assume that change is continuous. The primary methods to represent the continuous change of spatial relations are the Conceptual Neighborhood Graph (CNG) [7] and the Closest Topological Relation Graph (CTRG) [8]. We give both the CNG and CTRG of RCC62, as shown in Fig. 6 (left) and Fig. 6 (right) respectively.

4 Conclusions

Topological relation is the most basic relation of spatial objects which is one of the elementary aspects of spatial reasoning. The most popular models of topological relation are RCC and intersection models. However, few contributions are available for topological relations between concave regions, in which one of the typical models is Cohn's RCC23. Nevertheless, RCC23 has some limitations in the practical fields especially for spatial query. Therefore it is essential to establish a more expressive model of topological relations between concave regions.

In order to represent the topological relations between simple concave regions in 2D space, an extension of 9-intersection matrix is made to 16-intersection matrix, and RCC23 is refined by adding 39 relations based on 16-intersection method, thus derived RCC62, in which more relations can be distinguished, and is more expressive than RCC23. The CNG and the CTRG of RCC62 which are bases for the further reasoning of RCC62 are also given.

Acknowledgments

This research was supported by NSFC Major Research Program No. 60496321, National Natural Science Foundation of China under Grant No. 60573073, the National High-Tech Research and Development Plan of China under Grant No. 2006AA10Z245 and 2006AA10A309 the Major Program of Science and Technology Development Plan of Jilin Province under Grant No. 20020303, the Science and Technology Development Plan of Jilin Province under Grant No. 20030523, European Commission under grant No. TH/Asia Link/010 (111084).

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