

# Measuring Sequence Quality



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## 1 Introduction: The Quality of Binary Sequences of Successes and Failures

Examination of binary sequences, that is, sequences containing only two distinct characters, each of which represents a class of separate states or events of the process observed, is of key interest in social science research. Binary sequences, which can be seen as a series of positive versus negative characters – that is, of successes versus failures – are abundant in health and applied social science research. For instance, in the study of the course of a therapy, patients may show several kinds of unfavorable reactions, the failures, as well as various kinds of positive reactions, the successes. Similarly, pupils may make various kinds of mistakes, as a result, for example, of failures of the teaching process; alternatively, they may produce correct responses, the successes of the teaching process. Yet, another example refers to individuals' labor market careers, which can be represented as the succession of favorable and unfavorable labor market states; employment and vocational training, for example, can be considered as successes, while unemployment and inactivity as failures.

The presence of successes and the absence of failures defines the quality of a sequence. Over the course of the treatment, therapy quality is higher when the patient's unfavorable reactions – the failures – gradually disappear from the gamut of observed behaviors; the teaching quality is higher when inadequate responses disappear from the pupil's repertoire; and career quality improves if

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unemployment is overcome by finding stable employment. However, available techniques to quantify sequence quality have been lacking with existing measures focusing predominantly on the comparison of ordered sequences (Abbott 1995) or the variability of sequences (Elzinga and Liefbroer 2007). Brzinsky-Fay (2007) and Gabadinho et al. (2011) already recognize the need of such a measure and make an attempt to quantify the quality of sequences. Our study builds on and expands this knowledge by proposing a new method which distinguishes between states of differing quality, that is, states with different characteristics. We operationalize the concept of positive and negative states, which define successful and less successful sequences, and take into account the variation in the frequency, duration, and recency of successes and failures over the course of a trajectory. We interpret binary sequences as series of Successes ( $S$ ) and Failures ( $F$ ) and encode them as strings consisting of the characters  $S$  and  $F$ . Of course, the quality or successfulness, of a therapy may also depend on various characteristics of the therapist, the therapy, and the patient; the quality of teaching may depend on the teacher, the teaching instructions, or the pupil; the quality of a career may depend on individual characteristics of the worker, or labor market opportunities, among other factors. Hence, in many situations, it would be interesting to model the quality of a binary sequence in terms of one or more independent variables.

In Sects. 2 and 3, we briefly review existing methods to compare and describe trajectories, outline elementary requirements of our proposed measure of sequence quality, and discuss its properties. Next, we show an application of our measure to model the quality of labor market careers. In doing so, we provide a direct test of unemployment “scarring” theories that expect the number, duration, and the most recent spell of unemployment to largely determine the path of subsequent employment career quality due to human capital depreciation and signaling processes (Arulampalam 2001; Mooi-Reci and Ganzeboom 2015). Specifically, we ask to what degree unemployment is negatively associated with the quality of one’s future employment career, and at what rate, if at all, previously unemployed individuals eventually recover from it. Using data from the Household, Income, and Labour Dynamics in Australia (HILDA) Survey, we investigate the evolution of employment career quality after the occurrence of an initial spell of unemployment and as a function of an individual’s attributes. We conclude with a discussion of potential extensions to our measure.

## 2 Common Methods for Studying Sequence Trajectories

Sequence analysis has advanced rapidly in the last few decades, and several developments have been proposed, among which extensions of Optimal Matching (Abbott 1995) based on alignment techniques (e.g., Lesnard 2008, 2010; Stark and Vedres 2006; Stovel et al. 1996; Stovel and Bolan 2004), as well as non-alignment techniques (e.g., Elzinga 2003, 2010); see Studer and Ritschard (2016) for a full review. Newly developed measures, such as for example the “turbulence”

or “complexity” measures (Elzinga and Liefbroer 2007; Elzinga 2010; Gabadinho et al. 2011) have offered a quantification of the variability within rather than between sequences, but fall short in distinguishing between “good” or “bad” events over the course of a trajectory. This results in treating a series of positive events (e.g., upward job mobility) equally as a series of negative events (e.g., downward mobility), which may lead to serious substantive misinterpretations about the evolution of trajectories. Failing to attach a quality connotation to the states results in measures that ignore the variation in sequence quality.

An attempt to quantify the quality of sequences is proposed by Brzinsky-Fay (2007). The study uses data from the European Community Household Panel (ECHP) over the period 1994 to 2011 to examine sequences of school-to-work transitions of school leavers across ten European countries. It draws on explorative methods of optimal matching and cluster analysis to identify positive from negative variation of sequences where transition types with high volatility are considered negative and those with low volatility positive. While valuable, this volatility index accounts for the sequencing and variability of the transitions to infer indirectly about the nature of the transitions. However, the index suffers from the same ‘old’ shortcoming in which upward and negative job mobility are quantified equally because the index captures the volatility rather than the quality of the transitions. To our knowledge, one of the studies that comes closest to our measure of sequence quality is the “precarity” index proposed by Ritschard et al. (2018) in this bundle. The index draws on the complexity index as proposed by Gabadinho et al. (2011) that combines entropy with the number of transitions in the sequence and then uses a correction factor that reflects the penalizing versus rewarding quality of a transition. This correction factor is derived from the proportions of negative and positive transitions in a sequence. In doing so, the measure allows the user to determine the negative or positive transitions based on theory or data.

Different from the measure proposed by Ritschard et al. (2018) that is based on the quality of the transitions between states, our sequence quality measure is based on the quality of the states themselves and takes into account various dimensions of a particular state, such as the frequency, duration and its recency.

In the following section, we will introduce our new measure and its properties.

### 3 Developing a Measure of Sequence Quality: Formal Properties

Here, we introduce some notations and concepts. First, we define an alphabet, i.e., the set of states or characters that we deal with, as  $A = \{F, S\}$ , where  $F$  denotes a failure of some sort and  $S$  denotes a success of some kind; the specific interpretation will depend on the substantive meaning of the sequences. By concatenating the characters from the alphabet, we obtain sequences which we may denote as  $x$ ,  $y$ , or  $z$ , for example. Let  $x$  denote such a sequence; then  $x^F$  denotes the same

sequence, elongated with a failure  $F$  and, similarly,  $xS$  denotes the same sequence, elongated with a success  $S$ . If  $y$  is another such sequence, then  $xy$  denotes the (right-)elongation of  $x$  with  $y$ . A run is a number of consecutive successes or failures. Thus, if  $x = FFSSSF$ , we say that  $x$  consists of three runs:  $F^2$ ,  $S^3$ , and  $F^1 = F$ . More generally, we say that e.g.,  $S^n$  denotes a run of  $n$  successes where  $n$  is a nonnegative integer, the run-length. Nonnegative, since it is convenient to have an empty sequence  $\lambda = S^0 = F^0$  that does not materially elongate any sequence: we have that  $x\lambda = x = \lambda x$  for all sequences  $x$ .

These simple concepts and notations suffice to discuss some fundamental requirements that all quality measures of successfulness should adhere to. First, we require that such a measure, say  $\Gamma(x)$ , increases when the number of successes in a given sequence increases, independent of sequence length. Hence, we require that:

- (1) For any  $xy \neq \lambda$ ,  $1 \geq \Gamma(xSy) \geq \Gamma(xy)$ , equality holding precisely when  $x = S^n$  and  $y = S^m$  for any nonnegative  $n$  and  $m$ .

Stating that there is an upper bound of 1 is a way of saying that  $\Gamma(x)$  is independent of the sequence length; however long the sequences,  $\Gamma(x)$  will not exceed the value of 1. Requirement (1) also states that wherever we put an extra success into a sequence, the result will be that  $\Gamma(x)$  increases, with the only exception that  $\Gamma(S^n) = 1$  for all positive  $n$ . Substantively this effect can be seen as a “compensating effect” in which a positive event or state (e.g., employment) counteracts a previous negative state (e.g., unemployment or inactivity).

Our second requirement is the mirror image of the first as it pertains to the effect of failures on the value of  $\Gamma(\cdot)$ :

- (2) For any  $xy$ ,  $0 \leq \Gamma(xFy) \leq \Gamma(xy)$ , equality holding precisely when  $x = F^n$  and  $y = F^m$  for any nonnegative  $n$  and  $m$ .

Requirement (2) implies that the lower bound of  $\Gamma(\cdot)$  equals zero, which is attained for any sequence that consists exclusively of failures or that contains only the empty character  $\lambda$ . Furthermore, requirement (2) states that the quality of a sequence diminishes when we add more failures. Hence, requirements (1) and (2) jointly imply that in a given sequence, a quality measure: (a) has a fixed range of  $[0, 1]$ ; (b) increases when the number of successes increases; and (c) decreases when the number of failures increases.

Now consider the sequences  $x = SSF$  and  $y = FSS$ . These two sequences only differ in the position of the failure  $F$ :  $x$  ends in a failure and  $y$  ends in a success. Whether  $x$  and  $y$  stood for sequences of responses of patients, pupils, or labor market states, in all cases we would consider  $y$  as the highest quality sequence because of the recency of the success. We believe this to be a general principle: the more recent the successes, the more positive the quality of the sequence. Therefore, we formulate a third requirement that precisely formalizes this principle:

- (3) For any  $xy$ ,  $\Gamma(xFSy) > \Gamma(xSFy)$ .

While frequency refers to how often states occur in a sequence, we may also be interested in “durability”, which is the consecutive frequency of states or, in other words, the length of spells in a sequence. Frequency refers to the occurrence of a state, independent of whether it is part of the same or of different spells. Durability refers to state frequency within a spell. Higher durability is manifested in higher frequency of a state. However, the reverse may not be the case, as high frequency of a state may be due not only to a single durable spell, but also to several spells of short duration, in which case we will see low durability despite high frequency. For example, take the sequence  $x = S^2F^2$ , which includes two consecutive states of success, that is a spell of success with a duration of two units, followed by a spell of failure with duration of two units. Then take a sequence  $y = S^1F^1S^1F^1$  including alternating spells of successes and failures, each of the duration of one unit, for a total frequency of two states of failures and two states of success. Sequences  $x$  and  $y$  will have the same frequency of successes. However, the durability of success differs, reaching a value of two consecutive successes in sequence  $x$  and only one in sequence  $y$ . Note that differences in duration at equal frequency will affect the recency of success. Therefore, a measure fulfilling the above requirements (1–3) and accounting for both frequency and recency will capture duration differences as well. In sequence  $x$ , consecutive success durability is higher compared to sequence  $y$ , although success is more recent in sequence  $y$ . In the next section, we will discuss an implementation of  $\Gamma$  that satisfies the above requirements.

#### 4 Using S-Positions: Successes Weighed by Frequency and Recency

For a binary sequence  $x$  over  $A = \{F, S\}$ , we write  $x = x_1, \dots, x_n$  when  $x$  has  $n$  characters and  $x_i$  stands for the  $i$ -th position in  $x$ . Thus  $x_i \in A$  for all  $i \in \{1, \dots, n\}$ . For example, with  $x = SSF SF$ ,  $x_1 = x_2 = x_4 = S$  and  $x_3 = x_5 = F$ . The length of  $x = x_1, \dots, x_n$  equals  $n$ ; we denote this by writing  $|x| = n$ . For a sequence  $x$  with  $|x| = n$ , the  $k$ -th prefix of  $x$  is the sequence  $x^k = x_1, \dots, x_k$  for  $0 \leq k \leq n$  and  $x^0 = \lambda$ .

We begin by discussing a simple example. Consider the sequence:  $x = x_1x_2x_3x_4x_5 = SSF SF$ . First, we note that the first, second and fourth characters are successes. Then, we add the position-indices of these characters:  $1 + 2 + 4 = 7$ . If these three successes had occurred later in the sequence, as for example in the sequence  $FFSSS$ , the sum of the position indices would have been bigger:  $3 + 4 + 5 = 12$ . Therefore, we see that the sum of the position indices of the  $S$ -observations quantifies the quality level: the more  $S$ -observations and/or the more recent these are, the bigger the sum will be. However, we cannot judge the size of this sum independently of the length of the sequence, since longer sequences will have larger position indices. To adequately quantify quality, we divide the sum of the observed  $S$ -positions by the sum of all positions and we denote this ratio as  $\gamma$  to distinguish it from the general *Gamma*. Hence, we obtain:

$$\Upsilon(1 + 2 + 4)/(1 + 2 + 3 + 4 + 5) = 7/15 = 0.47.$$

Clearly, this ratio is always in the range  $[0, 1]$  as the numerator is non-negative and at most as big as the denominator. Formally and more generally, it is convenient to first define a position variable:

$$p_i = \begin{cases} i & \text{if } x_i = S \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

and then define:

$$0 \leq \Upsilon^w(x^n) = \frac{\sum_{i=1}^n p_i^w}{\sum_{i=1}^n i^w} \leq 1 \tag{2}$$

for some nonnegative exponent  $w$ . Note that in the above example, we set  $w = 1$ . Clearly,  $\Upsilon^1$  quantifies sequence quality in the required way: the more failures and the later in the sequence, the smaller its numerator. As the numerator of Eq. 2 is nonnegative and cannot exceed the size of the denominator,  $\Upsilon^w$  will be tightly bound by 0 and 1; the upper bound will be attained when all states are  $S$ -states, while  $\Upsilon^w = 0$  when the sequence shows no successes at all. Table 1 illustrates the behavior of  $\Upsilon^w$ . Specifically, the left part of the table illustrates the behavior of  $\Upsilon^w$  when failures get more recent for three different values of the parameter  $w$ ; the right part illustrates the combined effect of recency and number of failures on  $\Upsilon^w$ .

Let us now turn to the meaning of the parameter  $w$  in Eq. 2. First, let us set  $w = 0$ . The denominator then reduces to  $\sum_{i=1}^n i^0 = \sum_{i=1}^n 1 = n$  and the numerator counts the number of  $S$ -states, regardless of their position in the sequence. Hence, we see that:

$$\Upsilon^0(x^n) = \frac{f(S)}{n}$$

i.e., the measure calculates the fraction of  $S$ -states in the sequence  $x^n$ .

**Table 1** Illustration of  $\Upsilon^w$  behavior with varying  $w$  and recency of failure

	$\Upsilon(x)$				$\Upsilon(x)$		
	$w = 0.5$	$w = 1$	$w = 2$		$w = 0.5$	$w = 1$	$w = 2$
FFFSS	0.62	0.71	0.85	SSSS	1	1	1
FFSFSS	0.59	0.67	0.77	SFSS	0.77	0.8	0.87
FSFSS	0.56	0.62	0.71	SSFS	0.72	0.7	0.7
SFFFSS	0.53	0.57	0.68	SSSF	0.67	0.6	0.47
SFFSFS	0.5	0.52	0.58	FSSF	0.51	0.5	0.43
SFSFFS	0.48	0.48	0.51	SFSF	0.44	0.4	0.33
SSFFFS	0.45	0.43	0.45	SSFF	0.39	0.3	0.17
SSFFSF	0.43	0.38	0.33	FSFF	0.23	0.2	0.13
SSFSFF	0.41	0.33	0.23	SFFF	0.16	0.1	0.03
SSSFFF	0.38	0.29	0.15	FFFF	0	0	0

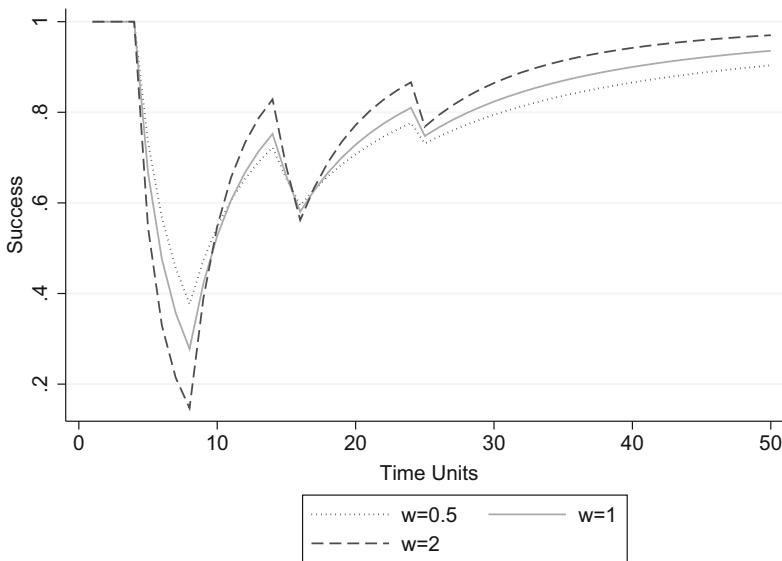
Next, we consider:

$$\gamma^1(x^n) = \frac{\sum_i p_i}{\sum_i i} = \frac{\sum_i p_i}{n(n+1)/2}$$

which is simply a formalization of the example above.

In Table 1, we illustrate how  $\gamma^w(x)$  reacts to changes in the recency of failures and to changes in the number of failures. To demonstrate the effect of the parameter  $w$ , Fig. 1 illustrates the dynamic behavior of  $\gamma^w$  for different values of  $w$ .

Specifically, we plot  $\gamma^w$  for various values of  $w$  applied to a fixed sequence  $x = S^4 F^4 S^6 F^2 S^8 F^1 S^{25}$  of length  $|x| = 50$ . To draw Fig. 1, we calculate, for each fixed value of  $w$ ,  $\gamma^w(x^n)$  for  $n = 1, 2, \dots, 50$ . Hence, the plots show how  $\gamma^w$  develops “over time”, as the sequence develops from a single state to a sequence of 50 states. The sequence chosen has three runs of failures, each next run shorter than the previous one. The plots demonstrate that the bigger the parameter  $w$ , the more severe is the effect of failures, but recovery from the failures due to subsequent successes is also faster for bigger  $w$ . This holds independent of the order of success and failure runs and their length. Moreover, the plots suggest that, given enough time and enough subsequent successes, the sequences will “fully recover” from previous failures. Here, “full recovery” means that  $\gamma$  will be arbitrarily close to 1. Formally, this can be seen from the fact that:



**Fig. 1** Plot of  $\gamma^w(x^n)$  (vertical axis), for various values of  $w$  and  $n$  (horizontal axis) ranging from 1 to 50, and  $x = S^4 F^4 S^6 F^2 S^8 F^1 S^{25}$

$$\gamma^w(x) = \frac{\sum_i i^w - S_F}{\sum_i i^w} = 1 - \frac{S_F}{\sum_i i^w} \quad (3)$$

wherein  $S_F$  denotes the sum of (the powers of) the positions on which failures occurred: given enough subsequent successes after the last failure, the fraction will converge to 1 since the quantity  $\sum_i i^w$  will increase while the quantity  $S_F$  will remain constant. Full recovery from failures is a phenomenon that happens, for example, in labor market careers of former criminals: after enough clean time (“Success”) after conviction (“Failure”), their chances of getting employed are equal to those never arrested (Blumstein and Nakamura 2009).

Calculating the quantity  $\gamma^w$  is straightforward; for the denominator of  $\gamma^w$ , closed expressions can be derived and  $w$  does not have to be an integer (Knuth 1993). However, in applications, such closed expressions are not required since the denominator is easily calculated simultaneously with the numerator while parsing the sequence. Thus, calculating  $\gamma^w$  is proportional to the length  $n$  of the sequences. The reader is aware that  $\gamma^w$  as defined in Eq. 2 is just one of the many possibilities to construct a quality measure on the basis of the position-numbers  $p_i$ : in fact, for any function  $f$  that satisfies  $f(a) > f(b)$  precisely when  $a > b$ , the quantity

$$0 \leq \Gamma(x^n) = \frac{\sum_{i=1}^n f(p_i)}{\sum_{i=1}^n f(p_i)} \leq 1$$

is a quality measure, i.e., satisfies the requirements (1) to (3) as formulated in the previous section. We believe that, by picking  $f(p_i) = p_i^w$ , we have covered sufficient potentially useful possibilities.

Theoretical reasons may lead a user to define a  $w$  of 0 and get an index that does not correct for the recency of failures or successes. Alternatively, a  $w$  of 1 quantifies sequence quality in the required way: the more failures and the later in the sequence, the smaller its numerator. However, should theory assign deeper penalties to failures and faster recoveries to most recent successes, then a  $w > 1$  would be preferable.

## 5 An Application: The Quality of Labor Market Careers Among the Unemployed

In this section, we apply  $\gamma^w$  to labor market careers in order to test whether unemployment has long-term negative effects on people’s career outcomes, also termed in the literature as unemployment “scarring” (Arulampalam 2001; Mooi-Reci and Ganzeboom 2015). Two prominent theoretical mechanisms have been suggested to drive unemployment scarring. First, from a human capital perspective, loss and depreciation of firm and occupation specific skills over a spell of unemployment is thought to make workers less productive. This translates into job offers of a poor quality (e.g., non-standard or temporary type of jobs) that are more likely



to be discontinued and result in recurrent unemployment spells in the future. This implies that the longer the elapsed time in unemployment the lower the chances of reemployment will be.

Signalling has also been suggested as an alternative mechanism driving scarring (Spence 1973). When employers have little information about one's qualifications or abilities they use a range of observable characteristics to infer an applicant's productive capabilities. Characteristics that are subject to individuals' own actions, such as previous unemployment spells, are likely to raise red flags about a worker's unobserved quality and the tasks that they are expected to perform. The views that employers hold of applicants with previous unemployment spells are most likely to translate into lower reemployment rates and wages for some age groups (Mooi-Reci and Muñoz-Comet 2016; Mooi-Reci and Wooden 2017; Pedulla 2016). However, not all "red flags" related to previous unemployment are of equal weight. When the reason for unemployment is believed to relate directly to a worker's poor performance with the previous employer, chances of reemployment will be much lower for reasons related to unobserved worker quality. Additionally, various studies provide considerable support that unemployment spells experienced far in the past – and thus no longer relevant to the current application – or during an economic downturn tend to carry less of a negative weight compared to spells experienced directly prior to the job application (Mooi-Reci and Ganzeboom 2015; Omori 1997). This means that more recent spells of unemployment will influence more negatively one's employment prospects.

These two potential scarring mechanisms raise the following two-fold question: if unemployment is negatively related to one's future career outcomes, do previously unemployed individuals eventually recover from it, and if so, at what rate? This question will guide our analyses in the next sections.

## 5.1 Data

We use the Household, Income, and Labour Dynamics in Australia (HILDA) Survey, one of the richest and longest running household longitudinal survey in Australia (Summerfield et al. 2015). The HILDA survey commenced in 2001 and data is collected every year among household members 15 years and over. In 2001, the initial sample counted 13,969 persons across 7,682 households, with the main sample remaining about the same in the subsequent data collection waves. The wave-by-wave response rates are exceptionally high in the HILDA survey, with retention rates varying between 86.9% in wave 2 to 96.4% in 2013. The retention rate of the first sample was 67.4% in 2013 (i.e. this is the number of people responding in both the initial wave and the 2013 wave), which is noticeably high. Every year respondents are asked to identify their labor market status at the time of interview, that we recoded into: employment (E), unemployment (U), non-participation/inactivity (N) and retirement (R) (see Summerfield et al. 2015 for a complete list of the questionnaires). Since the focus of this study is on career

sequences, we have restricted the sample to those men and women who are not in full-time education and for whom information on labor force status is available at any interview date over the period 2001–2013. This selection leaves us with a sample of 22,081 person-year observations. Of these 22,081 observations, by far the largest concentration (i.e. 44.42%) is among the group for which we have complete labor force status information for the entire observation period between 2001 and 2013. All the analyses are performed using Stata 14.<sup>1</sup>

*Measures:* Our dependent variable, *binary sequence quality*, is a time-varying measure defined as discussed in Sects. 3 and 4 and captures the quality of career sequences starting from a first unemployment experience up until  $t$ . To investigate the evolution of employment careers after the occurrence of an initial spell of unemployment, we define a variable counting the years elapsed since the first unemployment experience (i.e. our reference category). As we observe respondents up to 12 years following a first unemployment experience, we construct 12 dummies capturing each of the years in which a respondent was observed in any of the labor market statuses. These time-specific dummies allow us to trace the rate of recovery since the initial episode of unemployment.

We also control for socio-demographic characteristics, including variables for age and age squared, and account for human capital with a variable capturing the highest attained educational level at the time of interview. Education is specified as a categorical variable with Year 11 and below (i.e., early childhood education and primary school) as the reference category and six additional categories for Year 12 (i.e., lower secondary school), Cert III or IV (upper secondary school), Advanced diploma (i.e., post-secondary non-tertiary education), Bachelor or honors, Graduate diploma, Postgraduate education. We also include a variable for Gross Domestic Product per capita as proxy of economic growth in a specific year (GDP). Finally, to guard against the possibility that observed career outcomes are driven by career fluctuations that existed prior to the first unemployment (e.g., due to periods of inactivity), we include a variable specifying the career quality before the first unemployment experience in our models. Table 2 shows descriptive statistics for our person-year sample.

## 5.2 Method

A key statistical challenge for our analysis is non-random selection into the initial employment state that is correlated with unobservable traits. To solve this issue, we estimate a model with correlated random effects, also known as the ‘hybrid’ model (Allison 2009). The hybrid model allows time-varying covariates to be decomposed into individual specific means and deviations from these individual-specific means.

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<sup>1</sup>The `sqsuccess` package in Stata implements our quality measure as described in this paper. We thank Ulrich Kohler for developing it.

**Table 2** Summary descriptive statistics

	Men		Women	
	Mean	SD	Mean	SD
Career quality				
$w = 1$	0.44	0.39	0.40	0.38
$w = 0.5$	0.41	0.37	0.38	0.36
$w = 2$	0.47	0.43	0.43	0.42
Career quality before unemployment				
$w = 1$	0.35	0.45	0.31	0.42
$w = 0.5$	0.62	0.43	0.47	0.43
$w = 2$	0.62	0.45	0.47	0.45
Age	35	14.74	36	13.99
GDP	1.5	0.67	1.5	0.68
	Percent		Percent	
Education				
Year 11 or less	35%		36%	
Year 12	21%		20%	
Certificate III or IV	20%		19%	
Advanced diploma	7%		7%	
Bachelor or honors	9%		11%	
Graduate diploma	2%		3%	
Postgraduate	3%		3%	
$N$ observations	11,106		10,975	
$n$ workers	2,137		2,026	

The advantage of the hybrid model is that it corrects for unobserved heterogeneity across the time-varying variables and allows for inclusion of time-constant variables which otherwise would have been dropped from a fixed effect specification (Allison 2009). The model takes the form:

$$\gamma_{it}^w = \alpha + \beta \gamma_{\text{BEFORE } i}^w + D(d_{it} - \bar{d}_i) + \gamma \bar{d}_i + \delta(x_{it} - \bar{x}_i) + \eta \bar{x}_i + \mu_i + \varepsilon_{it} \quad (4)$$

where  $\gamma_{it}^w$  is the measure for sequence quality of worker  $i$  in year  $t$  (which covers the period 2001 to 2013);  $\gamma_{\text{BEFORE } i}^w$  controls for the career quality of individual  $i$  before the time of the first unemployment occurrence and  $\beta$  captures its effect.  $D$  is a vector of coefficients associated with the deviations of the specific time dummies from the overall time cluster mean denoted in the bracket ( $d_{it} - \bar{d}_i$ ), which gives us the within-effect estimates. In addition to these time deviations, the model adds a vector of coefficients ( $\gamma$ ) associated with the time cluster mean ( $\bar{d}_i$ ) to control for dependency of the repeated observations. Next, within-effect estimates for individual characteristics that are supposed to influence the overall sequence quality are captured through  $\delta$ , while ( $x_{it} - \bar{x}_i$ ) refers to the difference between time-varying variables expected to be associated with one’s sequence quality (including

age, education, and GDP) and their individual specific cluster means. Next, a vector of coefficients ( $\eta$ ) associated with the individual specific cluster means  $\bar{x}_i$  is added to the model. As described by Allison (2009) and later by Schunck (2013) this addition is necessary to estimate the evolution of the variations around the mean and to control for correlations between level 1 (i.e., respondents) and level 2 errors (i.e., occasions). Note that interpretation of cluster means is not of interest because we aim at predicting career quality based on the variations of individual characteristics around the mean, which in essence, mimics the logic behind fixed effect models. Finally,  $\mu_i$  refers to the individual specific error (i.e., level 2 error) with  $\varepsilon_{it}$  referring to the level 1 error. To capture sex-specific unemployment effects, we estimate separate models for men and women.

### 5.3 Findings

Table 3 shows, separately for men and women, the estimated coefficients from three hybrid models that are based on three weight specifications of our career quality measure, that is:  $w = 1$ ,  $w = 0.5$  and  $w = 2$ . In each model, the year coefficients show how the rate of career quality of previously unemployed workers evolves in each year following unemployment. To ease the interpretation, Fig. 2 plots the key results from such models. As we clarified above, the weight parameter determines the extent to which the quality measure is affected by a failure, with stronger penalties for failures, but faster recovery for successes the bigger the parameter  $w$ .

Although coefficient estimates from both individual-specific means and their deviations are shown in Table 3, only coefficients from deviation specific means are used for interpretation, because coefficients pertaining to individual specific means have no substantive interpretation (see above and for a review, see Allison 2009).

The large positive coefficient estimates for the year dummies in all three models suggest that, for both men and women, there is a trend of recovery. Specifically, using a  $w$  of 1 among the male sub-sample in Model 1, coefficient estimates indicate an improvement in the career quality with 0.36 points in the first year following the initial unemployment spell. This improvement in the career quality continues in the second and third year with 0.49 and 0.55 points, respectively, before flattening nine years after the first unemployment spell. We find a similar progressively improving trend among women. Interestingly, in the longer run women's patterns of recovery exceed men's. However, from the confidence intervals shown in Fig. 2 (and from tests of interactions, not shown but available upon request) we can establish that gender differences are not statistically significant. Other model specifications with different weights follow expected trends of recovery: using a weight of 0.5 (Model 2) coefficient estimates are lower, indicating less penalizing failures but also slower recovery, while weights of 2 show faster recovery in the years following first unemployment. As mentioned earlier, the choice of the  $w$  estimator is determined by theory and varies depending on the purpose of research. All other parameters included in the model act in the expected direction. Age, which can be interpreted as a proxy for experience, improves the career quality, which is also positively

**Table 3** Coefficients from hybrid models predicting career quality ( $Y_{it}$ ) over time since first unemployment, by sex

	Men			Women		
	Model 1 ( $w = 1$ )	Model 2 ( $w = 0.5$ )	Model 3 ( $w = 2$ )	Model 1 ( $w = 1$ )	Model 2 ( $w = 0.5$ )	Model 3 ( $w = 2$ )
<i>Deviations</i>						
Years since first unemployment						
1	0.364**	0.319**	0.437**	0.349**	0.307**	0.416**
2	0.489**	0.444**	0.550**	0.466**	0.424**	0.520**
3	0.549**	0.508**	0.596**	0.531**	0.492**	0.573**
4	0.587**	0.550**	0.624**	0.569**	0.535**	0.601**
5	0.616**	0.583**	0.646**	0.602**	0.571**	0.628**
6	0.634**	0.604**	0.659**	0.623**	0.595**	0.643**
7	0.653**	0.626**	0.673**	0.645**	0.620**	0.662**
8	0.669**	0.646**	0.685**	0.672**	0.648**	0.688**
9	0.678**	0.658**	0.687**	0.699**	0.675**	0.715**
10	0.688**	0.670**	0.694**	0.715**	0.692**	0.728**
11	0.688**	0.674**	0.689**	0.734**	0.711**	0.744**
12	0.687**	0.678**	0.681**	0.728**	0.711**	0.728**
Education (ref: year 11 or less)						
Year 12	0.047**	0.034*	0.069**	0.093**	0.074**	0.124**
Certificate III or IV	0.019	0.012	0.031	0.118**	0.101**	0.144**
Advanced diploma	0.072*	0.056*	0.099**	0.172**	0.151**	0.200**
Bachelor or honors	0.094**	0.075**	0.126**	0.208**	0.181**	0.246**
Graduate diploma	0.067*	0.056	0.087*	0.192**	0.166**	0.231**
Postgraduate	0.130*	0.108*	0.163*	0.241**	0.212**	0.286**
Age	0.024**	0.023**	0.025**	-0.006	-0.006	-0.005
Age squared	-0.000**	-0.000**	-0.000**	-0.000*	-0.000*	-0.000*
GDP	-0.001	-0.001	0.001	0.003	0.002	0.005
<i>Means</i>						
Years since first unemployment						
1	0.320**	0.280**	0.386**	0.324**	0.283**	0.390**
2	0.529**	0.479**	0.598**	0.419**	0.382**	0.468**
3	0.661**	0.612**	0.716**	0.573**	0.531**	0.621**
4	0.433**	0.406**	0.460**	0.53**	0.492**	0.572**
5	0.811**	0.759**	0.862**	0.432*	0.405*	0.458*
6	0.377	0.366	0.380	0.757**	0.734**	0.768**
7	0.631*	0.618*	0.634*	0.381	0.376	0.373
8	1.211**	1.126**	1.311**	0.898**	0.844**	0.956**
9	0.360**	0.352	0.355	0.773*	0.722*	0.831*
10	0.645**	0.607	0.690	0.170	0.193	0.127
11	0.745**	0.726	0.749	0.192	0.167	0.236
12	0.760**	0.728	0.787	0.390	0.404	0.353

(continued)

**Table 3** (continued)

	Men			Women		
	Model 1 ( $w = 1$ )	Model 2 ( $w = 0.5$ )	Model 3 ( $w = 2$ )	Model 1 ( $w = 1$ )	Model 2 ( $w = 0.5$ )	Model 3 ( $w = 2$ )
Education (ref: year 11 or less)						
Year 12	0.061**	0.056**	0.068**	0.037	0.033	0.041
Certificate III or IV	0.059**	0.053**	0.067**	0.063**	0.057**	0.073**
Advanced Diploma	0.106**	0.098**	0.117**	0.125**	0.112**	0.143**
Bachelor or honors	0.118**	0.109**	0.130**	0.111**	0.101**	0.125**
Graduate dipl	0.070	0.066	0.076	0.144**	0.134**	0.158**
Postgraduate	0.067*	0.063*	0.073*	0.089*	0.084*	0.096*
Age	0.011**	0.010**	0.012**	0.011**	0.010**	0.013**
Age squared	0.000**	0.000**	0.000**	0.000**	0.000**	0.000**
GDP	0.037*	0.033*	0.043*	0.034*	0.031*	0.039*
<i>Other control variables</i>						
Career quality before unemployment						
$w = 1$	0.138**			0.135**		
$w = .5$		0.130**			0.128**	
$w = 2$			0.146**			0.140**
Intercept	-0.271**	-0.247**	-0.304**	-0.285**	-0.255**	-0.326**
R-squared-between	0.541	0.543	0.533	0.424	0.427	0.415
R-squared within	0.583	0.616	0.526	0.509	0.545	0.451
Sigma u	0.166	0.159	0.171	0.188	0.179	0.196
Sigma e	0.183	0.162	0.218	0.191	0.168	0.228
Rho	0.450	0.490	0.381	0.492	0.532	0.425
Chi-squared	6283.534	6153.035	6443.005	4654.766	4643.767	4657.001
$N$	11,106	11,106	11,106	10,975	10,975	10,975

\*\* =  $p < 0.01$ ; \* =  $p < 0.05$

associated with higher levels of education and economic growths; that is, the higher the education level and economic prosperity the higher the probability for previously unemployed men and women to find re-employment. Finally, a high level of career quality before the first observed unemployment is related to higher career quality later in careers.

## 6 Conclusion and Discussion

In this study, we proposed a novel measure of sequence quality, which differs from existing methods of sequence analysis in that it is the first to quantify sequence quality by accounting for the variation in the frequency, duration and recency of failures and successes over the course of a trajectory. The possibility to differently weigh such measure also allows researchers the flexibility to adjust it based on their theoretical considerations. While quality may encapsulate more than positive or negative states, our measure captures a major dimension of quality.

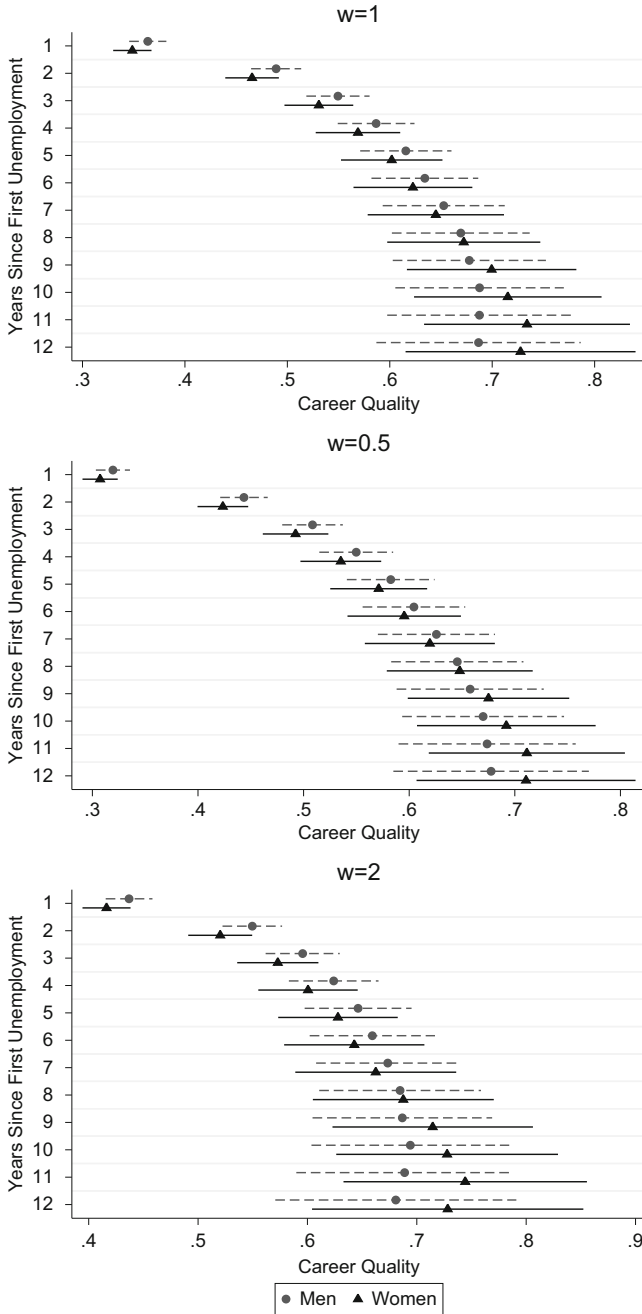


Fig. 2 Post unemployment career quality since first unemployment. Coefficient estimates from hybrid models with different weights, by sex

Drawing on theories about unemployment scarring, we defined states of unemployment and inactivity as failures and those of employment as successes to predict whether prior unemployment lead into descending spirals into inactivity and joblessness or whether patterns of full career recovery exist. We used data from the HILDA survey in Australia over the period 2001–2013 to illustrate the usefulness of our measure and investigate whether unemployment has adverse effects on future careers. Our results reveal no full career recovery among previously unemployed men and women, even over an extended period of 12 years. These results can be explained in the light of human capital depreciation and job mismatching, but signaling-related factors may be equally important in driving labor market disparities.

Some limitations with regard to the application of our measure should be mentioned. First, we applied our measure to the labor force status reported at the time of interview. Though labor force information at the time of interview is less likely influenced by inaccuracy and recall errors than retrospective types of data, it misses important information about labor force changes that occurred in-between the interview dates. Therefore, the measure of failure may be underestimated if workers experienced unemployment or inactivity spells in-between interviews. Further, the data used here are illustrative of binary sequences in which we distinguish between four possible crude labor force states. With more detailed data on people's careers it is possible to study more fine-grained labor force outcomes such as job-to-job changes in the same hierarchical position or moves to better or worse positions. Finally, an interesting issue for future research is to move beyond binary sequences to include more categories and variation in the sequences. Not all outcomes are binary. People can have neutral outcomes or outcomes that are more continuous. Therefore, quantifying the quality of categorical or continuous types of outcomes remains an important avenue for future research.

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