Omnidirectional Vision: Unified Model Using Conformal Geometry

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Abstract. It has been proven that a catadioptric projection can be modeled by an equivalent spherical projection. In this paper we present an extension and improvement of those ideas using the conformal geometric algebra, a modern framework for the projective space of hyperspheres. Using this mathematical system, the analysis of diverse catadioptric mirrors becomes transparent and computationally simpler. As a result, the algebraic burden is reduced, allowing the user to work in a much more effective framework for the development of algorithms for omnidirectional vision. This paper includes complementary experimental analysis related to omnidirectional vision guided robot navigation.

1 Introduction

Living beings inhabit complex environments. In order to survive in these environments, they should be able to perceive the surrounding objects. One of the most important senses for object perception is vision. This sense is characterized by the ability to focus in a particular object with high precision, but it is also capable of simultaneously observing most of the changing surrounding medium.

In the case of robotic navigation it would be convenient if the robot could have a wide field of vision; but the traditional cameras are limited since they have a narrow field of view. This is a problem that has to be overcome to ease robotic navigation.

An effective way to increase the visual field is the use of a catadioptric sensor which consists of a conventional camera and a convex mirror. In order to be able to model the catadioptric sensor geometrically, it must satisfy the restriction that all the measurements of light intensity pass through only one point in the space (fixed view-point). The complete class of mirrors that satisfy this restriction where analyzed by Baker and Nayar [1].

To model the catadioptric sensor we can use an equivalent spherical projection defined by Geyer and Daniilidis [3]. In this paper, we present a new proposal of the spherical projection using conformal geometric algebra. The advantage of this algebra is that this system has the sphere as the basis geometric object and all the other objects are defined in terms of it (e.g., the intersections between entities can be computed using the *meet* operator). Hence we obtain a more transparent and compact representation due to the high-level symbolic

T. Pajdla and J. Matas (Eds.): ECCV 2004, LNCS 3021, pp. 536–548, 2004.

language of geometric algebra. In this new representation the user can compute and derive conclusion much easier. As a result the development of algorithms for omnidirectional vision becomes simpler and effective.

2 Unified Model

Recently, Geyer and Daniilidis [3] presented a unified theory for all the catadioptric systems with an effective viewpoint. They show nicely that these systems (parabolic, hyperbolic, elliptic) can be modeled with a projection through the sphere. In their paper they define a new notation where A $\vee$ B denotes the line joining the points A and B, and $l \wedge m$ denotes the intersection of the lines l and m. Also, this operator is used to denote the intersection of the line l and the conic c in the form $l \wedge m$ (note that this can result in a point pair). When the intersection is a point pair they distribute over the $\vee, \wedge$; for example $A \vee (l \wedge c)$ is the pair $(A \vee P_1, A \vee P_2)$, where $P_{1,2}$ are points obtained from the intersection of l and c.

Definition of a quadratic projection. Let c be a conic, A and B two arbitrary points, ℓ any line not containing B and P a point in the space. The intersection of the line and the conic is a point pair R_1 and R_2 (possibly imaginary). The quadratic projection is defined as

$$P^{q(c,A,B,\ell)} \rightarrow (((P \vee A) \wedge c) \vee B) \wedge \ell \; . \tag{1}$$

Definition of a catadioptric projection. This projection is defined in terms of a quadratic projection where the points A and B are the focus F_1, F_2 of the conic c respectively, and ℓ is a line perpendicular to $F_1 \vee F_2$. The catadioptric projection is defined as

$$P^{q(c,F_1,F_2,\ell)} \rightarrow (((P \vee F_1) \wedge c) \vee F_2) \wedge \ell \; . \tag{2}$$

The important question now is: given a catadioptric projection with parameters (c, F_1, F_2, ℓ), which are the parameters (c', A, B, ℓ') that result in an equivalent quadratic projection? This is not answered in general, but in a more restricted form we can ask ourselves: are there any parameters (c', A, B, ℓ'), where c' is a circle with a unit radius and center in A, B is some point and $\ell \parallel \ell'$, that produce an equivalent projection? To obtain equivalent projections they must have the same effective viewpoint and therefore $A = F_1$. Thus it is required to find ℓ' and B such that

$$q = (c, F_1, F_2, \ell) = q(c', F_1, B, \ell') \; . \tag{3}$$

Derivation of $q(c, F_1, F_2, \ell)$. The quadratic form of c in terms of its eccentricity ϵ and a scaling parameter $\lambda > 0$ is given by the equation

$$Q_{\epsilon,\lambda} = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 4 - 4\epsilon^2 & -4\epsilon\lambda \\ 0 & -4\epsilon\lambda & -4\lambda \end{pmatrix} \; . \tag{4}$$

Hence $F_1 = (0,0,1)$ and $F_2 = (0,-2\epsilon,\lambda^{-1}(\epsilon^2-1))$ are the foci of c with *latus rectum* 2λ, assuming that the intersection of the line ℓ and the y-axis is μ, so the line has the coordinates $[0,1,-\mu]$. The first part of the catadioptric projection of a point P is $(P \vee F_1) \wedge c$, and can be expressed as

$$R_i = F_1 + \theta_i P, \tag{5}$$

where θ_i are the roots of the quadratic equation

$$0 = R_i Q_{\epsilon,\lambda} R_i^T = F_1 Q_{\epsilon,\lambda} F_1^T + 2\theta_i F_1 Q_{\epsilon,\lambda} P^T + \theta_i^2 P Q_{\epsilon,\lambda} P^T \ , \tag{6}$$

$$\theta_i = \frac{\lambda}{(-1)^i \sqrt{x^2+y^2} - \epsilon y - \lambda w}, \ . \tag{7}$$

Later, the projection of the points R_i to the line $\ell = [0,1,-\mu]$ from the point F_2 is

$$T_{\epsilon,\lambda,\mu} = \begin{pmatrix} -2\epsilon\lambda + \mu(1-\epsilon^2) & 0 \\ 0 & 1-\epsilon^2 \\ 0 & -2\epsilon\lambda \end{pmatrix} . \tag{8}$$

Finally the projected points Q_i are

$$Q_i = R_i T_{\epsilon,\lambda,\mu} = \left(x(2\epsilon\lambda - \mu(1-\epsilon^2)), -(1+\epsilon^2)y - 2(-1)^i \epsilon \sqrt{x^2+y^2} \right) . \tag{9}$$

Now, the projection $q(c', A, B, \ell')$ is the spherical projection — or in the transversal section, the projection to the circle. Let c' be a unit circle centered in F. The points R'_i are the intersections of the line $F_1 \vee P$ with this circle, and can be found by

$$R'_i = (-1)^i \sqrt{x^2+y^2} \ , \tag{10}$$

the projection of the points R'_i to the line ℓ' is

$$U_{l,m} = \begin{pmatrix} l-m & 0 \\ 0 & -1 \\ 0 & l \end{pmatrix} , \tag{11}$$

and finally the projected points are

$$Q'_i = R'_i U_{l,m} = ((l-m)x, -y + l(-1)^i \sqrt{x^2+y^2}) \ . \tag{12}$$

Once the catadioptric and spherical projections have been calculated, the question it arises is for which B and ℓ' are $q(c, F_1, F_2, \ell') = q(c', F_1, F_2, \ell')$? If l and m can be chosen freely, independent of x, y, w then (9) and (12) are the same up to scale factor. The projections are equivalent if we choose

$$l = \frac{2\epsilon}{1+\epsilon^2} \ , \tag{13}$$

$$m = \frac{\mu - \epsilon(\epsilon\mu + 2\lambda - 2)}{1+\epsilon^2} \ . \tag{14}$$

Interesting enough with (13) and (14) we can model any catadioptric projection through the spherical projection, it is just a matter of calculating the parameters l and m according to the eccentricity and scaling parameters of the mirror. Next section outlines a brief introduction into the mathematical system which will help us to handle effectively omnidirectional vision in a new framework.

3 Geometric Algebra

The mathematical model used in this work is the geometric algebra (GA). This algebra is based on the Clifford and Grassmann algebras and the form used is the one developed by David Hestenes since late sixties [5].

In the n-dimensional geometric algebra we have the standard interior product which takes two vectors and produces a scalar, furthermore we have the wedge (exterior) product which takes two vectors and produces a new quantity that we call bivector or oriented area. Similarly, the wedge product of three vectors produces a trivector or oriented volume. Thus, the algebra has basic elements that are geometric oriented objects of different grade. The object with highest grade is called *pseudo-scalar* with the unit pseudo-scalar denoted by I (e.g. in 3D the unit pseudo-scalar is $e_1 \wedge e_2 \wedge e_3$). The outer product of r vectors is called an *r-blade* of grade r. A multivector is a quantity created by a linear combination of r-blades. Also, we have the geometric product which is defined for any multivector. The geometric product of two vectors is defined by the inner and outer product as

$$ab = a \cdot b + a \wedge b \tag{15}$$

Where the interior product $(\cdot)$ and wedge product $(\wedge)$ of two vectors $a, b \in \langle G_{p,q} \rangle_1 \equiv R^{p+q}$ can be expressed as

$$a \cdot b = \frac{1}{2}(ab + ba) \quad \text{and} \tag{16}$$

$$a \wedge b = \frac{1}{2}(ab - ba) \; . \tag{17}$$

As an extension, the interior product of an r-blade $a_1 \wedge ... \wedge a_r$ with an s-blade $b_1 \wedge ... \wedge b_s$ can be expressed as

$$(a_1 \wedge ... \wedge a_r) \cdot (b_1 \wedge ... \wedge b_s) = \\ \begin{cases} ((a_1 \wedge ... \wedge a_r) \cdot b_1) \cdot (b_2 \wedge ... \wedge b_s) & (\text{if } r \geq s) \; , \\ (a_1 \wedge ... \wedge a_{r-1}) \cdot (a_r \cdot (b_1 \wedge ... \wedge b_s)) & (\text{if } r < s) \end{cases} \tag{18}$$

with

$$(a_1 \wedge ... \wedge a_r) \cdot b_1 = \textstyle\sum_{i=1}^{r} (-1)^{r-i} a_1 \wedge ... \wedge a_{i-1} \wedge (a_i \cdot b_1) \wedge a_{i+1} \wedge ... \wedge a_r \; ,$$

$$a_r \cdot (b_r \wedge ... \wedge b_s) = \textstyle\sum_{i=1}^{s} (-1)^{i-1} b_1 \wedge ... \wedge b_{i-1} \wedge (a_r \cdot b_i) \wedge b_{i+1} \wedge ... \wedge b_s \; . \tag{19}$$

The dual X^* of an r-blade X is

$$X^* = XI^{-1} \; . \tag{20}$$

The *shuffle* product $A \vee B$ satisfies the "DeMorgan rule"

$$(A \vee B)^* = A^* \wedge B^* \ . \tag{21}$$

3.1 Conformal Geometric Algebra

For a long time it has been known that using a projective description of the Euclidean 3D space in 4D has many advantages, particularly when the intersection of lines and planes are needed. Recently, these ideas where re-taken mainly by Hestenes [5], where he represents the Euclidean 3D space by a conformal space of 5D. In this conformal space the projective geometry is included, but in addition it can be extended to circles and spheres.

The real vector space $R^{n,1}$ or $R^{1,n}$ is called the Minkowski space, after the introduction of Minkowski's space-time model in $R^{3,1}$. The Minkowski plane $R^{1,1}$ has the orthonormal basis $\{e_+, e_-\}$ defined by the properties

$$e_+^2 = 1, \ \ e_-^2 = -1, \ \ e_+ \cdot e_- = 0 \ . \tag{22}$$

Furthermore, the basis null vectors are

$$e_0 = \frac{1}{2}(e_- - e_+), \quad \text{and} \quad e = e_- + e_+, \tag{23}$$

with properties

$$e_0^2 = e^2 = 0, \ \ e \cdot e_0 = -1 \ . \tag{24}$$

We will be working in the $R^{n+1,1}$ space, which can be decomposed in

$$R^{n+1,1} = R^n \oplus R^{1,1} \ . \tag{25}$$

This decomposition is known as the conformal split. Therefore any vector $a \in R^{n+1,1}$ admits the split

$$a = \mathbf{a} + \alpha e_0 + \beta e \ . \tag{26}$$

The conformal vector space derived from R^3 is denoted as $R^{4,1}$, its bases are $\{e_1, e_2, e_3, e_+, e_-\}$. The unit conformal pseudo-scalar is denoted as

$$I_c = e_{+-123} \ . \tag{27}$$

In the conformal space the basis entities are spheres

$$s = \mathbf{p} + \frac{1}{2}(\mathbf{p}^2 - \rho^2)e + e_0 \ . \tag{28}$$

A point x is nothing more than a sphere with radius $\rho = 0$, yielding

$$x = \mathbf{x} + \frac{1}{2}\mathbf{x}^2 e + e_0 \ . \tag{29}$$

The dual form s^* of a sphere s has the advantage that it can be calculated with four point in the sphere

$$s^* = a \wedge b \wedge c \wedge d \ . \tag{30}$$

The definition of the entities, its dual representation and its grade is shown in the Table 3.1.

Table 1. Entities in conformal geometric algebra

Entity	Representation	Grade	Dual Representation	Grade
Sphere	$s = \mathbf{p} + \frac{1}{2}(\mathbf{p}^2 - \rho^2)e + e_0$	1	$s^* = a \wedge b \wedge c \wedge d$	4
Point	$x = \mathbf{x} + \frac{1}{2}\mathbf{x}^2 e + e_0$	1	$x^* = (-Ex - \frac{1}{2}x^2 e + e_0)I_E$	4
Plane	$P = nI_E - de$ $n = (a - b) \wedge (a - c)$ $d = (a \wedge b \wedge c)I_E$	1	$P^* = e \wedge a \wedge b \wedge c$	4
Line	$L = rI_E - emI_E$ $r = (a - b)$ $d = (a \wedge b)$	2	$L^* = e \wedge a \wedge b$	3
Circle	$z = s_1 \wedge s_2$	2	$z^* = a \wedge b \wedge c$	3
Point Pair	$PP = s_1 \wedge s_2 \wedge s_3$	3	$PP^* = a \wedge b$, $X^* = e \wedge x$	2

3.2 Rigid Motion in the Conformal Geometric Algebra

In $G_{4,1}$ the rotations are represented by rotors $R = \exp(\frac{\theta}{2}l)$. Where the bivector l is the screw of rotation axis and the amount of rotation is given by the angle θ. An entity can be rotated by multiplying from the left with the rotor R and from the right with its reverse $\widetilde{R}$ (e.g., $x' = Rx\widetilde{R}$).

One entity can be translated with respect to a translation vector t using the translator $T = 1 + \frac{et}{2} = \exp(\frac{et}{2})$. The translation takes place by multiplying from the left with the translator T and from the right with $\widetilde{T}$ (e.g., $Tx\widetilde{T}$).

To express the rigid motion of an object we can apply a rotor and a translator simultaneously, this composition is equals to a motor $M = TR$. We apply the motor similarly as the rotor and the translator (e.g. $x = Mx\widetilde{M}$). Surprisingly this formulation of the rigid motion can be applied not only to lines and points but also to all the entities of Table 3.1.

4 Omnidirectional Vision Using Conformal Geometric Algebra

The model defined by Geyer and Daniilidis [3] is used to find an equivalent spherical projection of a catadioptric projection. This model is very useful to simplify the projections, but the representation is not ideal because it is defined in a projective geometry context where the basis objects are points and lines and not the spheres. The computations are still complicated and difficult to follow.

Our proposal is based in the conformal geometric algebra where the basic element is the sphere. This is, all the entities (point, point pair, circle, plane) are defined in terms of the sphere (e.g., a point can be defined as sphere of zero radius). This framework also has the advantage that the intersection operation between entities is mathematically well defined (e.g. the intersection of a sphere with a line can be defined as $L \wedge S$). In short, the unified model is more natural and concise in the context of the conformal geometric algebra (of spheres) than the projective algebra (of points and lines). However the work of Geyer and Daniilidis [3] was extremely useful to accomplish the contribution of this paper.

4.1 Conformal Unified Model

We assume that the optical axis of the mirror is parallel to the e_2 axis, then let $\mathbf{f}$ be a point in the Euclidean space (which represents the focus of the mirror which lies in such optical axis) defined by

$$\mathbf{f} = \alpha_1 e_1 + \alpha_2 e_2 + \alpha_3 e_3 \tag{31}$$

with conformal representation given by

$$F = \mathbf{f} + \frac{1}{2}\mathbf{f}^2 e + e_0 \; . \tag{32}$$

Using the point F as the center, we can define a unit sphere S (see Fig. 1) as follows

$$S = F - \frac{1}{2}e \; . \tag{33}$$

Now let N be the point of projection (that also lies on the optical axis) at a distance l of the point F, this point can be found using a translator

$$T = 1 + \frac{l e_2 e}{2} \tag{34}$$

and then

$$N = TF\widetilde{T} \; . \tag{35}$$

Finally, the image plane is perpendicular to the optical axis at a distance $-m$ from the point F and its equation is

$$\Pi = e_2 + (\mathbf{f} \cdot e_2 - m)e \; . \tag{36}$$

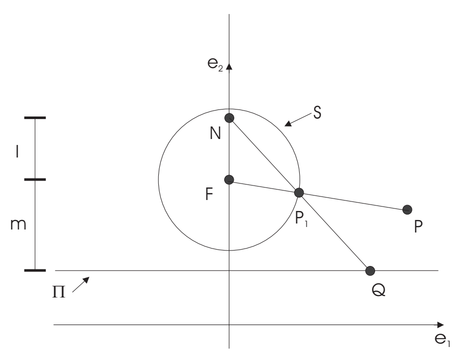


Fig. 1. Conformal Unify Model.

4.2 Point Projection

Let $\mathbf{p}$ be a point in the Euclidean space, the corresponding homogeneous point in the conformal space is

$$P = \mathbf{p} + \frac{1}{2}\mathbf{p}^2 e + e_0 \ . \tag{37}$$

Now, for the projection of the point P we trace a line joining the points F and P, using the definition of the line in dual form we get

$$L_1^* = F \wedge P \wedge e \ . \tag{38}$$

Then, we calculate the intersections of the line L_1 and the sphere S which result in the point pair

$$PP^* = (L_1 \wedge S)^* \ . \tag{39}$$

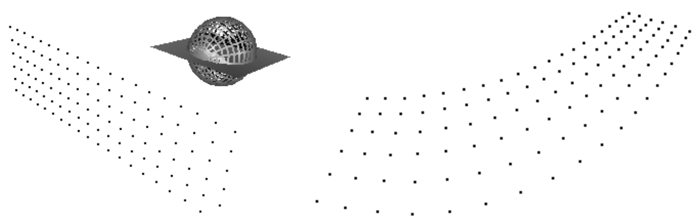
From the point pair we choose the point P_1 which is the closest point to P, and then we find the line passing through the points P_1 and N

$$L_2^* = P_1 \wedge N \wedge e \tag{40}$$

Finally we find the intersection of the line L_2 with the plane Π

$$Q = (L_2 \wedge \Pi)^* \ . \tag{41}$$

The point Q is the projection in the image plane of the point P of the space. Notice that we can project any point in the space into any type of mirror (changing l and m) using the previous procedure (see Fig. 2). The reader can now see how simple and elegant is the treatment of the unified model in the conformal geometric algebra.



a) b)

Fig. 2. a) Unified Model and points in the space. b) Projection in the image plane.

4.3 Inverse Point Projection

We have already seen how to project a point in the space to the image plane through the sphere. But now we want to back-project a point in the image plane into 3D space. First, let Q be a point in such image plane, the equation of the line passing through the points Q and N is

$$L_2^* = Q \wedge N \wedge e \;, \tag{42}$$

and the intersection of the line L_2 and the sphere S is

$$PP^* = (L_2 \wedge S)^* \;. \tag{43}$$

From the point pair we choose the point P_1 which is the closest point to Q, and then we find the equation of the line from the point P_1 and the focus F

$$L_1^* = P_1 \wedge F \wedge e \;. \tag{44}$$

The point P lies on the line L_1^*, but it can not be calculated exactly because a coordinate has been lost when the point was projected to the image plane (a single view does not allow to know the projective depth). However, we can project this point to some plane and say that it is equivalent to the original point up to a scale factor (see Fig. 3).

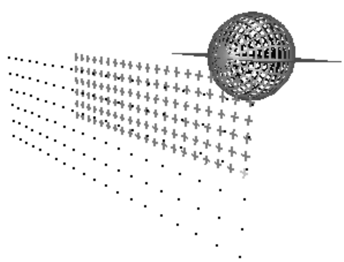


Fig. 3. Inverse point projection (from the image to the space). The crosses are the projected points and the dots are the original points.

4.4 Line Projection

Suppose that L is a line in the space and we want to find its projection in the image plane (see Fig. 4.a). First we find the plane were L and F lies, its equation is

$$P^* = L^* \wedge F \;. \tag{45}$$

The intersection of the plane and the sphere is the great circle defined as

$$C^* = (P \wedge S) \;. \tag{46}$$

The line that passes through the center of the circle and is perpendicular to the plane P is

$$U^* = (C \wedge e) \;. \tag{47}$$

Using U as an axis we make a rotor

$$R = exp(\frac{\theta}{2}U) . \tag{48}$$

We find a point pair PP^* that lies on the circle with

$$PP^* = (C \wedge e_2)^* . \tag{49}$$

We choose any point from the point pair, say P_1, and using the rotor R we can find the points in the circle

$$P_1' = RP_1\widetilde{R} , \tag{50}$$

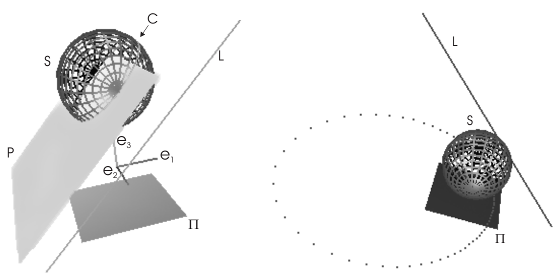
for each point P_1' we find the line that passes through the points P_1 and N defined as

$$L_2^* = P_1' \wedge N \wedge e . \tag{51}$$

Finally for each line L_2 we find the intersection with the plane Π

$$P_2 = (L_2 \wedge \Pi)^* , \tag{52}$$

which is the projection of the line in the space to the image plane (see Fig. 4.b).



a) b)

Fig. 4. a) Line projection to the sphere. b) Line projection to the image plane (note that in this case results an ellipse).

5 Experimental Analysis

In this experiment a robot was placed in the corridor and the goal was that the robot passes trough the corridor using purely the information extracted from the omnidirectional image of a calibrated parabolic system (see Fig. 5). As we know, parabolic mirror projects lines in the space into circles in the image and due to the conformal projection angles are preserved. In this experiment we take advantage of these properties in order to control the robot by means of circles in the omnidirectional image. With the axes of the circles we can calculate the

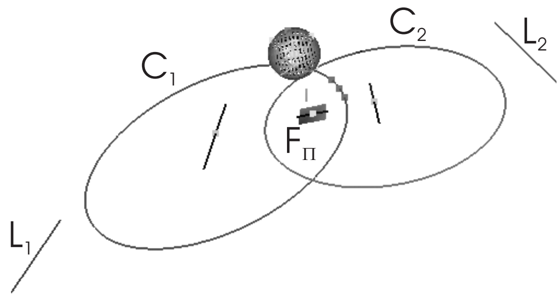


Fig. 5. Robot navigation control using circles in the image plane (dark lines represent the axes of the circles).

robot heading. If we want the robot in the center of the corridor the radius of the circles must be the same, and then the distances to the corridor lines will also be the same. Note that Π is the image plane (36) and F_Π is the focus F of the mirror projected in the plane Π.

The next computations are illustrated in figure 6. The first circle C_1^* (dual representation according to Table 3.1) is defined with the wedge of three points

$$C_1^* = X_1 \wedge X_2 \wedge X_3 \ , \tag{53}$$

similarly for the second circle C_2^*

$$C_2^* = X_4 \wedge X_5 \wedge X_6 \ . \tag{54}$$

The center of the each circle (i=1,2) is calculated by

$$N_i^* = (C_i \wedge e) \cdot \Pi \ . \tag{55}$$

The first axis of each circle is defined as

$$A_{1,i}^* = N_i \wedge F_\Pi \wedge e \ , \tag{56}$$

an the second axis is

$$A_{2,i}^* = \{[((A_{1,i} \wedge e) \cdot e_0) \cdot I_e] \cdot (e_{12})\} \wedge N_i \wedge e \ . \tag{57}$$

The angle in the image is

$$\theta = \arccos(A_{1,1} \cdot e_{3+-}) \tag{58}$$

The sphere with center and radius of the circle is

$$S_i = C_i / \Pi \ . \tag{59}$$

and the radius of the circle is

$$r_i = S_i \cdot S_i \ . \tag{60}$$

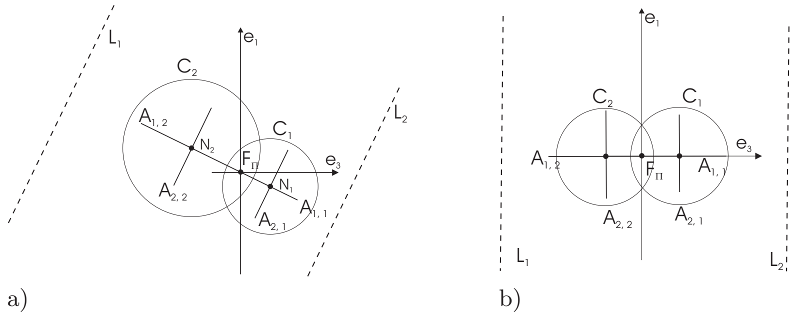


Fig. 6. a) In this image the robot is not parallel to the corridor nor centered. b) In this image the robot is parallel to the corridor and centered.

The control strategy for the navigation is based (as we said previously) on the angles of the circles and its radius. With the angles we correct the parallelism of the robot with the corridor. Furthermore, we place the robot at the center of the corridor using the radius of the circles. The position error is defined by

$$\alpha = (r_1 - r_2)\pi/2 \; . \tag{61}$$

The heading error angle is calculated as

$$\beta = \theta - \pi/2 \; . \tag{62}$$

The robot angular velocity is calculated with the combination of the robot heading error, the position error and the proportional gain κ

$$\omega = \kappa(\beta + \alpha) \; . \tag{63}$$

6 Conclusions

The major contribution of this paper is the refinement and improvement of the use of the unified model for omnidirectional vision. To achieve this goal the authors used the conformal geometric algebra, a modern framework for the projective space of hyper-spheres. This framework is equipped with homogeneous representations of points, lines, planes and spheres, operations of incidence algebra and conformal transformations expressed effectively as versors. The authors show how the analysis of diverse catadioptric mirrors becomes transparent and computationally simpler. As a result, the algebraic burden is reduced for the users who can now develop more efficient algorithms for omnidirectional vision. The paper includes complementary experimental analysis of omnidirectional vision guided robot navigation.

Acknowledgment. We are thankful to CONACYT and CINVESTAV for supporting this work.

References

1. Baker, S., Nayar, S.: A theory of single-viewpoint catadioptric image formation. International Journal of Computer Vision (1999) 35(2):1-22.
2. Gaspar, J., Santos-Victor, J.: Visual path following with a catadioptric panoramic camera. In Proceedings of the 7th International Symposium on Intelligent Robotic Systems (SIRS'99), Coimbra, Portugal (1999) 139-147.
3. Geyer, C., Daniilidis, K.: Catadioptric projective geometry. International Journal of Computer Vision (2001) 223-243.
4. Hartley, R., Zisserman, A.: Multiple View Geometry in Computer Vision. Cambridge University Press, UK (2001).
5. Hestenes, D., Li, H., Rockwood, A.: New algebraic tools for classical geometry. In Sommer, G.(ed.): Geometric Computing with Clifford Algebras. Springer-Verlag, Berlin Heidelberg (2001) 3-23.
6. Winters, N., Santos-Victor, J.: Omni-directional visual navigation. In Proceedings of the 7th International Symposium on Intelligent Robotic Systems (SIRS'99), Coimbra, Portugal (1999) 109-118.