Chapter 2
Embedded Observers and Self-expression

Empirical evidence can solely be drawn from operational procedures accessible to embedded observers. Embeddedness means that intrinsic observers have to somehow inspect and thus interact with the object, thereby altering both the observer as well as the object inspected.

Physics shares this feature with computer science as well as the formalist, axiomatic approach to mathematics. There, consistency requirements result in limits of self-expressivity relative to the axioms [326, 573] (if the formal expressive capacities are "great enough"). Indeed, as expressed by Gödel (cf. Ref. [549, p. 55] and [210, p. 554]), *"a complete epistemological description of a language A cannot be given in the same language A, because the concept of truth of sentences of A cannot be defined in A. It is this theorem which is the true reason for the existence of undecidable propositions in the formal systems containing arithmetic."*

A generalized version of Cantor's theorem suggests that non-trivial (that is, non-degenerate, with more than one property) systems cannot intrinsically express all of its properties. For the sake of a formal example [573, p. 363], take any set S and some (non-trivial, non-degenerate) "properties" P of S. Then there is no onto function $\mathsf{S} \longrightarrow \mathsf{P}^{\mathsf{S}}$, whereby[1] P^{S} represents the set of functions from S to P. Stated differently, suppose some (nontrivial, non-degenerate) properties; then the set of all conceivable and possible functional images or "expressions" of those properties is strictly greater than the domain or "description" thereof.

[1]An equivalent function is $\mathsf{S} \times \mathsf{S} \longrightarrow \mathsf{P}$. Every function $f : \mathsf{S} \longrightarrow \mathsf{P}^{\mathsf{S}}$ can be converted into an equivalent function g, with $g : \mathsf{S} \times \mathsf{S} \longrightarrow \mathsf{P}$, such that $g(a_1, a_2) = [f(a_2)](a_1) \in \mathsf{P}$. One may think of a_2 as some "index" running over all functions f.

A typical example is taken from Cantor's proof that the (binary) reals are non-denumerable: Identify $\mathsf{S} = \mathbb{N}$ and $\mathsf{P} = \{0, 1\}$, then $\{0, 1\}^{\mathbb{N}}$ can be identified with the binary reals in the interval $[0, 1]$. Any function $f(n) = r_n$ with $n \in \mathbb{N}$ and $r_n \leq\in [0, 1]$ representable in index notation as $r_n = 0.r_{n,1}r_{n,2}\ldots r_{n,k}\ldots$ can be rewritten as $[f(n)](k) = g(n, k) = r_{n,k}$.

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For the sake of construction of a "non-expressible description" relative to the set of all functions $f : \mathbf{S} \longrightarrow \mathbf{P}^{\mathbf{S}}$, let us closely follow Yanofsky's scheme [573]: suppose that, for some non-trivial set of properties $\mathbf{P}$ we can define (that is, there exists) a "diagonal-switch" function $\delta : \mathbf{P} \longrightarrow \mathbf{P}$ without a fixed point, such that, for all $p \in \mathbf{P}$, $\delta(p) \neq p$. Then we may construct a non-f-expressible function $u : \mathbf{S} \longrightarrow \mathbf{P}^{\mathbf{S}}$ by forming

$$u(a) = \delta(g(a, a)), \tag{2.1}$$

with $g(a, a) = [f(a)](a)$.

Because, in a proof by contradiction, suppose that some function h expresses u; that is, $u(a_1) = h(a_1, a_2)$. But then, by identifying $a = a_1 = a_2$, we would obtain $h(a, a) = \delta(h(a, a))$, thereby contradicting our property of δ. In summary, there is a limit to self-expressibility as long as one deals with systems of sufficiently rich expressibility.