Chapter 1
Felix Klein—Mathematician, Academic Organizer, Educational Reformer

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Abstract Having been a full professor at the University of Erlangen, the Technical University in Munich, and the University of Leipzig, Klein joined the University of Göttingen in 1886. He had gained international recognition with his significant achievements in the fields of geometry, algebra, and the theory of functions. On this basis, he was able to create a center for mathematical and scientific research in Göttingen. This brief biographical note will demonstrate that Felix Klein was far ahead of his time in supporting all avenues of mathematics, its applications, and instruction. It will be showed that the establishment of new lectures, professorships, institutes, and curricula went hand in hand with the creation of new examination requirements for prospective secondary school teachers. Felix Klein's reform of mathematical instruction included all educational institutions from kindergarten onward. He became the first president of the International Commission on Mathematical Instruction in 1908 at the Fourth International Congress of Mathematicians in Rome.

Keywords Felix Klein · Biographical note

Max Born (1882–1970), who received the Nobel Prize in Physics for his contributions to quantum mechanics, once reminisced as follows about Felix Klein (1849–1925) in Göttingen: "Klein commanded not only mathematics as a whole but also all of the natural sciences. Through his powerful personality, which was complemented by his handsome appearance, he became a leading figure in the faculty and at the entire university. […] Over the years, Klein became more and more of a Zeus, enthroned above the other Olympians. He was known among us as 'the Great Felix', and he controlled our destinies" (Born and Born 1969, p. 16).

How did Klein develop into this Zeus-like figure? By the time Max Born was completing his studies in Göttingen during the first decade of the twentieth century, Klein had already reaped the fruits of his mathematical accomplishments and achieved an international reputation. In 1904, while attending the Third International Congress of Mathematicians in Heidelberg, he expressed what might be called his guiding

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words: “In order for science to flourish, it is necessary for all of its components to be developed freely” (Klein 1905, p. 396). With this motto in mind, he aspired to promote all aspects of mathematics equally, including its practical applications and instruction. He was also an admirer and supporter of newly formulated approaches to mathematics and the natural sciences, including actuarial science, aerodynamics, the theory of relativity, modern algebra, and the didactics of mathematics.

Of course, Klein’s wide-reaching program expanded gradually into its mature form. Yet even as a young scholar, he was characterized by the breadth of his interests, the tendency to systematize and unify things, his desire to create an overview of the whole, and his concern for pedagogy. The present contribution will concentrate on three aspects: the centers of activity that defined Klein’s life; the characteristic features of his work; and the way in which he integrated educational reform with his broader ideas about reorganization in order to transform the University of Göttingen into an internationally renowned center for mathematical and natural-scientific research.

1.1 Felix Klein’s Upbringing, Education, and Academic Career

Felix Klein was born on April 25, 1849 in Düsseldorf, which was then the seat of government for the Rhine Province of the Prussian kingdom. He was the second of four children born to Peter Caspar Klein (1809–1889), a senior civil servant and chief treasurer of the Rhine Province, and his wife Sophie Elise Klein (née Kayser; 1819–1890), who came from a family of fabric manufacturers.

After being tutored at home by his mother, he spent two and half years at a private elementary school before transferring, in the fall of 1857, to the Humanistisches Gymnasium in Düsseldorf, which continues to exist today. In August of 1865, just sixteen years old, he completed his *Abitur*, for which he was examined in nine subjects: German, mathematics, Latin, Greek, Hebrew, French, Protestant theology, natural history, as well as the combined subject of history and geography. He decided to pursue further studies in mathematics and the natural sciences, a fact that is already noted on his *Abitur* diploma. His interest in the natural sciences was aroused less by the curriculum of his humanities-based *Gymnasium* than it was by his earlier experiences in elementary school and by his extra-curricular activities.

On October 5, 1865, Klein applied to the nearby University of Bonn, which had been founded through the sponsorship of the Prussian king in 1818. There were not many students enrolled at the time, so it did not take long for Julius Plücker (1801–1868), a professor of physics and mathematics, to recognize Klein’s talent. Plücker chose Klein, who was just in his second semester, to be his assistant for his course on experimental physics. However, because Plücker’s own research at the time was devoted to his concept of “line geometry” (*Liniengeometrie*), he involved his assistant in this work as well. By the time Plücker died—on May 22, 1868—Klein

had thus been educated on two fronts. Regarding his achievements in physics, it is documented that he received an award for his work on theoretical physics during the celebration of the university's fiftieth anniversary (see Tobies 1999). Firm evidence for his mathematical abilities is the faith that Plücker's family placed in him as a young man; they entrusted him with the task of preparing the second volume of Plücker's *Liniengeometrie* (Klein 1869). By way of this work, Klein independently developed a topic for his doctoral dissertation, about which he sought advice from Alfred Clebsch (1833–1872) and Rudolf Lipschitz (1832–1903). Under Lipschitz's supervision, Klein defended his dissertation in Bonn on December 12, 1868, and he received the highest grade for his work. In January of 1869, he moved to Göttingen to continue his studies with Clebsch and participate in the latter's school of algebraic geometry. During the winter semester of 1869/70, Klein studied in Berlin, after which he travelled with the Norwegian mathematician Sophus Lie (1842–1899) to Paris, where they published two short papers together in the *Comptes Rendus hebdomadaires des séances de l'Académie de sciences de Paris* and prepared additional publications. In July of 1870, his time in Paris was brought to an end by the outbreak of the Franco-Prussian War.

Declared unsuitable for military service, Klein applied to serve as a paramedic. After a few weeks on the front, he contracted typhus and returned to his parents' home in Düsseldorf. In January of 1871, he completed his *Habilitation* with Clebsch in Göttingen, where he remained for three semesters as a lecturer (*Privatdozent*). His work during this time yielded significant results on the relation between linear and metric geometry and in the areas of non-Euclidian geometry, equation theory, the classification of third-order surfaces, and the systematization of geometrical research, which would form the basis of his "Erlangen Program". As a *Privatdozent*, too, he supervised his first doctoral student. Recommended by Clebsch, and at the age of just twenty-three, Klein was soon hired as a full professor by the small University of Erlangen in Bavaria.

A unique feature at the University of Erlangen was that every newly appointed professor had to produce an inaugural work of scholarship outlining his research program. Klein's work, which he completed in October of 1872, bore the title *Vergleichende Betrachtungen über neuere geometrische Forschungen* (Klein 1872) and later appeared in English as "A Comparative Review of Recent Researches in Geometry." The key novelty of this much-discussed "Erlangen Program," lay in Klein's insight that geometries could be classified by means of their associated transformation groups, each of which determines a characteristic collection of invariants. This fundamental idea is still cited and used by mathematicians today (see, for example, Ji and Papadopoulos 2015). Klein also had to deliver an inaugural lecture for his new position. This took place on December 7, 1872 before a university audience of largely non-mathematicians. In his lecture, he spoke about his ideas concerning teaching activity, which, in addition to lectures, also included practica, seminars, and working with models. Because mathematical education in Germany at the time was primarily intended for future teachers at secondary schools, he was sure to underscore the following point: "If we create better teachers, then education will improve

on its own and its traditional form will be filled with new and vital content!" (Jacobs 1977, pp. 15–16).

During his short time in Erlangen (1872–1875), Felix Klein supervised six doctoral dissertations and managed a number of affairs brought about by early death of Alfred Clebsch, who passed away in November of 1872. For instance, Klein arranged for one of his students, Ferdinand Lindemann (1852–1939), to edit Clebsch's lectures on geometry. Clebsch's death also resulted in a vacancy on the editorial board of the journal *Mathematische Annalen*, which he had founded in 1868 with Carl Neumann (1832–1925); this was filled in 1873 by two of Clebsch's students, Felix Klein and Paul Gordan (1837–1912). One year later, Klein secured an associate professorship for Gordan so that they could work together in Erlangen. While in Erlangen, too, Klein met his wife Anna Hegel (1851–1927), the eldest daughter of the historian Karl Hegel (1813–1901) and granddaughter of the great philosopher Georg Wilhelm Friedrich Hegel (1770–1831). From this marriage, which was consecrated on August 17, 1875, one son and three daughters would be born.

On April 1, 1875, Klein accepted a more challenging position at the Polytechnical School in Munich (as of 1877, a Technical College or *Technische Hochschule*), which, after its reorganization in 1868, began to educate teachers as well as engineers. His appointment there was as a professor of analytic geometry, differential and integral equations, and analytical mechanics. In order to manage the growing number of students at the college, the creation of an additional professorship had been authorized, and Klein ensured that this position was offered to another of Clebsch's former students, Alexander Brill (1842–1935). At Klein's initiative, they founded a new Institute of Mathematics, created a workshop for producing mathematical models, and reorganized their teaching duties so that time remained for their own research. It was here that, as Klein himself believed, he developed his own mathematical individuality—as well as that of many students. To earn doctoral degrees, however, Klein's talented students had to submit their dissertations to the *University* of Munich (see Hashagen 2003); the *Technical College* in Munich did not receive the right to grant doctorates until 1901. This and other reasons led Klein to seek a position elsewhere.

This transition was made possible by Adolph Mayer (1839–1908), a professor of mathematics at Leipzig with whom Klein had been editing the journal *Mathematische Annalen* since 1876 (see Tobies and Rowe 1990). In October of 1880, Klein was appointed a professor of geometry at the University of Leipzig (Saxony). While there, he founded a new institution, the so-called *Mathematisches Seminar* (1881), and began to give lectures on geometric (Riemannian) function theory. Noting that the French mathematician Henri Poincaré (1854–1912) had started to work in the same field, Klein began a fruitful correspondence with him (see Rowe 1992; Gray 2012). This resulted in the development of a theorem for the uniformization of algebraic curves by means of automorphic functions, something that Klein regarded among his most important findings and that would further occupy him and other mathematicians later on. After this intensive period of research (1881–82) Klein felt somewhat exploited and began to reorient his work. He turned to writing textbooks.

In 1884, the desirable opportunity arose for Klein to return to the small university town of Göttingen; Moritz Abraham Stern (1807–1894) had resigned from his professorship there. Encouraged by the physicist Eduard Riecke (1845–1915), with whom Klein had already had a good working relationship as a lecturer (*Privatdozent*), the majority of the Philosophical Faculty (which was then still a single unit) voted in Klein's favor. He was offered the position in the summer semester of 1886, despite official opposition from the other professors of mathematics at Göttingen, Hermann Amandus Schwarz (1843–1921) and Ernst Schering (1833–1897) (see Tobies 1991, 2002). Before Klein left Leipzig, he had managed to ensure that he would be replaced there by Sophus Lie. This move intensified the aversions and differences that already existed between Klein and a number of other German mathematicians, who disapproved of granting the position to a foreigner.

While in Göttingen, Klein gradually developed the Zeus-like status mentioned by Max Born. It was not until 1892, when he rejected an invitation from the *University* of Munich and when Hermann Amandus Schwarz took a new position in Berlin, that Klein became increasingly free to make his own decisions and began to hold some sway at the Prussian Ministry of Culture in Berlin. With the support of the influential civil servant Friedrich Althoff (1839–1908), Klein was finally able to initiate and realize a sweeping reorganization and renovation of the University of Göttingen's institutions, personnel, curricula, and research programs. He justified many of these changes by referring to his experiences during visits to the United States in 1893 and 1896 (see Parshall and Rowe 1994; Siegmund-Schultze 1997). By this time, Klein's influence had spread even further throughout Germany and beyond.

1.2 The Characteristics of Klein's Methods

Klein's growing influence can only be understood by examining the way in which he worked, which David Hilbert (1862–1943) once described as selfless and always in the interest of the matter at hand.

(1) The young Felix Klein internalized, from his upbringing and early education, a strong work ethic, which he maintained throughout his life. Stemming from a family of Westphalian tradesmen and farmers, his father had risen high through the ranks of the Prussian civil service and had impressed upon his children such virtues as unwavering discipline and thriftiness. That such lessons continued to be imparted throughout Klein's time at secondary school is evident from his following recollection: "We learned to work and keep on working" (Klein 1923). The essay that Klein wrote for his *Abitur* contains the following sentence, with a reference to Psalm 90:10: "Indeed, if a life has become valuable, it has done so, as the Psalmist says, on account of labor and toil" [Gymnasium Düsseldorf]. This creed increasingly defined his daily approach to work.

Whereas, in his younger years, Klein was known to meet up with colleagues and hike in the mountains, and although he continued take walks with colleagues

and with his family into old age, over time he refrained, on account of his health, more and more from participating in pleasantries unrelated to his work. He devoted every possible minute to pursuing his research and to helping his (male and female) doctoral students and post-doctoral researchers, from Germany and abroad, advance their own work. To this end, he met with each of them on a regular basis. The number of projects and positions that he took on reduced his free time to such an extent that his supportive wife was able to remark that they could hardly ever spend their wedding anniversary or birthdays together because priority was always given to his duties at the university. This tendency to overwork took its toll. After a long stay in a sanatorium, Klein retired early at the age of sixty-three. Even in retirement, however, he remained highly active. He gave lectures on the history of mathematics, made contributions to the theory of relativity, and continued to exert influence over hiring decisions, the formation of new committees, and book projects, among them his own collected works (Klein 1921–1923). Collaborators and colleagues would visit him at home where, though confined to a wheelchair, he refused to waste any time.

(2) Klein was aware that he could not work without *cooperation*, and this pertained to both his scientific and organizational undertakings. On October 1, 1876, for instance, he wrote the following words to Adolph Mayer: "It is a truly unfortunate scenario: When, as on this vacation, I only have myself to consult, then I am unable to complete anything of value. […] I need scholarly exchange, and I have been yearning for the beginning of the semester for some time now" (quoted from Tobies and Rowe 1990, p. 76). Already accustomed, while studying under Plücker, to developing new ideas through discussion, he had carried on this practice while working with his second teacher, Clebsch. Clebsch's ability to find connections between distinct areas of mathematics that had hitherto been examined in isolation became a point of departure for Klein's own research methods.

During his time studying in Berlin, Klein cooperated with the Austrian mathematician Otto Stolz (1842–1905) to develop the idea of combining non-Euclidian geometry with the projective metric devised by the British mathematician Arthur Cayley (1821–1895). With Ludwig Kiepert (1846–1934), a student of Karl Weierstraß (1815–1897), Klein made his first attempt to delve into the theory of elliptic functions. His most fruitful collaboration, however, was with the aforementioned Sophus Lie. They supported one another, published together, and maintained an intensive mathematical correspondence. Klein, moreover, went out his way to promote Lie's career (see Rowe 1989; Stubhaug 2002). Even though they came to disagree over certain matters later in life, Klein took these differences in stride and, in 1897, even endorsed Lie's candidacy to receive the inaugural Lobatschewski Prize (see Klein, GMA 1923).

Beginning in 1874, Klein also enjoyed a strong collaborative relationship with Paul Gordan, who had likewise studied under Clebsch. Both Lie and Gordan found it difficult to formulate their own texts, and so Klein was often asked to help them by editing their writing and systematizing their ideas. By recording their thoughts, he

immersed himself in them and expanded his own knowledge. Through his discussions with Gordan, and on the basis of the latter's knowledge of algebra, Klein entered into a wide—ranging field of research. Working together with students and colleagues at home and abroad, he combined the methods of projective geometry, invariant theory, equation theory, differential equations, elliptic functions, minimal surfaces, and number theory, thus categorizing various types of modular equations.

Klein applied this cooperative approach wherever and whenever he worked, vacations and research trips included. Even if not every mathematician from within Klein's sphere in Leipzig and Göttingen was willing to collaborate with him, everyone who sought his advice benefited from it. Here there is not enough space to list all of these beneficiaries. Prominent examples include Robert Fricke (1861–1930) and Arnold Sommerfeld (1868–1951), who edited books based on Klein's lectures and took his ideas in their own creative directions. Another mathematician worthy of mention is David Hilbert, who profited in Königsberg from the tutelage of Klein's student Adolf Hurwitz (1859–1919) and earned his doctoral degree under the supervision of Klein's student Lindemann, who was mentioned above. Klein personally supported Hilbert beginning with the latter's first research stay in Leipzig (1885/86); he recommended Hilbert to travel to Paris, maintained a correspondence with him (see Frei 1985), and secured a professorship for him in Göttingen (1895). There they conducted several research seminars together, and Hilbert, despite many enticing invitations to leave, remained Klein's colleague at that university.

Klein's skill at cooperating was also reflected in his activities as an editor: for the aforementioned *Mathematische Annalen*; for the *Encyklopädie der mathematischen Wissenschaften mit Einschluss ihrer Anwendungen* (B. G. Teubner, 1898–1935), which appeared in an expanded (and partially incomplete) French edition (see Tobies 1994; Gispert 1999); for the project *Kultur der Gegenwart* (see Tobies 2008); and for the *Abhandlungen über den mathematischen Unterricht in Deutschland, veranlasst durch die Internationale Mathematische Unterrichtskommission* (5 vols., B. G. Teubner, 1909–1916). Klein was able to connect a great number of people who collaborated on these projects.

Ever since Klein's years at the Technical College in Munich (1875–80), engineers and business leaders also numbered among his collaborative partners. While a number of engineers and technical scientists in the 1890s were initiating an anti-mathematics movement (Hensel et al. 1989), Klein was able to keep things in balance. In 1895, he joined the Association of German Engineers (*Verein deutscher Ingenieure*) as a mathematician; and, regarding mathematical instruction, he instituted a more applications-oriented curriculum that included actuarial mathematics and teacher training in applied mathematics. In order to finance the construction of new facilities in Göttingen, Klein followed the American model and sought funding from industry. His solution, which was novel in Germany at the time, was the Göttingen Association for the Promotion of Applied Physics and Mathematics (*Göttinger Vereinigung zur Förderung der angewandten Physik und Mathematik*). Initially founded exclusively for applied physics in 1898 and extended to include mathematics in 1900, this organization brought together Göttingen's professors of mathematics, physics, astronomy, and chemistry with approximately fifty financially

powerful representatives of German industry. In this way, Klein convinced industrial leaders that one of their goals should be to improve the application-oriented education of future teachers. The Ministry of Culture supported this initiative by introducing a new set of examinations—developed by Klein—that, for the first time, included the field of applied mathematics (1898). This, in turn, provided the impetus for establishing new institutes and professorships for applied mathematics, technical mechanics, applied electricity research, physical chemistry, and geophysics (see Tobies 1991, 2002, 2012, ch. 2.3). With these developments in mind, Klein began to shift the focus of his teaching more and more toward applications and questions of pedagogy. In his seminars, he no longer only cooperated with Hilbert and others on teaching "pure" mathematics but rather also with newly hired professors and lecturers to teach applied fields as well mathematical didactics (see [Protocols]).

(3) From the beginning, Klein's approach was distinguished by its *internationality*. He profited early on from the international networks of his teachers Plücker and Clebsch, and he came away with the general impression "that we restrict ourselves to a level that is far too narrow if we neglect to foster and revitalize our international connections" (a letter to M. Noether dated April 26, 1896; quoted from Tobies and Rowe 1990, p. 36). Klein lived by these words even when the officials at the Prussian Ministry of Culture did not yet value such things: "We have no need for French or English mathematics," or so the ministry responded in 1870 when, at his father's prompting, he sought a recommendation for his first trip abroad (see Klein 1923).

Proficient in French since his school days and an eager learner of English, Klein developed his own broad network of academic contacts beginning with his first research trips to France (1870), Great Britain (1873), and Italy (1874). This served his research approach well, which was to become familiar with and integrate as many areas of mathematics as possible, and it also benefited the *Mathematische Annalen*, for which he sought the best international contributions in order to surpass in prestige the competing *Journal für die reine und angewandte Mathematik* (Crelle's *Journal*), which was edited by mathematicians based in Berlin. His international network also helped to the extent that many of his contacts sent students and young scientists to attend his courses. Even while Klein was in Erlangen, Scandinavian students (Bäcklund, Holst) came to study with him at the recommendation of Lie; while in Munich, he was visited by several Italian colleagues, and after his second trip to Italy (1878), young Italian mathematicians (Gregorio Ricci-Curbastro, Luigi Bianchi) came to study under him (see [Protocols], vol. 1; Coen 2012). Gaston Darboux (1842–1917), with whom Klein had corresponded even before his first trip to Paris and with whom he had collaborated on the review journal *Bulletin des sciences mathématiques et astronomiques*, sent young French mathematicians to work with him both in Leipzig and in Göttingen. Darboux was the first person to commission a translation of one of Klein's works into a foreign language—*Sur la géométrie dite non euclidienne* (1871)—and they would go on to work together for many years,

work that included their participation on prize committees, teaching committees, and bibliographies (Tobies 2016).

During Klein's first semester in Leipzig (1880/81), the following international students (among others) came to work with him: Georges Brunel (1856–1900), recommended by Darboux; the Englishman Arthur Bucheim (1859–1888), who had been educated at Oxford by Henry John Stephen Smith (1826–1883); Guiseppe Veronese (1854–1917), at the instigation of Luigi Cremona (1830–1903); and Irving W. Stringham (1849–1917), who had already earned a doctoral degree under James Joseph Sylvester (1814–1897) at Johns Hopkins University in Baltimore. Under Klein's direction, they produced findings that were published in the *Mathematische Annalen* (Veronese in 1881 and 1882, Brunel in 1882) or in the *American Journal of Mathematics* (Stringham in 1881). To Daniel Coit Gilman, the president of Johns Hopkins, Stringham wrote enthusiastic letters about Klein's critical abilities and about the international nature of his seminars. When Stringham's former teacher Sylvester left his position in Baltimore, Klein was invited in 1883 to be his successor. Klein declined the offer for financial reasons, which was itself a sign of his international reputation. Ever since his time in Leipzig, Klein also made conscious efforts to enhance his relations with Russian and other Eastern European mathematicians. Wishing to foster exchange, he would always request his students from these areas to provide him with an overview of the institutions there, their staff, and their research trends.

In Göttingen, and thus back under the purview of the Prussian Ministry of Culture, Klein had to decline an invitation in 1889 to work as a visiting professor at Clark University in Worcester, Massachusetts (USA) because the Ministry did not approve ([UBG] Ms. F. Klein I, B 4). After securing his position, however, he ultimately travelled in 1893 with the official endorsement of the Ministry to Chicago for the World's Fair, which included an educational exhibit and which was being held in conjunction with a mathematics conference. While there, Klein gave twelve presentations on the latest findings in mathematics. He spoke about the work of Clebsch and Sophus Lie, algebraic functions, the theory of functions and geometry, pure and applied mathematics and their relation, the transcendence of the numbers e and π, ideal numbers, the solution of higher algebraic equations, hyperelliptic and Abelian functions, non-Euclidean geometry, and the study of mathematics at Göttingen (Klein 1894). In his talks, Klein gave particular weight to his own recent findings and to those of his students and collaborators, thus waging a successful publicity campaign for studying at the University of Göttingen (see Parshall and Rowe 1994). With these lectures, which were later translated into French at the instigation of Charles Hermite (1822–1901), Klein did much to increase his international profile.

During the 1890s, Hermite occasioned additional translations of Klein's work (on geometric number theory, the hypergeometric function, etc.), most of which appeared in the *Nouvelles annales de mathématiques, journal des candidats aux écoles polytechnique et normale*, which was then edited by Charles-Ange Laisant (1841–1920). Hermite gushed that Klein was "like a new Joshua in the Promised Land" (*comme un nouveau Josué dans la terre promise*) and nominated him, in 1897, to become a corresponding member of the Académie des Sciences in Paris (Tobies 2016). By

this time, Klein was already a member of numerous other academies in Germany, Italy, Great Britain, Russia, and the United States. When, in 1899, Laisant and the Swiss mathematician Henri Fehr (1870–1954) founded the journal *L'Enseignement mathématique*, Klein was made a member of its Comité de Patronage, which consisted of twenty mathematicians from sixteen countries. As the first international journal devoted to mathematical education, it published several reports concerning educational reforms, including essays by Klein (in French translation). Fehr reviewed Klein's books for the journal, among them his *Elementarmathematik vom höheren Standpunkte aus* ("Elementary Mathematics from an Advanced Standpoint," as the work would be known in English).

L'Enseignement mathématique became the official organ of the International Commission on the Teaching of Mathematics, which was founded in 1908 at the Fourth International Congress of Mathematicians in Rome. Klein's election to the board of this commission, which took place despite his absence from the conference, was a testament to his international reputation (see Coray et al. 2003). As president of this commission (from 1908 to 1920), Klein initiated regular conferences and publications devoted to the development of mathematical education not only in Germany but in all of the countries involved.

(4) Felix Klein followed a principle of *universality*. When asked to characterize his efforts, he himself spoke about his universal program. As a young researcher, he wanted to familiarize himself with all branches of mathematics and to contribute to each of them in his own work, an approach that gave rise to his principles of transference (*Übertragungsprinzipien*) and his "mixture" of mathematical methods. Inspired by Clebsch, he also attempted from quite early on to bring together people with different areas of mathematical expertise in an effort to overcome disciplinary divides (see Tobies and Volkert 1998). This end was likewise served by his large-scale undertaking of the *Encyklopädie der mathematischen Wissenschaften*, for which he recruited international experts to provide an overview of all of mathematics and its applications (Tobies 1994). Klein's participation in the preparations for the *International Catalogue of Scientific Literature* (1902–21), which was directed by the Royal Society of London, can also be interpreted in this way.

Klein's universal program not only involved supporting and advancing new and marginal disciplines. He applied his universal approach to teaching as well. He promoted talented scholars regardless of their nationality, religion, or gender. Although a university professor, he was deeply interested in improving and fostering mathematical and scientific education from kindergarten onward. In this regard, Klein operated according to one of the guiding pedagogical mottos of the nineteenth century: "Teach everything to everyone."

1.3 Educational Reform and Its Institutional and International Scope

From early on, Klein felt that the mathematical education being offered at secondary institutions, which neglected applied mathematics and was based primarily on synthetic geometry, was in need of reform. Even while still a doctoral student, he argued that new geometric methods ought to be introduced into the curriculum to complement Euclidian geometry. In this matter, he found an ally in Gaston Darboux, as is documented in their correspondence from the 1870s (Richter 2015).

In 1890, the teachers of mathematics and the natural sciences at secondary schools founded an Association for the Promotion of Mathematical and Natural-Scientific Education (*Verein zur Förderung des mathematischen und naturwissenschaftlichen Unterrichts*) in order to be on equal footing with their colleagues in the philological and historical disciplines. When public discussions began to be held about designing new curricula, Klein felt that the time was ripe for reform. He developed a course of study for educating teaching candidates at the university level; he began to teach, as of 1892, continuing education courses for teachers who were already working; and he soon developed contacts with the association named above (Tobies 2000). For the year 1895, Klein invited the association to hold its annual conference in Göttingen. Here he was sure to showcase the university's modern facilities, and he celebrated the event by presenting the attendees with a book concerned with question of elementary geometry (Klein 1895), a work which was soon translated into French (1896), Italian (1896), and English (1897).

The Prussian Ministry of Culture honored Klein with decorations and titles. Althoff turned to Klein as an advisor in matters of hiring and other affairs. In Göttingen, two additional professors were hired to join Klein and Hilbert: Hermann Minkowski (1864–1909) in 1901, who was succeeded in 1909 by the number theorist Edmund Landau (1877–1938); and Carl Runge (1856–1927) in 1904, who was appointed as the first professor of applied mathematics at a German university. Under Klein's guidance, further expansions were made in the fields of technical mechanics, applied electricity theory, and geophysics.

In 1899, and with the backing of the Ministry of Culture, Klein supported an initiative that would allow Prussian technical colleges to grant doctoral degrees. By preparing a series of commissioned reports and by participating in a school conference in Berlin in 1900 (see Schubring 1989), Klein contributed to an imperial decree (issued that same year) which mandated that the diplomas (*Abitur*) granted by the three existing types of secondary schools for boys (the so-called *Humanistisches Gymnasium*, *Realgymnasium*, and *Oberrealschule*) would henceforth be regarded as equal. Until then, the graduates of *Oberrealschulen* had been at a disadvantage. At the same time, a process was begun to modernize mathematical and scientific education at all sorts of schools. The principle aims were to accord a central position of the notion of the function, to teach of analytic geometry, and to in corporate elements of differential and integral calculus, application-oriented instruction, and genetic methods. Having served three terms (1897, 1903, 1908) as the chairman

of the German Mathematical Society (*Deutsche Mathematiker-Vereinigung*), which was founded in 1890, Klein also took advantage of this venue to enhance discussions about pedagogical issues.

In the wake of the school conference in Berlin, Klein also came to be regarded as an expert by biologists, who requested his assistance in reintroducing the subjects of botany and zoology as components of higher education (the latter had been banned in Prussia since 1879 on account of the Darwinian theory of evolution). In response, Klein convened a meeting of Göttingen professors on the philosophical faculty in order to weigh the demands of the biologists without disadvantaging any other fields. This led to the creation of an additional organization within the framework of the Society of German Natural Scientists and Physicians (*Gesellschaft deutscher Naturforscher und Ärzte*), which, at its annual meeting in 1904, formed a twelve-member education committee in order to develop reformed curricula for all types of schools. Klein deployed his friend August Gutzmer (1860–1924) as the director of this committee, while Klein himself acted on behalf of the German mathematical society and spoke to audiences of philologists and historians in order to win their support for the proposed reforms to the mathematical and scientific curricula.

Plans for the reform were presented and discussed at conferences in Merano (1905), Stuttgart (1906), and Dresden (1907), and they were ultimately published. In order to implement them, a board was formed in 1908 in Cologne—the German Commission for Mathematical and Natural-Scientific Education (*Deutscher Ausschuss für mathematisch-naturwissenschaftlichen Unterricht*)—and Klein was asked to lead its division concerned with teacher education. In the same year, Klein was not only made the president, as mentioned above, of the International Commission on the Teaching of Mathematics; on February 17, 1908, he was also named a member of the upper chamber (House of Lords) of the Prussian House of Representatives (Tobies 1989). The invitation to join the House of Lords was an expression of Klein's status at the University of Göttingen, for his mandate as a member was to representative the university. Klein, who was nonpartisan, succeeded Göttingen's previous representative, the professor of ecclesiastical law Richard Wilhelm Dove (1833–1907), in this lifelong position (which, for Klein, ended in 1918 with the end of the German Empire). Here he took advantage of the alliances formed by the Göttingen Association for the Promotion of Applied Physics and Mathematics between science, industry, and the government to abet the implementation of educational reforms. In the speeches that he delivered in House of Representatives, he advocated for improving educational standards at all types of schools, including primary schools, schools for girls, and trade schools.

Klein was a firm believer in the equal abilities of men and women, and he accordingly believed that they should have access to the same educational opportunities. As early as 1893, he arranged for the first women to study under his supervision, even though women were officially not allowed to enroll in Prussian universities until 1908. By 1895, the Englishwoman Grace Chisholm (1868–1944) and the American Mary F. Winston (1869–1959) had submitted their dissertations to him. Numerous additional students—both men and women, from Germany and abroad—would come

to study under him (Tobies 1991/1992, 2019); in all, he supervised more than fifty dissertations.

The fact that Klein took a parliamentary position—and that he was the first German mathematician to do so—is best understood from an international perspective. In this matter, his role models were colleagues from Italy and France. According to Hilbert, Darboux influenced Klein's interest in educational reform in a particular way. Since 1888, Darboux had been a member of the French High Council for Public Education (*Conseil supérieur de l'instruction publique*), and in 1908 he was made the vice president of the Council's standing committee for advising the government in educational affairs (Richter 2015, p. 20). Darboux directed the French branch of the International Commission on the Teaching of Mathematics while the German subcommittee was being led by Klein.

As originally planned, the aforementioned *Encyklopädie der mathematischen Wissenschaften mit Einschluss ihrer Anwendungen*, which appeared in six comprehensive volumes, was intended to contain a seventh volume devoted to the history, philosophy, and didactics of mathematics. After initial plans were discussed in May of 1896, publications in *L'Enseignement mathématique* and further studies commissioned by the International Commission on the Teaching of Mathematics promoted the preparation of the volume. As late as April of 1914, Klein arranged for Heinrich Emil Timerding (1873–1945), the intended editor of the work, to attend the Congrès de philosophie mathématique in Paris. The First World War, however, prevented the project from being completed (Tobies 1994, pp. 56–69), just as it had stalled so many international collaborations (see Siegmund-Schultze 2011).

On March 15, 1915, the Académie des Sciences in Paris annulled Klein's membership because he had signed the so-called "Manifesto of the Ninety-Three," a nationalistic proclamation in support of German military action. In a detailed study, Tollmien (1993) has demonstrated that Klein, like a number of other German scientists, had not been fully aware of what he was signing, that he regretted doing so, and that—unwilling to repay like with like—he discouraged German academies from expelling French scientists. As a member of the Prussian House of Lords, Klein issued a memorandum in March of 1916 that called for a thorough investigations of conditions abroad after war's end. To the international boycott of German scientists after the war, Klein responded with the motto "Keep quiet and work." In his memoirs, he looked back fondly on his strong contacts with foreign scientists, and he lamented the period of nationalistic antagonism (Klein 1923).

When, in 1920, the Emergency Association of German Science (the German Research Foundation today) was formed as an organization for funding research, Klein was elected as the first chairperson of the committee (*Fachausschuss*) for mathematics, astronomy, and geodesy. While an anti-technical mood was setting in after the defeat in the First World War, and while the number of lessons in mathematics and the natural sciences at secondary schools were being reduced, Klein supported a nationwide union, the *Mathematischer Reichsverband* (1921), to counter such trends. When, in the same year, Richard von Mises (1883–1953) founded the *Zeitschrift für angewandte Mathematik und Mechanik*, which is still in circulation today, Klein applauded this achievement and saw in it the realization of one of his own goals,

which he had attempted to achieve in 1900 by coordinating the specializations of German mathematical journals.

Klein's vision was to accommodate all branches of mathematics and to secure a firm place for mathematics within the "culture of the present," that is, to make it a necessary component of other sciences, technology, and general education. He had been pursuing this vision with greater and greater vigor and detail ever since he had delivered his Erlangen inaugural lecture in 1872. To realize it, he endeavored to cater his arguments to the interests of his audiences, which included industrialists and government officials, and to underscore the importance of international connections to developments in Germany (see Siegmund-Schultze 1997). In light of Klein's integrative approach to mathematics, its applications, and its instruction, it might be appropriate to end with the following remark about him by Richard von Mises: "We see that the value and dignity of the works that he accomplished are in perfect harmony with the significance of the man behind them" (1924, p. 86).

Translated by Valentine A. Pakis

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