Elementary Mathematicians from Advanced Standpoints—A Cultural Perspective on Mathematics Education

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Abstract Many challenges face those of us for whom mathematics education research is our life's work. In some countries where significant attempts are continually being made to reform mathematics teaching, it is often a highly politicised field. While rational arguments and relevant data-gathering are valid parts of a democratic research process, awareness of the broad cultural context is paramount. Despite the challenges that adopting a new cultural perspective brings, episodes and analyses from our sociocultural research field do demonstrate much promise for advancing mathematics educational practices. In particular the relatively new ideas of values and valuing show much research promise. In this paper, referencing Felix Klein's fundamental ideas, I will analyse the twin pluralised notions of 'elementary mathematicians' and 'advanced standpoints'. In addition research focussed on a third key notion, 'pedagogical practices' will be discussed. Finally some of the implications of this three dimensional and culturally oriented research will be presented.

Keywords Culture · Elementary mathematicians · Advanced standpoints

Introduction

The main aim of this paper is to broaden the discussion about the future of research in mathematics education. It derives from the parallels between the ideas of Felix Klein and the growth in research approaches in our current era However despite teachers' best efforts over many years mathematics is still rated as one of the most difficult subjects to teach and thus to learn. This is despite good arguments for making mathematics one of the key STEM subjects to teach in the modern world

Felix Klein award.

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(Educational Council, 2015). It would surely therefore be one of the most important subjects to research?

However many challenges face those of us for whom mathematics education research is our life's work. Cuts to University funding, general economic pressures, and unnecessary standards-based evaluations are all contributing to a sense of unease and disillusion in many countries. In this paper, as well as referencing Klein's ideas, I will explore my versions of his twin notions of 'elementary mathematics' and 'advanced standpoint', focussing on their humanistic side. Thus in the title of my paper 'elementary mathematicians' and 'standpoints' are the key ideas. In addition a third key notion, 'pedagogical practices' will complete the basic three-dimensional trio of culturally-based constructs which I believe should structure mathematics education research today.

In a sense this paper will be loosely based around my academic career, as the activities and explorations that engaged me reflected my research involvement with many others in our field. However as the old saying goes: "No man is an island" and in accepting the Felix Klein award for 2015 I am conscious of the many collaborators who have helped me structure and develop my ideas.

Our field of research is not like a highly abstract field of theoretical physics, for example where one mathematical mind can achieve much, as Stephen Hawking's has and indeed as Felix Klein's did. Education is naturally much more inclined to the sociocultural fields of people, socio-political groups and multidisciplinary thinking, as is mathematics education.

As the recognition of the team-work of Jill Adler and her colleagues shows, by her being awarded the Hans Freudenthal medal for 2015, quality research in our field today lies with teams and groups of researchers rather than individuals. So if I deserve any reward for my achievements, it is that I have been able/allowed to take advantage of, and have access to, opportunities for research in local, national and international contexts.

I have often been fortunate to have been at the right place at the right time, and working with the right people. So in that spirit and throughout the paper I would like to name especially some key people without whom my research might never had happened.

Thus I initially recognise the contribution of Sir Wilfred Cockcroft in the late 1960s to my early academic career. He was my mathematics Tutor at Southampton University, UK, an intelligent and perceptive academic (Cockcroft, 1982[1]).

He encouraged me to apply for a scholarship to study in the USA. Prof Frank Land who gave me my first research position, was also influential in my time at Hull University (Land, 1962). These leaders, together with another raw research student Donald McIntyre, made it possible for me and others to begin opening up the field of research in mathematics education (Morrison & McIntyre, 1973). They were in a sense my first 'significant others'.

[1]In naming key people, I have chosen to include at least one reference to their work that impinged on my own, thus giving some indication of their contribution to my thinking.

Klein and Culture

For me, personally the 1960s were an exploratory time and I had not yet started serious academic writing. I had just returned to the UK from three years at Harvard University, studying with Jerome Bruner and Ed Moise—and having experimented with various new teaching ideas in the schools nearby.

Structurally the systems of teacher education, school mathematics curricula, etc. in the UK, USA, and Europe were growing both politically and in terms of awareness of the need for research. Sputnik also appeared and focussed the minds considerably! It was an exciting international time to be a young researcher. This was where my first ideas of mathematics as a culture were born and where I paid my first respects to my cultural elders.

Moreover, I believe that Felix Klein also had this strong feeling for the idea of mathematics as a culture. He too was clearly concerned to explore mathematics as a form of cultural knowledge, with deep meanings to be understood and valued rather than just as routine knowledge to be accumulated and memorised for examination purposes.

This is how I read his invocation of "elementary mathematics from an advanced standpoint" (Klein, 2004). He did not use the word 'culture'; it had not yet been widely discussed or explored in his context and in his time of the late 1800s and the beginning of the 1900s. But we can see in retrospect from our 'advanced standpoint' of cultural knowledge, that this clearly was what was challenging him—how to choose the right elementary mathematics with which to induct the young students into the language, the world, and the culture of advanced mathematics?

The continuing problem for us, with all the advantages of modern technologies, well-educated teachers and parents etc., is that students are still failing in examinations, still unaware of the wider mathematical world, and still ignorant of the values of mathematical understanding.

This is now our time to take up the challenge from the legacy left by Klein. We can ask: What does the cultural metaphor offer us who are working to improve the mathematical education of young people around the world? In our context, and making the task for researchers like us even harder is that the very people who ought to be the inductees of the elementary mathematicians, namely schoolteachers, are just as unaware of this core idea as their students are.

So with these preliminary thoughts let me build on Klein's ideas, and explore how a cultural perspective on mathematics education can literally change the mathematical world for the better.

Elementary Mathematicians

Initially the field of mathematics education research in those early 1970s when I was really starting out as a researcher concerned the learners, as opposed to "students", their different attributes, the main mathematical challenges for them, their differing skill levels, and their so-called 'abilities'. The research constructs and methodology were principally adapted from the general field of educational psychology, and there was little reference to specifically mathematics learning, nor to theory or data that could relate to the internationally perceived problems of mathematics education.

Klein's analysis and ideas were summed up by his notion of "elementary mathematics" and this reflected the debates that were going on in his university context. In particular he incurred much criticism from within school and university mathematics departments, on the grounds that there was nothing elementary about his mathematical agendas.

Nor was it clear how his ideas could solve the major problems of the so-called 'dropout' curriculum. That is where everyone drops out of the curriculum at some stage in their mathematics education, either through the curriculum being too difficult, or too irrelevant to the rest of their lives, or whatever.

Others attending this ICME conference are more qualified than I to comment on the validity of what I would call the mathematical arguments underlying his choices and sequences of mathematical subjects. However, what is clear to me is that he was focussed on what he saw as the chief missing cultural ideas needing to be included in the school and university mathematics curriculum. For example he discussed the role to be played by functions within algebra, the structural power of group theory, and the whole field of different geometries.

The paradox was that at this very same time the popular slogan was "Mathematics for all" and mathematics education researchers were starting to tackle the difficult questions of how best to teach mathematics for "all"? In fact at the same time university mathematicians were trying to redefine what should be the "mathematics" to be taught to all?

So what was happening to those young learners who I have called "elementary mathematicians"? Basically and conceptually they were in the middle of this debate! Moreover the choice of this label I have made not just to reference Klein's ideas, but also to demonstrate the sociocultural nature of our current educational research.

My argument is that if we want to make more progress in our research field, we need to address the social and cultural positioning of the various players in the main game, and see the fundamental commonality of the notion of "elementary". Whether they are Aboriginal elders, school groups, immigrants, or second language learners of all ages, they all belong in sociocultural communities and in "a place" wherever and of whatever kind. Understanding that idea is crucial for beginning to understand what "elementary" can mean in any context.

The sociocultural nature of this research did help to mend the fences between the mathematics communities and the growing mathematics educator communities. The title of my book called "Mathematical enculturation, a cultural perspective on mathematics education" (Bishop, 1988), grew out of the belief that all students are being enculturated into the mathematics culture. In that sense, for good or for ill, they are learning the mathematical culture by being elementary mathematicians.

Advanced Mathematical Education Standpoints

From the research standpoint we have now, advanced standpoints if you like, gives us opportunities to revisit some old, and ask some new, research questions, just as Felix Klein's analysis did in his context. But for this generation the research focus has not been on the mathematical topics and the curriculum nor on the mathematical topics that Klein and others have identified.

Of course I recognise that the curriculum is a vitally important part of the change process. But for me, and others, the focus of the 1970s and 1980s was firmly on mathematical education as a sociocultural field, with a broad enculturating perspective. Moreover, despite facing the necessary challenges that adopting the new cultural perspective brought, episodes and analyses from our sociocultural research field demonstrated much promise for advancing educational practices.

I now propose to explore the notion of advanced standpoints by documenting some of the major research trends from the 1970s through to the present day. This also allows me to pay my respects to some of the influential colleagues with whom I have collaborated.

I have already noted that in the 1970s there was a growing awareness of the deficiencies of the research models and approaches based on chiefly psychological methodologies. In particular the voices of academics in several developing countries were being heard as they brought to the attention of the mainstream (Western in the main) researchers the view that there were other ways to think of mathematics.

Particularly for me this was a dramatic time as I visited Papua New Guinea to work with a colleague there, Glen Lean (Clarkson, 2008). This was the time when his research work came to be recognised worldwide. He had collected data on over 2000 different counting systems from around Papua New Guinea, a task of huge importance for the region and for the world of mathematics education in general (Lean, 1994).

It was not just a matter of documenting the counting systems, Lean was concerned to note the many ways in which counting was embedded in the local cultures and languages. Far from being a simplified abstract system, counting was revealed by Lean as one of the key mathematical foundational constructs of the societies in Papua New Guinea, and we can claim, everywhere.

This ethnographic database still exists thanks to the very difficult task undertaken by Kay Owens in Dubbo, Australia (Bishop, 1999; Owens, 2001). Allied to the work on counting there was a huge interest in other anthropological data, and later its methods. Ken Clements and Lloyd Dawe (my first Ph.D. student) were active in this developmental work in Australia and the South Pacific (Clements & Ellerton, 1996; Dawe, 1986).

The educational implications of this research work had still to be worked out and colleagues in Mozambique and Brazil took great interest in it. Paulus Gerdes led one of these very effective teams and in particular they were accumulating other aspects of anthropological data that could be related to a more localised and relevant mathematics education instead of the out-dated colonial versions of mathematics instruction. It showed that subjects such as practical geometry and probability and statistics were likely to be much more relevant (Gerdes, 1986).

More than that however it showed that mathematical knowledge was/is not an abstract, universal field—it did/does have strong connections to the particulars of the society, especially when education is being considered. In fact it is important to recognise here that all societies have developed their own mathematics. "Ethnomathematics" was the term developed by Ubiratan D'Ambrosio to encompass both local mathematical ideas and so-called universal Western ideas (D'Ambrosio, 1985). This work was part of a much broader push to focus on other student groups that were not succeeding with the standard mainstream version. This issue relates back to the earlier work based on psychology, and convinced many researchers that if they were to have any influence on their country's mathematics education they would need to address the issue of how to relate "Street mathematics and school mathematics"; the title of an influential book by Nunes, Schliemann, and Carraher (1993).

Research such as this also showed the many positive ways that the local mathematics enabled, and was part of, the "normal" everyday life in those societies. This research of the 1970s blossomed in the 1980s and culminated in the 1988 ICME held in Hungary. In my view the great achievement of that decade was the so-called "Fifth Day Special" in that ICME conference, which brought together for one day more than 200 people and resulted in the UNESCO publication "Mathematics, Education, and Society" (Keitel, Damerow, Bishop, & Gerdes, 1989). Groups worked on papers concerned with political issues, societal relationships and particularly on sociocultural aspects.

Working in planning this Fifth Day Special with colleagues Christine Keitel, Peter Damerow, and Paulus Gerdes was an inspiration to me. Although we did not achieve all that we had hoped for, nevertheless the fact that we could bring together this number of colleagues to work on alternatives to the mainstream, marks the 1980s and the early 1990s as the turning point of change completely.

Political and critical dimensions of mathematics education started to come to the fore as a part reaction to this day's debate. In particular some felt the political had not been foregrounded enough. This gave rise to the Political Dimension of

Mathematics Education (PDME) group that was active for some years through the 1990s. Key figures in this group included Richard Noss and Celia Hoyles (Noss et al., 1990). In some ways this PDME group was a forerunner of the now robust Mathematics Education and Society group.

There was still some way to go in changing the mainstream system, but nevertheless at ICME in Budapest there were many images of progressive ways to develop the systems and to democratise mathematics education more completely. Furthermore, I would like to think that Felix Klein would have found himself thoroughly immersed in the debates and discussions concerning elementary mathematicians from advanced standpoints!

Pedagogical Practices in Relation to Values and Valuing

As the cultural metaphor became widespread, so did the growth of contact between educational researchers. Thus international comparative studies of mathematics education in different countries became a significant form of study. In fact this kind of comparative study was something that ICMI had done at its inception and carried on for many years.

Indeed, it was always of great interest to see how the "neighbours" were dealing with many similar problems that were emerging. Comparisons of textbooks and examination papers were also a source for research ideas. Teacher education institutions also played a strong role at this stage when it was clear that just random collections of textbook practices were not enough as data for serious, even scientific study.

Nevertheless, the roles of journals and books were highly significant in that phase. When Hans Freudenthal invited me in 1983 to take over the editorship of the journal Educational Studies in Mathematics (ESM), I could not believe what it would lead me into, with my own research. Suddenly I found myself deeply involved with international colleagues working throughout the world who were writing excellent research papers, and I had to be the one who finally had to make the decision on whether or not to publish them! What this did for me was to emphasise the social and cultural contexts of mathematics education research and of course I could not have done this work without the help of a skilled and dedicated Editorial Board.

But then I became aware of another significant issue. Many colleagues were now writing excellent research papers but, at that time, there were few opportunities to publish books in our field. I approached Kluwer (the then publisher of ESM before they were taken over by Springer) and we began the Mathematics Education Library, with the first book being by Hans Freudenthal. The publishers were understandably nervous about this venture especially as the title of Freudenthal's

book was "Didactical phenomenology of mathematical structures" (Freudenthal, 1983). However, my argument was "Trust me" and they did. We now have more than 50 books in that Mathematics Education Library series.

Sometime later, Kluwer developed another publication initiative, which was the idea of a Handbook of Mathematics Education. The first one was a collaboration between myself as lead editor and Jeremy Kilpatrick, Christine Keitel, Ken Clements and Colette Laborde (Bishop, Clements, Keitel, Kilpatrick, & Laborde, 1996), and later in the second edition Frederick Leung joined the ranks of editors (Bishop, Clements, Keitel, Kilpatrick, & Leung, 2003). Once again it was a huge pleasure to work with these impressive scholars and we realised then the significant role that such publications could play in shaping the social and cultural contexts of our field. Since then the publication scene has matured and grown to reflect the multiplicity of research approaches in our field.

Turning to one of the significant groups that I was a part of during the 1980s, called BACOMET, was a powerful intellectual group, mainly European based, whose research activity spread over several years. The leaders were Bent Christiansen, Geoffrey Howson and Michael Otte and the full title of the group was "Basic Components of Mathematics Education for Teachers" (Bishop, Mellin-Olsen, & van Dormolen, 1991; Christiansen, Howson, & Otte, 1986). My specific interest in that group was with the teaching of values and multicultural aspects of teacher education, and these came in through chasing my ideas of mathematical enculturation. In fact the idea of 'mathematical acculturation' also came to the fore in this period.

Originally this idea seemed to relate most strongly to the elementary mathematicians working in a culture different from the mainstream; for example in Papua New Guinea, Brazil or Mozambique or indeed with significant indigenous groups in South Africa or New Zealand, where the extraordinary work of colleagues such as Bill Barton are noted with thanks (Barton, 2008). Also Tamsin Meaney's work with indigenous Australians is another significant contribution (Meaney, Trinick, & Fairall, 2012).

However, one could argue that for any school learner the teaching and learning process would be one of acculturation. Immigrant students were and are a prime example. This profound idea engaged me in some exciting new research, thanks to working closely with colleagues such as Guida d'Abreu, Norma Presmeg, Nuria Gorgorio and Marta Civil (Abreu, Bishop, & Presmeg, 2002). One consequence of this re-focus on learners' situations developed the idea of the learners being in a culture conflict situation. School and home cultures are often in conflict, particularly for learners who are new to the country concerned.

Over the years I have had some interesting and challenging interactions with a number of other colleagues from the Scandinavian countries beginning with Stieg Mellin-Olsen from Norway (Mellin-Olsen, 1987), and continuing with Ole Skovsmose from Denmark (Skovsmose, 1994). They have continued to open up the

political dimensions of the debate as have my two colleagues at Monash University whose research is related to this political discussion, more specifically the politics of gender; namely Gilah Leder and Helen Forgasz (Leder & Forgasz, 1996).

With another group currently I have preferred to focus more on values and valuing—normally ignored, and only partially understood educationally. My original collaborators Philip Clarkson and Wee Tiong Seah have brought their own perspectives to this work on values (Clarkson, Bishop, & Seah, 2010); Clarkson with his various research studies on language and mathematics learning (Clarkson, 1991), and Seah with his wide intercultural perspective (Seah, 2008). More recently other colleagues have joined us in our values research: Annica Andersson from Denmark and Penelope Kalogeropoulos from Monash University, Australia. Andersson's work is based on socioculture and place (Andersson, 2012) while Kalogeropoulos is focussing more on students who are disengaged from school mathematics learning (Kalogeropoulos, 2016). In each case we can see how powerful is the notion of values—what values are controlling the actions of those students who are engaged or disengaged from the learning tasks? Indeed are they pursuing different values and if so what are they? As I noted above many challenges and tensions face those of us who work in the field of mathematics education and how they influence the mathematics discussions of the day. One area of these challenges concerns developing appropriate methodologies for our research. This is an issue that we as a group are still wrestling with, exploring at present whether using role-playing will give us, research colleagues and fellow teachers more insights into values and valuing (Clarkson in press).

Once again it is important to state that Felix Klein did not choose to discuss the idea that mathematics is a culture with its own values. However, it is also clear to me that he would have embraced that idea. He was neither anthropologist nor historian but once again he was ahead of his time, and once again I am sure he would have fitted into the sociocultural genre of our current educational struggles touched on here.

And Finally

The challenge of creating a satisfactory mathematics education for all learners has a long and fundamental history—indeed one can say that the issues and challenges are always with us, only the manner of understanding and dealing with them changes.

Felix Klein had little formal training in mathematics education but he was not short of ideas about the teaching of the subject. He had heard about the teaching methods of Maria Montessori and he recommended using models and small objects as vehicles for developing important geometrical notions and images.

So we can see that mathematics education has embraced diverse theoretical 'standpoints' in the quest to improve the teaching of mathematics. The focus on sociocultural values is one of the latest standpoints, and those of us working in this field believe strongly in its power. However, the best research is not just reflective, though that is important, but should also be projective—offering leadership to the mathematics education community in a parallel approach with medical research, which always has its empirical 'eye' on any new research.

I stated at the start of this paper that my chief aim was to broaden the discussion about the future of research in mathematics education. I have based my ideas upon the notions of "elementary mathematicians" and "advanced mathematical standpoints". I hope I have shown that reworking the ideas of Felix Klein this way could make a significant contribution to your own research. The ideas are that mathematics is a culture with its own norms, values, language and customs.

Finally I feel honoured not just to have been awarded this Klein medal but also to see how much of our current research on culture and values resonates with Klein's ideas. In this paper I have described the nature of some of the groups working in the sociocultural field of mathematics education. I have also noted some of the key colleagues with whom I have worked over my lifetime.

The danger in naming colleagues like this is that some others will inevitably feel excluded! They know who they are and I know who they are, and I apologise to them. I am grateful for their collaboration. It is only space and time that have prevented me from referring specifically to them. Thank you, and good luck to all researchers out there!

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I also wish to recognize the work of my colleague and friend Phil Clarkson with whom I have shared many creative debates both on and off the golf course!

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