Chapter 1
Building a Strong Foundation Concerning Whole Number Arithmetic in Primary Grades: Editorial Introduction

Maria G. Bartolini Bussi and Xu Hua Sun

Il ne s'agit pas là de philosophie comparée, par mise en parallèle des conceptions; mais d'un dialogue philosophique, où chaque pensée, à la rencontre de l'autre, s'interroge sur son impensé
('This is not about comparative philosophy, about paralleling different conceptions, but about a philosophical dialogue in which every thought, when coming towards the other, questions itself about its own unthought' (Jullien 2006, p. vi))

他山之石,可以攻玉 *(tā shān zhī shí, kěyǐ gōng yù)*
('The stone from another mountain can be used to polish one's own jade' (Xiao Ya, Shijing: He Ming, 1000 A. C.))

1.1 Introduction

After more than five years of collaboration on whole number arithmetic (WNA), we summarise our experiences, focusing on the process, the merits and the limits of the ICMI Study 23, together with the potential for future activity and for addressing different kinds of audience. We have not worked alone. A very knowledgeable and helpful International Program Committee (IPC) shared the whole process of preparation of this volume. We wish to thank them all for their long-lasting (and not yet finished) collaboration; although, obviously, the responsibility for some delicate choices and possible mistakes and misunderstandings is left to the two of us.

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The two epigraphs above, from the French philosopher and sinologist François Jullien and from an ancient Chinese saying, summarise our attitude now. This international study has offered us the opportunity to increase our knowledge and start two complementary processes:

– Becoming aware of some deep values of our own culture (our 'unthought') which we may have considered in the past the only possible choice or, at least, the most suitable choice for an ideal 'human nature'.
– Considering the possibility of introducing into our own practices, beliefs and values (our 'jade stone') the processes of innovation, not copied from but influenced by practices, beliefs and values of another culture.

The Study Volume is an account of the collective memory of participants offered to the wider community of primary mathematics educators, including researchers, teachers, teacher educators and policymakers. It is a product of fruitful collaboration between mathematicians and mathematics educators, in which, for the first time in the history of ICMI, the largely neglected issue of WNA in primary school has been addressed. The volume reports all the activities of the Conference. Many co-authors, who were involved in a collective co-authorship, are listed at the end of the volume.

1.2 The ICMI Study 23

1.2.1 The Rationale of the Study

Primary schooling is compulsory in all countries, with different facilities and opportunities for children to take advantage of it. Mathematics is a central subject in primary mathematics education, and the delivery of the mathematics curriculum is important in all countries for the different kinds of citizens and the different kinds of competences each seeks to produce. In the proceedings of a recent workshop organised by the National Academies of Science, Engineering and Medicine, held in November 2016, to explore the presence and the public perception of the Social and Behavioural Science (SBS) in K-12 education, a research survey was conducted that compared public knowledge and attitudes towards the natural sciences and social sciences, using a representative national sample of 1000 adults (balanced in terms of age and gender). Besides questions on SBS, the survey included questions about STEM (Science, Technology, Engineering and Mathematics). More than 30% of respondents opined that mathematics and science education should begin in elementary school or earlier with a strong preference for mathematics in both preschool and elementary school.[1]

[1] http://nap.us4.list-manage.com/track/click?u=eaea39b6442dc4e0d08e6aa4a&id=99397b4537&e=f0cb5232c5

WNA and related concepts form the basis of mathematics content covered in later grades. WNA in primary school lays the foundation for secondary school. It is one of the goals of education for all and a part of the UN Global Education First Initiative (UNESCO 2012). Consistently, the volume titled *Building the Foundation: Whole Numbers in the Primary Grades* aims to convey the message of the importance of laying a solid foundation of WNA as early as possible for further mathematics learning.

1.2.2 The Launch of the Study

A reflection on primary school mathematics was considered timely by the ICMI Executive Committee (EC) (term 2010–2012). The theme of the study was defined as follows:

The beginning of the approach to whole numbers, including operations and relations, and the solution of arithmetic word problems, in schools (and possibly preschool environments), up to Grade 3 or more, according to the various education systems

Although it is not the only topic relevant to primary school mathematics, WNA was chosen by the EC of ICMI to focus on a shared centrality in primary school mathematics curricula all over the world.

The study was launched by ICMI at the end of 2012, with the appointment of two co-chairs and the IPC, which, on behalf of ICMI, was responsible for conducting the study:

Maria (Mariolina) G. Bartolini Bussi, Italy, and Xu Hua Sun, Macao SAR, China (co-chairs);

Berinderjeet Kaur, Singapore; Hamsa Venkat, South Africa; Jarmila Novotná, Czech Republic; Joanne Mulligan, Australia; Lieven Verschaffel, Belgium; Maitree Inprasitha, Thailand; Sybilla Beckmann, USA; Sarah Inés González de Lora Sued, Dominican Republic; Abraham Arcavi, Israel (ICMI Secretary General); Ferdinando Arzarello, Italy (ICMI President); Roger E. Howe, USA (ICMI liaison)

1.2.3 The Discussion Document

During 2013, an intense mail exchange within the IPC established and shared the rationale, the goals and the steps of the forthcoming study. In January 2014, an IPC meeting took place in Berlin, at the IMU Secretariat, which generously supported the costs. The IPC members were welcomed by Prof. Dr. Jurgen Sprekels, Director of the Weierstrass Institute for Applied Analysis and Stochastic (WIAS, Berlin), and by the then ICMI President Prof. Ferdinando Arzarello, who participated in the meeting and, later, in the entire Study Conference.

The meeting in Berlin took place in a productive and collaborative atmosphere.

A Discussion Document (this volume, Appendix 2) including a call for papers for the Study Conference was prepared, with the Study Conference announced for June 2015 in Macao (SAR China). This document summarised issues that were considered important to discuss in the study. Emphasis was given to the importance of cultural diversity and its effects on the early introduction of whole numbers. In order to foster understanding of the different contexts in which authors had developed their studies, each applicant for the study was required to fill a specific *context form* in order to include background information about their submission's context (this volume, Chap. 2).

Five themes (each corresponding to a working group in the Conference) were identified and assigned to pairs of members of the IPC:

1. *The why and what of whole number arithmetic*
2. *Whole number thinking, learning and development*
3. *Aspects that affect whole number learning*
4. *How to teach and assess whole number arithmetic*
5. *Whole numbers and connections with other parts of mathematics*

Three plenary panels were identified:

1. *Traditions in whole number arithmetic*, chaired by Ferdinando Arzarello
2. *Special needs in research and instruction in whole number arithmetic (WNA),* chaired by Lieven Verschaffel
3. *Whole number arithmetic and teacher education,* chaired by Jarmila Novotná

Three plenary speakers were invited: Hyman Bass, Brian Butterworth and Liping Ma.

The intention of the IPC was to offer a map of some important issues related to WNA, crossing the borders of countries and regions. The aim was to foster reflections among participants (and, subsequently, among the readers of the volume) on their own cultural contexts, with representation in the Conference and the volume of sources from a wide range of geographical and socio-economic contexts. Cole's (1998) book on *Cultural Psychology* affirms the need for this kind of range:

> *In recent decades many scholars whose work I discuss have sought to make the case for a culture-inclusive psychology. They argue that so long as one does not evaluate the possible cultural variability of the psychological processes one studies, it is impossible to know whether such processes are universal or specific to particular cultural circumstances. For examples, John and Beatrice Whiting, anthropologists with a long-term interest in human development, wrote: 'If children are studied within the confines of a single culture, many events are taken as natural, or a part of human nature, and are therefore not considered as variables. It is only when it is discovered that other people do not follow these practices that have been attributed to human nature that they are adopted as legitimate variables'.* (p. 2)

The temptation of a narrow and local perspective is a risk for mathematics educators too, given the enormous advantages that mathematics developed in the West in recent centuries has had on the development of science, engineering and technologies. This study aimed at challenging some of these beliefs with a short, yet lively, immersion in an atmosphere where a more open mind is needed, at least when

discussing early year mathematics and where the strong links with everyday life and cultural traditions come into play.

1.2.4 The Study Conference

By the end of the selection process, 67 papers were accepted and distributed over the five themes. For each accepted paper, a maximum of two co-authors were invited to participate in the Study Conference. A volume of proceedings was edited by Xu Hua Sun, Berinderjeet Kaur and Jarmila Novotná (Sun et al. 2015).

Thanks to generous support from the University of Macau, the Education and Youth Affairs Bureau, Macao SAR and ICMI, for the first time the ICMI Study 23 was able to invite observers from developing countries. A choice was made to privilege *Capacity and Networking Project (CANP)* participants who comprise the major developmental focus of the international bodies of mathematicians and mathematics educators (this volume, Appendix 1). Other observers came from the Great Mekong Area and China. The total number of participants was 91 from 23 countries.

The Study Conference was held on June 3–7, 2015, in Hengqin Campus, University of Macau, leased to Macao by the State Council of the People's Republic of China in 2009 for the construction of the new campus. The Conference was opened by Prof. Zhao Wei, Rector of the University of Macau. Addresses were given by Mr. Wong Kin Mou, Representative of Director of the Education and Youth Affairs Bureau and Chief of Department of Research and Educational Resources of Macao SAR; Prof. Shigefumi Mori, President of IMU; Prof. Ferdinando Arzarello, President of ICMI; and the co-chairs (the co-authors of this chapter).[2]

1.2.5 The Study Volume

The ICMI Study Conference served as the basis for the production of this Study Volume, edited by the two co-chairs of the study. The five themes identified in the Discussion Document (this volume, Appendix 2) were assigned to pairs of members of the IPC, who took part in the selection of the submitted papers and the organisation of the five working groups in the conference. As is the tradition with the ICMI studies, the IPC members who led the working groups proceeded to lead the writing of the corresponding chapter and to synthesise and integrate the papers presented in the group alongside the subsequent discussions. Unfortunately, due to health reasons, Sarah Inés Gonzáles de Lora Sued was not able to take part in the Conference. During the writing process, Christine Chambris kindly accepted to take Sarah's role.

[2] A gallery of photos from the Study Conference is available at: www.umac.mo/fed/ICMI23/photo.html.

A short summary of the volume follows.

The introductory part addresses some background issues.

The diversity of contexts (Chap. 2) addresses the growing importance of understanding the role of the social and cultural context in which the teaching and learning of mathematics is situated. The process that led the IPC to prepare a *context form* for each submitted paper is reported, together with a short analysis of the collected forms. This information is important to understand the perceptions of the contributors involved in writing this volume.

The diversity of languages (Chap. 3) addresses a feature that emerged in working groups and plenary panels as well. The richness of cultural contexts allowed participants to discuss possible linguistic supports or limitations that may interfere with students' mathematics learning and teacher education. The participants were informed by the working group leaders (when appropriate) that their contribution to the language discussion would have been summarised in an editorial chapter by the co-chairs, mentioning their contribution in the proceedings. A large part of the chapter is devoted to the Chinese case that is different from many other languages.

Chapter 4 is a commentary paper prepared by an acknowledged scholar in the field, David Pimm. He was not able to participate in the Conference, but was kindly willing to write a commentary chapter.

The working groups' part comprises 10 chapters, organised in pairs. The working groups' chapters are co-authored by the IPC members who led the group together with listed participants, and the different levels of collaboration during the writing process are acknowledged as mutually agreed. The odd-numbered chapters (Chaps. 5, 7, 9, 11 and 13) report, in order, the outputs of the discussions of the five working groups. Each of these chapters is followed (in the even-numbered chapters) by a commentary paper authored by an acknowledged scholar with expertise in the field of whole number arithmetic, who did not take part in the Conference and thus offered a different perspective on the study's key themes: Roger Howe (Chap. 6), Pearla Nesher (Chap. 8), Bernard Hodgson (Chap. 10), Claire Margolinas (Chap. 12) and John Mason (Chap. 14).

The panel part includes three panels (Chaps. 15, 16 and 17), which aimed to address some transversal issues (traditions, special needs and teacher education) that cut across the working group foci, with the participation of most members of the IPC exploiting their areas of expertise and of some other invited participants, including a discussant for each panel.

The plenary presentation part includes three plenary presentations (Chaps. 18, 19 and 20) which aimed at addressing WNA from three different perspectives: that of a professional mathematician and past ICMI president (Hyman Bass), that of a neurocognitive scientist with research on developmental dyscalculia (Brian Butterworth) and that of a scholar in mathematics education with expert knowledge of Chinese and US traditions (Liping Ma).

Three appendixes are included in the volume – the first related to the CANP participants' reflections, the second related to the Discussion Document of the ICMI Study 23 and the third related to the electronic supplementary material (videos).

1.3 Merits of the Study

The ICMI Study 23 has seen merits from both organisational and scientific perspectives.

The Study Conference was located in Macao SAR, the right place for many reasons. First, in recent years, the outstanding performance of Chinese students in the OECD PISA mathematics assessment was debated all over the world. In particular, Macao SAR's performance rose from 15th position in 2009 to 3rd position in 2015. Knowing more about this performance is of interest to all mathematics educators.

But there are other reasons. Macao is known as the place of a dialogue between Portugal and China, between European and Eastern cultures. Contacts between Asia and the West started along the Silk Road even before the Common era (BCE).[3] As from the thirteenth century, numerous traders – most famously the Italian Marco Polo – had travelled between Eastern and Western Eurasia. In the sixteenth century (1552), St Francis Xavier, a Navarrese priest and missionary and co-founding member of the Society of Jesus, reached China. Some decades later, the Italian Jesuit Matteo Ricci reached Macao. He introduced Western science, mathematics, astronomy and visual arts into China and carried on significant intercultural and philosophical dialogue with Chinese scholars, particularly representatives of Confucianism.

> Matteo Ricci (1552–1610) is known as the initiator of the Catholic missions in China and one of the earliest members of the Society of Jesus. Others before him ventured towards China, but did not succeed in remaining there for life, let alone to receive the respect and admiration from the Chinese people that Ricci enjoys even to this day. The root of Ricci's success lies in his achieved integration as a human person that made it possible for him to enter so fully into another culture without losing himself. The Society of Jesus and Macao, in many ways, share together a common beginning and 450 years of history. The Jesuits in Macao have always been at the service of the human person, either in need of education or material help, but always at the very deepest level of ideals and hopes, where culture finds its roots. This Jesuit tradition continues even today in Macao at the Macao Ricci Institute.[4]

The Ricci Institute was visited by the participants in the Study as a part of the social programme, with lectures by Man Keung Siu about the role of Matteo Ricci in introducing elements of European mathematics into China. Among these were the first six books of *Euclid's Elements* and the first arithmetic book on European pen calculation. These translations changed Chinese mathematics education and gave Chinese people their first access to real images of Western mathematics (Chap. 15).

This intercultural dialogue is evident not only in the architecture of the old city, the parallel entrance corridor of the Macao museum[5] and the road signs (written in Chinese and Portuguese), but also on the new Hengqin Campus,[6] where the Study

[3] www.ancient.eu/Silk_Road/

[4] www.riccimac.org/eng/introduction/index.htm

[5] www.macaumuseum.gov.mo/w3ENG/w3MMabout/MuseumC.aspx

[6] www.umac.mo/about-um/introduction/about-the-university-of-macau.html

Fig. 1.1 The customised *suàn pán* for the participants in the conference

Conference was held. Hence, the participants were physically embedded in intercultural dialogue. We believe that this heritage of mixed traditions under the influence of the Confucian educational heritage can provide a resource for new thinking in global mathematics education development. In all the working groups and the panels, the discussion was lively, and the presence of the Chinese culture was evident: the colleagues from the Chinese areas discussed their own perspectives, often different from the others' and still connected with the classical tradition. Interestingly, a special gift was offered to all participants: a *suàn pán* (算盘), the famous Chinese abacus, added in 2013 to UNESCO's intangible heritage list (see Fig. 1.1).

A central part of the social programme was the visit to two first grade classrooms to observe lessons on addition and subtraction, according to the typical Chinese tradition of *open classes* (*guānmó kè*, 观摩课), where many observers (several dozens in our case) observed a lesson, with a carefully organised teaching plan distributed in advance, and discussed with the teachers later in order to improve the lesson for the future. The participants showed great interest in this lively observation of a Chinese classroom, it is described at length in one chapter of the Study Volume (Chap. 11), and commented on from a Western perspective (Chap. 12). The immersion in a culture so different from that of most participants led to a sharing of some features of a range of different traditions, providing a much broader and deeper airing of what is known in the literature. Comparisons between Chinese and Western cultures of education have become relatively common in the international literature (e.g. Gardner 1989; Stevenson and Stigler 1992), but most participants at the Study Conference had never had personal experience in this field. The meeting of different *cultural traditions* was reconsidered in a specific panel chaired by Ferdinando Arzarello (Chap. 15).

An innovation related to our central attention to culture was that during the Study Conference, in some working groups, short *video clips* about classroom episodes were shown by the participants, who had agreed to prepare them with English subtitles. The vivid impression that a video clip can give of classroom life and of the implicit culture is different from what is discernible in a written paper. While access to video clips was constrained by the need to meet permission, privacy and ethical rules (where these too are culturally dependent on different countries' laws and

norms), and by the resources available to prepare the English transcripts required to make the video clips understandable in the context of an international conference, we collected a small gallery of video clips that can be enlarged in the future. References to particular video clips appear across the volume as electronic supplementary material (see also Appendix 3) and are available on the publisher website.

Our attention to *contexts* and different cultural traditions is one of the major merits of this study, in place from the beginning in the Discussion Document (this volume, Appendix 2). It is worth noting our increased emphasis on what previously was considered more as a 'special interest' rather than a core feature: for instance, a plenary panel on *Cultural contexts for European research and design practices in mathematics education* (Jaworski et al. 2015) was hosted by CERME 9 (the Conference of the European Society for Research on Mathematics Education, held in Prague in 2015) and a plenary address was given by Bill Barton on *Mathematics education and culture: a contemporary moral imperative* at ICME 13.[7] The direction seems to be right but the way remains long.

The issue of *languages* and their influence on WNAs was considered in different working groups and was summarised in a specific chapter (Chap. 3). Perspectives on WNA in relation to history, language and societal changes were also discussed in Chap. 5 and Chap. 6.

During the process, the IPC felt that the traditional limits on how WNA is perceived did not afford adequate recognition to the *connections* existing between different mathematical areas, for instance, the connection between arithmetic and algebra. Two chapters (Chaps. 13 and 14) address this issue.

Teacher education and development in relation to WNA was addressed in a panel (Chap. 17), complementing the ICMI Study 15 (Even and Ball 2009), thereby filling a gap in that the earlier study made little reference to primary level in the Study Volume.

S*pecial needs* were addressed in a panel (Chap. 16) that drew on the contribution of Chap. 7, reporting on neurocognitive, cognitive and developmental approaches. It represents a first step into a desirable and better dialogue between scholars from different communities, that is mathematics educators and (neuro-)cognitive scientists. WNA has been a hot topic in the field of psychology. Yet, studies carried out from the perspective of classroom teaching are relatively rare, and most studies are conducted in experiment rooms, with risks of limited application to classroom teaching and instruction. This study has thus started to build important discussions.

The issue of *early childhood* settings is considered in the chapters focusing on observation studies (Chap. 7) and intervention studies (Chap. 9). The importance of supporting literacy in these early childhood settings is widely accepted; but, historically, mathematics has often been viewed by many as unimportant to, or developmentally inappropriate for, young children's learning experiences: for example, current US state standards for early childhood do not include much mathematics (National Research Council 2009). More generally, many early childhood pro-

[7] https://lecture2go.uni-hamburg.de/l2go/-/get/v/19757

grammes spend little focused time on mathematics and are accompanied by concerns of low instructional quality. Many opportunities are therefore missed for learning mathematics. A key exception is represented by the proactive stance of the European Society for Research in Mathematics Education, which has, from 2009, included a specific working group on *Early Years Mathematics* meeting every second year (Levenson et al. in preparation). A plenary talk on *Towards a more comprehensive model of children's number sense* by Lieven Verschaffel, member of the IPC of the ICMI Study 23, was also presented at CERME 10[8] in Dublin.

Last but not least, a further merit of the Study is the involvement of *CANP representatives* as observers. This group has acknowledged (Appendix 1) the importance of the Study Conference where each of them was assigned to a working group, ensuring dialogue between them and the other participants. They also had a formal meeting with the ICMI President, Ferdinando Arzarello, during which, for the first time, experiences across CANPs were shared. Veronica Sarungi (personal communication), representative of CANP4, noted in her reflections:

> One of the major contributions of the ICMI Study 23 was to enable the CANPs to build networks beyond their regions. As a result of connections formed in Macao, a discussion group proposal was submitted and accepted for ICME-13 that will focus on CANPs. Apart from networking, the meeting in Macao enhanced the individual capacity of the representatives that had an effect on their respective institutions, national and regional associations.

This friendly and supported introduction into the international community of mathematics educators has already contributed to broadened participation in other ICMI activities and regional conferences and meetings of affiliated organisations such as CERME.

1.4 Impact of the Study

Overall, the impact of the study is promising. Some communities indicated their interest before the Study Conference (e.g. Bartolini Bussi and Sun 2014; Beckmann 2015). After the Conference, reports (by invitation) have appeared in key journals (*European Mathematical Society Newsletter*, in English; *Mathematics Education Journal*, in Chinese; the *Bulletin of CFEM*, in French) and conference proceedings (*Copirelem*, Bartolini Bussi and Sun 2015; *SEMT 2015*, Novotná 2015). An official report has been published by *L'Enseignement Mathémati*que (Bartolini Bussi and Sun 2016). A report on "ICMI Study 23 on Whole Number Arithmetic" was given by Roger Howe at NCTM 2017. A presentation of the Study Volume was also held in 2016 in a special timeslot at ICME 13 in Hamburg.[9]

The intercultural dialogue between mathematics educators interested in WNA for the primary school continues in international conferences (such as SEMT, taking

[8] www.cerme10.org

[9] https://lecture2go.uni-hamburg.de/l2go/-/get/v/19768

place every second year in Prague[10]) and at ICME, where specific groups are organised every fourth year. Moreover, the Inter-American Conference on Mathematics Education (IACME), taking place every 4 years, has a special section on primary mathematics education, and WNA is an important part of it.

1.5 Limits of the Study

The aim of constructing a map of the main educational aspects of WNA has been partially fulfilled in the Study Conference and in this Volume, with a wide multicultural approach. Some themes have been deepened and some others have been opened up as new avenues that currently are simply sketched.

The issue of *textbooks* within the teaching of WNA is touched upon in some of the chapters (Chaps. 9 and 11), but would deserve a whole study in its own (Jones et al. 2014[11]).

The issue of *assessment* of and for WNA learning too has been touched (Chap. 11), but the theme deserves further exploration. The ICMI Study 6 on assessment is as yet not updated (Niss 1993a, b) with changes internationally influencing practices at the country and classroom levels (see, for instance, Suurtamm et al. 2016).

The issue of *gifted students' needs* was only skimmed within the consideration of challenging mathematical tasks (Chaps. 9, 10 and 14). Hence, in this case too, there is space for further development (see, for instance, Singer et al. 2016).

The participation in the Study Conference deserves some comments. It was not surprising that China was well represented in the Conference, because of the proximity to the venue. Yet, in spite of the significant efforts of the IPC members, a limitation of the study was the failure to involve mathematics educators from a wider range of countries and regions (e.g. Russia, India, Japan, Korea, several parts of Africa and Latin America). Equity imperatives for participation in the ICMI Study 23 therefore remained far from being reached, although the themes of the Study had the potential to involve mathematics educators and policymakers from developing countries. Key obstacles that we identified included:

- Ineffective dissemination (international mailing lists and journals continue to reach a limited portion of the mathematics education community across the world).
- Language issues (the choice of English as the study language, although inescapable, may well have inhibited some authors from applying).
- Costs (while airfares tend not to be strictly related to the distance from countries, commercial constraints continue to apply).

[10] www.semt.cz

[11] www.sbm.org.br/icmt2/

1.6 The Implications of This Study

1.6.1 A Message for Practitioners

The multifaceted aspects considered in the many chapters of the Study Volume have the potential to attract mathematics teachers and teacher educators from all over the world: there are collections of tasks, activities and artefacts (see, for instance, Chaps. 9 and 10), addressing WNA. Approaches and models for teacher education and development are also broadly represented in the study, with input from many acknowledged scholars in the field of mathematics education, making the study attractive for researchers in primary school arithmetic.

From the many examples, we pick up some:

- Balancing ordinal, cardinal and measurement aspects of (and approaches to) number sense.
- Connecting the three core concepts of addition, subtraction and number together.
- Exploiting the potential of cultural artefacts (e.g. abaci, Dienes blocks, Cuisenaire rods, pascalines, devices from multitouch technologies).
- Focusing on structural approaches to early number development.
- Focusing on the make-a-ten method of addition and subtraction within 20.
- Storytelling to borrow the completely regular number names in those cultures where irregular names are present.
- Emphasising the importance of figural and spatial representations.
- Fostering bodily involvement such as counting with fingers, dancing or jumping on the number line.

1.6.2 A Message for Curriculum Developers and Policymakers

The attention to the social and cultural contexts and to the importance of native languages in mathematics learning has the potential to attract curriculum developers and policymakers. Around the world, 250 million children either fail to complete more than 3 years of basic education or lack basic numeracy skills for ongoing learning despite finishing 3 years of basic education (Matar et al. 2013). In one region of Morocco, one assessment showed that 20% of Grade 2 students could not solve any simple addition problems and 44% could not answer any simple subtraction problems (Matar et al. 2013). Furthermore, children who start school with a poorly developed understanding of number tend to remain low achievers throughout school (Geary 2013). This contrast between acknowledged needs and existing instructional programmes should be a major preoccupation of curriculum developers and policymakers.

Some policies and approaches that this study offers as suggestions include:

- Taking seriously the influence of early grade instruction on success in later education.
- Promoting early childhood mathematics in schools.
- Considering globalisation and the roots of mathematics in local cultures in a dialectic way.
- Taking the particular language and cultural constraints into account.
- Addressing the use of cultural artefacts and cognitively demanding tasks as teaching aids.
- Acknowledging the professional status of primary school teachers.
- Designing primary teacher education and development in order to make them highly educated professionals.

1.7 Concluding Remarks

Our hope is that the special focus on WNA in primary school mathematics within ICMI Study 23 lays the ground for further attention to primary mathematics topics, curricula and pedagogies to be addressed in future studies and in the conferences of organisations affiliated to ICMI, because *building the foundation*, as the title of volume reads, is critically important for the development of mathematics teaching and learning in secondary/high schools and beyond.

The interest shown by the participants from many different countries and regions and their engagement in authoring parts of this volume, as well as the early impact of the study, suggest strong potential and opportunities for organising a follow-up study in a few years' time. We, as co-chairs of the ICMI Study 23, will continue to collaborate in order to ensure a long-lasting influence of this study in our regions and, more generally, at the international level.

ICMI conferences and studies are examples of attempts to improve communication between different communities. However, it is misleading to claim that ICMI Study 23 achieved a *shared perspective*. This volume does not present a single coherent discourse, nor did the mathematics educators and the mathematicians converge to a common discourse of teaching WNA. A better description would be *sharing perspectives*, in the following sense: the various communities were given ample opportunities to present and elaborate their perspectives; others listened attentively and respectfully; there were opportunities for participants to discuss commonalities and differences and to develop new insights, yet eventually each participant was free to adopt, reject, modify or integrate parts of the others' perspectives into his/her own discourse of WNA.

In a world increasingly driven by questions about borders and migrations across them, what this volume has succeeded in collecting are overviews and discussions that are of interest to mathematics educators across phases and across borders. The volume provides illustrations of interventions and developments that share, across

different cultural contexts, a concern with broadening access to foundational mathematical ideas that are important if we are to contribute to progress and participation into higher-level mathematics. Diversity of language, artefacts and approaches to this endeavour of broadening access strengthens the field's ability to address this goal. We conclude by offering our thanks for the cross-cultural interaction processes that have culminated in this work. The broader global discourses that prevail at the time of the publication of this volume resonate with talk of walls and of borders. This volume stands as a testament to the strength of cross-national and cross-cultural collaboration – the dissolution of borders – and this study, like previous ICMI studies, is enriched by the international collaboration.

1.8 Processes and Acknowledgements

The chapters were reviewed internally by the IPC and by the co-chairs. Through this process, cross-referencing was developed as much as possible, and there was careful examination of any overlaps. Where different chapters have treated similar ideas, we have tried as far as possible to indicate cross-references.

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