

Chapter 10

Imagination Acts



In various learning situations, mathematics students are required to perform what I call “imagination acts.” For instance: When studying Euclidean geometry they are required to think about a Euclidean point, which has no dimensions (no width and no length). This is in contrast to a physical point, which has dimensions, regardless of the sharpness of the pencil you use to draw it. Thus, when they perform a geometrical construction with a ruler and a compass they should bear in mind that what they draw is only a physical approximation of the “real” thing, the Euclidean configuration.

Moreover, we are asked to imagine a straight line which can be extended infinitely in both directions. However, in our physical world all straight lines are finite. When dealing with the density axiom (between any two points on a straight line there is another point and therefore, as a consequence, there are infinitely many points between any two given points on a straight line) we should bear in mind that the density of points on a Euclidean line is essentially different from the density of points on a physical line. In any given segment of a physical line, there are only a finite number of points.

Unfortunately, there are some students who are not capable of performing these imagination acts. These students should be advised to study a minimal amount of mathematics. We should be extremely careful not to consider them stupid or unintelligent. From my own experience, I can tell that these students can be very successful in history, literature, art, music, and many other domains in which mathematics is not required.

Some more examples of imagination acts which will be required in the next chapter: Can you imagine an infinite hotel whose rooms are across an infinite straight line (like the number line)? Can you imagine a group of infinitely many guests who come to stay in this hotel? If you cannot—just skip the chapter about the weirdness of infinity.