Fourth Order Nonlinear Diffusion Filters for Multiplicative Noise Removal

Mahipal Jetta(✉), Pradeep Nalluri, Preetham Dasari, and Sai Hitesh

Mahindra École Centrale, Hyderabad, India
mahipal.jetta@mechyd.ac.in

Abstract. The second order partial differential equations based multiplicative noise removal filters produce step edges (staircase artifacts) in the filtered image. This paper proposes two fourth order nonlinear diffusion filters, an isotropic filter and an anisotropic filter, which do not allow these artifacts. Through numerical simulations it is shown that the proposed isotropic filter produces the filtered image in a relatively shorter time, with noticeable improvement in the quality, when compared to a second order filter and the proposed anisotropic diffusion filter.

Keywords: Fourth order partial differential equation
Multiplicative noise · Speckle removal · Staircasing artifacts

1 Introduction

The coherent imaging introduces speckle noise in an image. The presence of this noise hinders the performance of high level algorithms like image segmentation and classification. Hence noise removal is an important step in better understanding the image.

The speckle noise present in an image is usually described with a multiplicative model $g = fn$, where g is the observed image, f is the original image and n is multiplicative noise. The aim of a noise removal algorithm is to obtain the best approximation of the original image f. To achieve this goal, initially window based filters were developed [1–3]. In these filters one estimates the noise free image $\hat{f}$ using local mean and local variance of the observed image and the noise variance. A typical model in this direction is

$$\hat{f} = \bar{g} + \kappa(g - \bar{g}) \tag{1}$$

where $\bar{g}$ denotes the local mean of the observed image, and κ is a function of second order moments of g and the noise variance σ_n^2. According to Lee filter [1] $\kappa = \dfrac{v_f}{v_g - \sigma_n^2 v_f}$ with $v_f = \dfrac{v_g - \sigma_n^2 \bar{g}^2}{1 + \sigma_n^2}$, local spatial variance of f, v_g being the local variance of g, at the current pixel location, and noise variance is assumed to be constant throughout the image. In case of Kuan filter [2] the estimated image

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is obtained using (1) with $\kappa = \frac{v_f}{v_g}$. The window based filters remove noise from homogeneous regions effectively but will not remove noise present in the edges.

To better preserve edges and other striking features of the image, while removing multiplicative noise, variational approaches [4,5], patch-based approaches see for example [6,7], and speckle reducing partial differential equations based filters were proposed. In this paper we focus on the partial differential equations based speckle removal filters. The first one among these filters is the speckle reducing anisotropic diffusion filter (SRAD) [8]. This filter was developed by comparing a window based filter with a discrete anisotropic diffusion model. This filter has been discussed briefly in the next section. Many improvised models of SRAD are proposed in the literature, see for example, [9–11], to obtain the best approximation of the underlying speckle free image. Note that these filters are different from the additive noise removal PDE based filters [12,13] as the latter estimates the edges based on gradient which will be biased in case of speckle image and hence produces singal-dependent results, see [10] and the references there in.

Most of these filters succeeded in achieving noise removal and better preservation of edges to some extent. However it is well known, in case of additive noise, that the second order filters produce undesired step edges in the filtered image [14]. In this paper we have shown through numerical simulations that the same conclusion holds for speckle reducing second order PDE filters. To address this issue we propose two new fourth order nonlinear diffusion filters, motivated by the additive noise removal fourth order filters [15], with a diffusivity function depending on the speckle statistics. The performance of these filters has been studied in terms of stair-case artifacts, quality of edge preservation and computation time.

The remainder of the paper is organized as follows. In Sect. 2 we briefly present some of the standard speckle reducing diffusion filters. The two fourth order models are presented in Sect. 3. In Sect. 4 we carry out the numerical simulations. Section 5 constitutes the concluding remarks.

2 Detail Preserving Anisotropic Diffusion Filter

The general speckle reducing diffusion filter takes the following form

$$\frac{\partial u}{\partial t}(\mathbf{x}, t) = \nabla.(D\nabla u(\mathbf{x}, t)) \tag{2}$$

with initial condition $u_0(\mathbf{x}) = u(\mathbf{x}, t = 0)$. Here $u(\mathbf{x}, 0)$ is a noisy image, and D is a symmetric positive definite diffusion tensor which is a function of local statistics of the image. The role of diffusion tensor D is to control the diffusion process depending on the local structure of the image. In general, in both additive and multiplicative noise cases, the quality of the filtered image depends on the choice of diffusion coefficient D [8,12,13].

To remove speckle noise from an image Yu and Acton [8] developed a similar diffusion filter by comparing the discrete form Perona-Malik filter [12] with the Lee's filter [1]. It is formulated as

$$u_t = \nabla.(c(q)\nabla u) \tag{3}$$

with homogeneous Neumann boundary condition and considered noisy image as the initial condition.

Here

$$c(q) = \frac{1}{1 + \dfrac{q^2 - q_0^2}{q_0^2(1+q_0^2)}}, \tag{4}$$

where $q(\mathbf{x};t) = q(x,y;t)$ is the instantaneous coefficient of variation determined by

$$q(x,y;t) = \sqrt{\frac{(1/2)(|\nabla u|/u)^2 - (1/16)(\nabla^2 u/u)^2}{[1 + (1/4)(\nabla^2 u/u)]^2}}, \tag{5}$$

and $q_0(t)$ is coefficient of variation of noise. In homogenerous regions $q \approx q_0$ therefore (3) behaves as a linear diffusion equation which is known to smooth the image. It has been realized that the estimation of $q_0(t)$ is crucial in obtaining the quality image [9–11]. The better version of this PDE, called the detail preserving anisotropid diffusion (DPAD) [9], can be obtained by considering

$$c(q) = \frac{1 + \dfrac{1}{q^2}}{1 + \dfrac{1}{q_0^2}}, \tag{6}$$

in Eq. (3).

We can see that the traditional anisotropic speckle reducing filters are of second order type. Hence they tend to generate stair-case artifacts in the filtered image, see Fig. 1. To address this issue we propose two new fourth order diffusion filters, incorporating the speckle statistics obtained using mode operator [9].

3 Proposed Filters

To remove noise from an image while preserving the edges and without generating the staircase artifacts, we propose two new filters.

3.1 Fourth Order Anisotropic Diffusion Filter

The first filter in achieving the desired goal is a fourth order anisotropic diffusion filter, motivated by Hajiaboli [15] additive noise removal model, which is given by

$$u_t = -\Delta(c(q)^2 u_{\eta\eta} + c(q)u_{\xi\xi}), \tag{7}$$

with the same initial and boundary conditions as in (3). Here Δ denotes the Laplacian operator and the diffusion coefficient $c(q)$ is as given in (6), and

$$u_{\eta\eta} = \frac{u_{xx}u_x^2 + 2u_x u_y u_{xy} + u_{yy}u_y^2}{u_x^2 + u_y^2} \tag{8}$$

and

$$u_{\xi\xi} = \frac{u_{xx}u_y^2 - 2u_x u_y u_{xy} + u_{yy}u_x^2}{u_x^2 + u_y^2}. \tag{9}$$

We will abbreviate this filter as prop1 in the rest of the paper. This is an anisotropic filter as it allows varying diffusion strengths in the directions of gradient $\left(\eta = \frac{[u_x\ u_y]}{\sqrt{u_x^2 + u_y^2}}\right)$ and level set $\left(\xi = \frac{[-u_y\ u_x]}{\sqrt{u_x^2 + u_y^2}}\right)$. The presence of second order directional derivatives in the directions of η and ξ, that is $u_{\eta\eta}$ and $u_{\xi\xi}$, in the filter steers the diffusion process unevenly in two orthogonal directions η and ξ. The diffusion is more in the level set direction than in the gradient direction thereby the diffusion process removes noise without harming the edges. Moreover the noise present in an edge will also be removed. To address the multiplicative noise, we consider the diffusivity coefficients of $u_{\eta\eta}$ and $u_{\xi\xi}$ in the filter as functions of noise level which can be estimated through, for example, the techniques provided in [9] or using principal component analysis (PCA) [16]. This filter performs well in preserving the edges and avoids stair-case artifacts to a larger extent than second order filter. However, the filtering of texture-rich images consumes more time to remove noise from the image. Moreover, in case of high noise level, it loses the sharpness of features of the image due to directional smoothing, see Fig. 1. Apart from these drawbacks, it involves computation of several partial derivatives which in turn increase the computational effort. Therefore we propose an alternative fourth order diffusion filter which is described in the following subsection.

3.2 Fourth Order Isotropic Diffusion Filter

To balance the trade-off between computational effort and positive attributes of prop1 we propose a fourth order isotropic diffusion filter, herein after called prop2 filter, which has a few partial derivatives. This filter is given by

$$u_t = \Delta(c(q)\Delta u) \tag{10}$$

where $c(q)$ is as mentioned in (6) and with homogeneous Neumann boundary condition. This filter removes noise and preserves ramp edges as Δu is minimum whenever we have planar approximation to the initial image. In homogeneous regions $c(q) \approx 1$ and hence becomes a biharmonic filter which is known to damp the high frequencies in a short time and hence eliminates noise efficiently. The performance of all the considered filters is demonstrated in the following section.

4 Numerical Experiments

In this section we demonstrate the performance of the proposed fourth order filters, and compare the results with the detail preserving anisotropic diffusion (DPAD), (3) along with (6). Here all the simulations were carried out on a workstation (Intel Xeon(R) @ 2.20 GHz × 20) for a fair comparison.

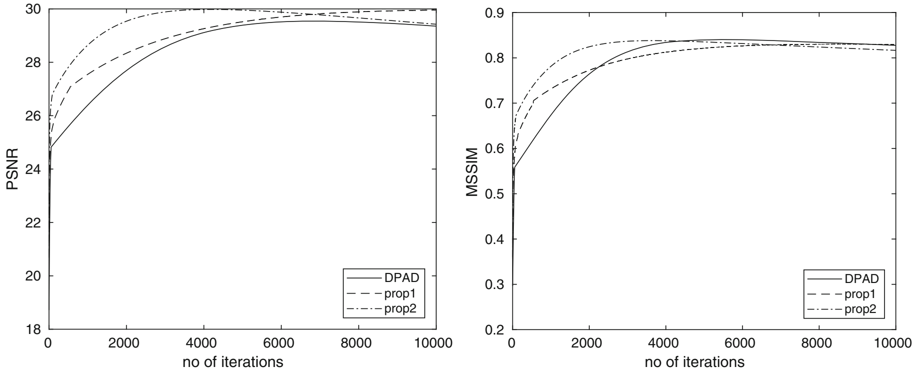
Fig. 1. Denoising results with various filters: (a) Lena image corrupted with a speckle noise of *variance* 0.05, (b) using DPAD, (c) prop1 (d) prop2

The discrete versions of DPAD, prop1, and prop2 are obtained by employing central difference approximation for space derivatives and forward difference approximation for time derivative. This type of discretization produces the explicit scheme for a given diffusion filter. It is well known that this scheme demands a heavy restriction on time step size to obtain stable results. Hence we have taken the time step sizes to be 0.15, 0.015 for DPAD and the proposed filters respectively.

In Fig. 1, we considered Lena image (f) and added multiplicative noise (n) to it, using the equation $g = f + n * f$, where n is uniformly distributed random noise with mean 0 and variance 0.05. Considering this as an initial condition and evolving according to DPAD, we can see the piecewise constant approximation of the image. The obtained filtered image does not look visually appealing as it contains false edges and sudden change of intensity values in homogeneous regions. Note that these artifacts cannot be seen in other two images which are obtained using proposed fourth order filters. The same can be observed in Fig. 2. We can also see that the edges are better preserved with prop1 filter than with prop2 filter. This is due to fact that the prop1 filter allows the diffusion along the level set (edge direction) and inhibits the smoothing across the edge. However, in case of high noise level and texture-rich images, this filter takes longer time to produce quality filtered image than the second order filter, see Table 1 and Fig. 3. The prop2 filter provides noticeable improvement in the filtered image in terms of reduction of step edges. Also the processed image is obtained in a very short time when compared with the other two filters, see (3). To check the performance of the proposed filters quantitatively we consider two well known quality checking measures, namely SSIM and PSNR. The higher the values of these measures means the better the quality. Table 1 shows that the prop2 filter produces a better quality image in terms of PSNR, for different noise levels, when compared DPAD. In all the simulations we have stopped the iteration process whenever the filtered image attains the maximum PSNR. It is clear from these figures that the prop2 filter is a better choice among all the three regarding computational effort and quality of filtered image.

(a) (b) (c) (d)

Fig. 2. Denoising results with various filters: (a) Peppers image corrupted with speckle noise of *mean* zero and *variance* 0.01, (b) using DPAD, (c) prop1 (d) prop2



(a) (b)

Fig. 3. Quantitative performance of diffusion filters using (a) PSNR (b) MSSIM

Table 1. Comparison of various filters

Image	Method	ssim (iterations)	psnr (iterations)	CPU time
Variance = 0.01				
Lena	dpad	0.8931 (1247)	33.14 (2069)	217
	prop1	0.8841 (1334)	33.31 (1496)	157
	prop2	0.8908 (665)	33.50 (908)	95
Peppers	dpad	0.9254 (927)	31.62 (1444)	137
	prop1	0.9221 (515)	31.97 (638)	66
	prop2	0.9266 (425)	32.06 (573)	57
Variance = 0.05				
Lena	dpad	0.8401 (5437)	29.59 (7112)	873
	prop1	0.8298 (8326)	29.95 (9981)	1086
	prop2	0.8378 (3668)	30.01 (908)	433
Peppers	dpad	0.8728 (3417)	27.75 (5120)	461
	prop1	0.8728 (2784)	28.31 (5461)	432
	prop2	0.8785 (1804)	28.28 (2322)	195

5 Conclusion

This paper is an attempt to overcome the artifacts of a second order speckle reducing anisotropic diffusion filter. The development of two new filters is achieved by incorporating speckle statistics into the diffusion process. The first one is an anisotropic diffusion filter which is shown to remove speckle noise effectively and preserve important features present in the image without creating false edges. But this filter requires high computational effort as it involves several partial derivatives. Hence we proposed an alternative isotropic filter which is shown to produce the speckle free image, without step edges, in a shorter time than the detail preserving anisotropic diffusion filter and the fourth order anisotropic diffusion filter. In future we wish to develop an efficient and reliable scheme for the proposed isotropic filter.

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