Chapter 6
Conceptual Objections

Abstract
There are two conceptual objections to the idea of justification by an infinite regress. First, there is no ground from which the justification can originate. Second, if a regress could justify a proposition, another regress could be found to justify its negation. We show that both objections are pertinent to a regress of entailments, but fail for a probabilistic regress. However, the core notion of such a regress, i.e. probabilistic support, leaves something to be desired: it is not sufficient for justification, so something has to be added. A threshold condition? A closure requirement? Both? Furthermore, the notion is said to have inherent problems, involving symmetry and nontransitivity.

6.1 The No Starting Point Objection

In the previous chapter we discussed the main pragmatic argument against justification by infinite chains, known as the finite mind objection. Perhaps even more serious, however, are the conceptual objections. They aim to show that even creatures with an infinite lifespan or with a mind that can handle infinitely long or complex chains will run into problems, because the very idea of justification is at odds with a chain of infinite length:

> conceptual arguments . . . appeal . . . to the incompatibility of the concept of epistemic justification and infinite series of support.[1]

Two conceptual objections in particular are often discussed. According to the first, *no* proposition can ever be justified by an infinite regress, since in such

[1] Aikin 2011, 51.

D. Atkinson, J. Peijnenburg, *Fading Foundations*, Synthese Library 383,
DOI 10.1007/978-3-319-58295-5_6

a regress justification is for ever put off and never materialized. This is the much raised *no starting point objection*, as Peter Klein has called it, which is based on the fact that an infinite chain is bereft of a source or a foundation from which the justification could spring.[2] The second conceptual objection goes beyond the first one, spelling out what would happen if the no starting point objection did not apply. If, *per impossibile*, a particular proposition q were justified by an infinite chain, then it can be demonstrated that *all* propositions could be justified in that manner, including the negation of q. This objection is known as the *reductio argument*, and it has been raised in different forms, notably by John Pollock, Tim Oakley, James Cornman, Richard Foley, and John Post.[3]

In the present section and in the next one we discuss the no starting point objection. We shall argue that a starting point is not needed if the regress is probabilistic — a conclusion which follows from the preceding chapters. In Sections 6.3 and 6.4 we shall deal with the reductio argument, showing that this objection, too, fails for a probabilistic regress. In the final section, 6.6, we note that the concept which is central to a probabilistic regress, viz. probabilistic support, is itself prone to problems. We elaborate on two properties of probabilistic support that are allegedly problematic for the concept of justification, namely that probabilistic support is symmetric and that it lacks transitivity.

The no starting point objection asserts that justification can never be created by inferences alone. The reason is that an infinite inferential chain blocks *ab initio* the possibility of justification. The only way to generate justification is by having a starting point, i.e. a proposition or a belief that is itself non-inferentially justified. Aikin phrases the objection as follows:

> ... if reasons go on to infinity, then as far as the series goes, there will always be a further belief necessary for all the preceding beliefs to be justified. If there is no end to the chain of beliefs, then there is no justification for that chain to inherit in the first place.[4]

The no starting point objection exploits the fact that in an infinite regress justification seems to be indefinitely postponed and never cashed out. It is as if we are given a cheque with which we go to a bank teller, who gives us a new cheque and directs us to another bank teller, who hands us a third

[2] Klein 2000, 204. *Cf.* Laurence Bonjour: "The result ... would be that justification could never get started and hence that no belief would be genuinely justified" (Bonjour 1976, 282).
[3] Pollock 1974, 29; Oakley 1976; Cornman 1977; Foley 1978; Post 1980.
[4] Aikin 2011, 52.

cheque, instructing us to go to yet another bank teller, and so on and so forth. Never do we encounter a bank teller who actually converts our current cheque into bars of gold.

Like the finite mind objection, this objection too has a long history, going back indeed to Aristotle. Aikin recalls some of the latest versions:

> William Alston captures the argument as follows: If there is a branch [of mediately justified beliefs] with no terminus, that means that no matter how far we extend the branch the last element is still a belief that is mediately justified if at all. Thus, as far as this structure goes, whenever we stop adding elements we have still not shown that the relevant necessary condition for mediate justification of the original belief is satisfied. Thus the structure does not exhibit the original belief as mediately justified [Alston 1986, 82].
>
> Henry Johnstone captures the thought: '*X* infinitely postponed is not an *X*' since the series of postponements shortly becomes 'inane stammering' [Johnstone 1996, 96].
>
> Romane Clark notes that such a series will produce only 'conditional justification' [Clarke 1988, 373], and Timo Kajamies calls such support 'incurably conditional' [Kajamies 2009, 532].
>
> The same kind of thought can be captured with an analogy. Take the one R.J. Hankinson uses in his commentary on Sextus: 'Consider a train of infinite length, in which each carriage moves because the one in front of it moves. Even supposing that fact is an adequate explanation for *the movement of each carriage*, one is tempted to say, in the absence of a locomotive, that one still has no explanation for *the motion as a whole. And that metaphor might aptly be transferred to the case of justification in general*' [Hankinson 1995, 189].[5]

In the same vein, Carl Ginet writes:

> Inference cannot *originate* justification, it can only *transfer* it from premises to conclusion. And so it cannot be that, if there actually occurs justification, it is all inferential ... [T]here can be no justification to be transferred unless ultimately something else, something other than the inferential relation, does create justification.[6]

Ginet cites Jonathan Dancy, who phrases the no starting point objection as follows:

> Justification by inference is conditional justification only; [when we justify *A* by inferring it from *B* and *C*] *A*'s justification is conditional upon the justification of *B* and *C*. But if all justification is conditional in this sense, then nothing can be shown to be actually, non-conditionally justified.[7]

[5] Ibid., 53 – misspellings corrected.
[6] Ginet 2005, 148.
[7] Dancy 1985, 55.

The no starting point objection is also at the heart of Richard Fumerton's "conceptual regress argument" against justificatory chains. On several occasions Fumerton has distinguished between two "regress arguments" in support of foundationalism: the epistemic and the conceptual regress argument.[8] The first boils down to the finite mind objection against infinite chains. It states that "having a justified belief would entail having an infinite number of different justified beliefs" while in fact "finite minds cannot complete an infinite chain of reasoning".[9] In the previous chapter we have explained why we think that this objection does not succeed. The conceptual regress argument, on the other hand, appears to be a rewording of the no starting point objection. Fumerton calls it "quite different" from the epistemic regress argument, and "more fundamental".[10] It states that an infinite justificatory chain is vicious because we can only understand the concept of inferential justification if we accept that of *non*inferential justification:

> [I]f we are building the principle of inferential justification into an analysis of the very concept of justification, we have a more fundamental vicious *conceptual* regress to end. We need the concept of a noninferentially justified belief not only to end the epistemic regress but to provide a conceptual building block upon which we can understand all other sorts of justification. I would argue that the concept of noninferential justification is needed ... in order to *understand* other sorts of justification ....[11]

In other words, the very idea of inferential justification does not make sense without assuming justification that is noninferential, or, as Fumerton formulates it later, the concept of inferential justification is "parasitic" on that of noninferential justification:

> To *complete* our analysis of justification we will need a base clause — we will need a condition sufficient for at least one sort of justification the understanding of which does not already presuppose our understanding the concept of justification. But that sort of justification is just what is meant by noninferential justification (justification that is *not* inferential). Our concept of inferential justification is parasitic upon our concept of noninferential justification. It doesn't follow, of course, that anything falls under the concept. But if nothing does, then there is no inferential justification either ...[12]

[8] Fumerton 1995, Chapter 3; Fumerton 2004; Fumerton and Hasan 2010; Fumerton 2014.

[9] Fumerton 1995, 89; 2004, 150; 2006, 40; 2014, 76.

[10] Fumerton 1995, 89; 2014, 76.

[11] Fumerton 1995, 89.

[12] Fumerton 2014, 76.

Not surprisingly, Fumerton's response to his conceptual regress argument echos the standard reply to the no starting point objection: the only way to inject justification into an inferential chain is to assume a source from which the justification springs. Without such a source, the very concept of inferential justification becomes unintelligible or even absurd, 'inane stammering' as Henry Johnstone would have it.

A particularly interesting and generalized version of the no starting objection has been put forward by Carl Gillett.[13] The problem with an infinite chain of reasons, Gillett says, does not lie in its epistemological character as such, but is more general: it has to do with its general metaphysical structure, which it shares with many vicious regresses outside epistemology. This structure is such that the relevant dependent property (which in the epistemological case is 'being justified') cannot be produced, because there is a relation of dependence, what Gillett calls the 'in virtue of' relation. If a proposition q is justified in virtue of A_1 being justified, which in turn is justified in virtue of A_2 being justified, then it is notoriously unclear how any of the propositions could be justified. Making the chain longer is of course no solution, for irrespective of the number of propositions we add, each proposition will only be justified because of another proposition. Thus, Gillett concludes, there is no number of propositions that can be added "that will suffice for any of its dependent properties to feed back to any members of the chain".[14] According to the 'Structural Objection', as Gillett has dubbed his particular version of the argument, the very structure of the epistemic regress prevents justification from arising.

In none of these different formulations of the no starting point objection is it made clear what exactly is meant by epistemic justification. When for example Dancy complains that, "if all justification is conditional ... then nothing can be shown to be actually, non-conditionally justified", it is not clear what he means by 'conditional' and 'non-conditional', since it remains open whether he sees justification as for example entailment or as involving probabilistic support (see Chapter 2). In the first case, his talk about conditional and non-conditional justification would refer to the difference between if-then statements and categorical statements; in the second case, it pertains to the difference between conditional and unconditional probability statements. The distinction is however vital in a discussion of the no starting point objection. For while the objection applies to justification as entailment, as applied to justification as probabilistic support it backfires completely. This result

[13] Gillett 2003.
[14] Ibid., 713.

was already intimated in the previous chapter, but we will explain it further in the next section.

6.2 A Probabilistic Regress Needs No Starting Point

It is not difficult to see why the no starting point objection applies if justification is interpreted as a kind of entailment. Consider the finite chain

$$A_0 \longleftarrow A_1 \longleftarrow A_2 \longleftarrow A_3 \longleftarrow \ldots \longleftarrow A_m \longleftarrow A_{m+1} \tag{6.1}$$

where the arrow represents entailment, where A_0 does duty for the target, q, and where A_{m+1} stands for the foundation or ground. Then of course the only way to know for sure if A_0 is true is by knowing that A_{m+1} is true. In the words of Aikin: "Conceptual arguments start from the deep, and I think right, intuition that epistemic justification should be pursuant of the truth".[15] But if we are ignorant of the truth or falsity of the ground, A_{m+1}, we are groping in the dark about the truth value of A_0. When we make chain (6.1) infinite, so that it looks like:

$$A_0 \longleftarrow A_1 \longleftarrow A_2 \longleftarrow A_3 \longleftarrow A_4 \longleftarrow \ldots \tag{6.2}$$

then the matter is worse: since there is no initiating A_{m+1}, there is no truth value that is preserved in the first place. For the only way in which the target can be justified is by receiving the property from its neighbour, which received it from its neighbour, and so on. If there is no origin from which the property is handed down, there is nothing to receive, so the no starting point objection applies in full force.

Things are very different when justification is interpreted probabilistically. Applied to a probabilistic chain, the no starting point objection means that the target can only be justified by a chain of conditional probabilities if we know the unconditional probability of the ground. That is, in order to know $P(A_0)$, we need to know not only all the $P(A_j|A_{j+1})$ and $P(A_j|\neg A_{j+1})$, but also the unconditional probability $P(A_{m+1})$. But if the chain is infinitely long, there is no A_{m+1}, and thus there is no probability of A_{m+1} that can be known in the first place. As a result, the no starting objection concludes, there is no way to know the value of $P(A_0)$.

In the previous chapters we have seen why this conclusion does not follow. In all but the exceptional cases, the value of $P(A_0)$ can be determined

[15] Aikin 2011, 51.

without having to know the value of some $P(A_{m+1})$. In fact, as we saw in Chapter 5, in many cases we do not even need to know the values of all the conditional probabilities; once we have fixed a particular level of accuracy with which we are satisfied, we can decide how many conditional probabilities we need to know in order to attain that accuracy. If the number of conditional probabilities turns out to be too big to handle, then we must adjust the accuracy level and make do with an approximation of the target's true value with an error margin that is somewhat bigger than we had initially envisaged. So while the no starting point objection implies that in an infinite regress the value of $P(A_0)$ either goes to zero or remains unknown, neither of these two options actually obtains when the probabilistic regress is in the usual class.

John Pollock has trenchantly criticized what he calls "the nebula theory" of justification: never can an infinite chain justify a target, since the chain's ground is for all future time hidden in "a nebula".[16] Pollock would be right that this is an insuperable problem so long as we are speaking about a regress of entailments; but in a probabilistic regress the difficulty does not arise at all. For all we care A_∞ may forever lie hidden in nebulae, in a probabilistic regress that does not matter since A_∞ is completely irrelevant to the question whether A_0 is probabilistically justified or not.

Rather than talk about a nebula, we could also use the metaphor of a borehole. Compare the justification of a target by an epistemic chain to the pumping up of water from a deep well. If the chain is non-probabilistic, then the relations of entailment serve as neutral conduits through which justification passes unhindered. The justification itself comes from the bottom of the borehole, whence it is pumped up and transferred along the chain, whither it streams to the target proposition. If the epistemic chain is infinite, there is no beginning, the borehole is bottomless, the pumping stations forever remain dry, and no justification will ever gush out to the target. But now imagine that the infinite chain is probabilistic. Then a bottom is not needed. For now justification does not surge up unchanged from source to target; rather it comes from the conditional probabilities, which jointly work to confer upon the target proposition an acceptable probability. The conditional probabilities are, as it were, the intermediate pumping stations which actively take a moeity of justification from the circumambient earth, rather than passively wait for what comes up through the borehole. In a probabilistic regress we deliver justification, albeit piecemeal, whereas in a non-probabilistic regress we are not able to produce anything at all. In the latter case there is nothing more

[16] Pollock 1974, 26-31.

than the pointing to a fathomless borehole, or to a bank teller beyond the end of the universe who is supposed to administer my fortune.

Yet another metaphor was suggested to us by an anonymous reviewer; it concerns the saga of the bucket brigade. Suppose there is a fire and Abby gets her water from Boris, and Boris gets it from Chris, and Chris from Dan, and so on *ad infinitum*. It would seem that the fire will never be put out, since there is no first member of the brigade who actually dips his or her bucket into the lake. However, once we assume that justification involves probabilistic support the dousing operation looks quite different. Under this assumption, the proposition 'Abby gets water from Boris' (A_0) is only probabilistically justified, and we can calculate the probability value of A_0 by applying the rule of total probability that we cited earlier:

$$P(A_0) = P(A_0|A_1)P(A_1) + P(A_0|\neg A_1)P(\neg A_1)\,, \tag{6.3}$$

where A_1 reads 'Boris gets water from Chris'. Of course, whether Boris gets water is also merely probable, and its probability depends on whether Chris gets water, and so on. We face here an infinite series of probability values calculated via the rule of total probability. As we know by now, we are perfectly able to compute the outcome of this infinite series in a finite time: with the numbers that we used in the uniform case of the bacterium example in Section 3.7, the probability that Abby gets water is $\frac{2}{3}$.

All four probabilities on the right-hand side of (6.3), the conditional as well as the unconditional ones, are supposed to have values strictly between zero and one (in the interesting cases). In contrast, the regress of entailments, in which justification is not probabilistic, can be modelled by restricting all four 'probabilities' to be 0 or 1. Within this non-probabilistic approach, Abby either gets water or she does not. According to the no starting point objection, the moral of the saga about the bucket brigade is precisely that she does not get water — if the number of brigadiers is infinite. Because this is unacceptable, it is concluded that there must be a first firefighter on the shore of the lake who starts off the whole operation. In the probabilistic scenario the existence of a primordial firefighter is not needed, since the problem that it is supposed to solve does not arise in the first place. The reason is, as we have seen, that now the relations between the propositions are not idle channels, but actively contribute to the probability value of A_0; they for example allow for a downpour somewhere along the line that fills the bucket. So if we take seriously that justification involves probabilistic support, then the probability that Abby extinguishes the fire can have a precise value, despite the infinite number of her team-mates. As in the examples that we considered above, this unconditional value is a function of all the conditional probabilities.

Note that the above reasoning is independent of whether we embrace an objective interpretation of probability (assuming, for example, that the firefighters have propensities for handing over the water only now and then) or a subjective interpretation (in which we specify our degree of belief in A_0). Both the objective and the subjective interpretation are bound by the rule of total probability, and that is all that counts here. This suggests that our approach is not restricted to epistemological series, but might be applied more generally to the metaphysical structures that Carl Gillet has been talking about. In fact, it might even be used to query similar reasonings in ethics. Richard Fumerton argued that his conceptual regress argument for foundationalism has a counterpart in the ethical realm. Suppose we are interested in whether an action, X, is good, and suppose we are being offered a series of conditional claims: if Y is good then X is good, if Z is good then Y is good, and so on, *ad infinitum*. Have we answered the original question? Fumerton believes we have not. At best we possess an infinite number of conditional claims, but this does not tell us whether X is good. Just as inferential justification only makes sense if there exists noninferential justification, instrumental goodness only makes sense if we assume that some things are intrinsically good:

> ...the view that there is only instrumental goodness is literally unintelligible. To think that something X is good if all goodness is instrumental is that X leads to a Y that is good by virtue of leading to a Z that is good, by virtue of ..., and so on ad infinitum. But this is a vicious conceptual regress. The thought that X is good, on the view that all goodness is instrumental, is a thought that one could not in principle complete. The thought that a belief is justified, on the view that all justification is inferential, is similarly, the foundationalist might argue, a thought that one could never complete.
>
> Just as one terminates a conceptual regress involving goodness with the concept of something being intrinsically good, so one terminates a conceptual regress involving justification with the concept of a noninferentially justified belief.[17]

> The concept of intrinsic goodness stands to the concept of instrumental goodness as the concept of noninferential justification stands to the concept of inferential justification. Just as there are no good things without there being something that is intrinsically good, so also there are no inferentialy justified beliefs unless there are noninferentially justified beliefs.[18]

Fumerton would be right that instrumental goodness implies intrinsic goodness if the conditional claims are of the form 'if Y is good then X is good'.

[17] Fumerton 1995, 90.
[18] Fumerton 2014, 76.

For then goodness is transferred lock, stock and barrel along the chain, and the no starting point objection, or rather Gillet's more general Structural Objection, applies in full force. However, we have been arguing that the situation changes radically if the claims take on the form 'if Y is good then there is a certain probability that X is good' and 'if Y is bad then there is a certain (lower) probability that X is good', and so on. For now goodness is not transferred in its entirety along the series. Rather it slowly emerges as we progress from the links Z to Y and Y to X. In this probabilistic scenario the original question would be how probable it is that a certain action, X, is good. And this question can indeed be answered; as we have seen, with the numbers chosen, it is $\frac{2}{3}$.

6.3 The Reductio Argument

According to the reductio argument, if an infinite chain could justify a target A_0, then another infinite chain could be constructed that would justify the target's negation, $\neg A_0$. Since it does not make sense for a proposition and its negation both to be justified, the proponents of this argument conclude that justification by an infinite chain is absurd.

Like the no starting point objection, the reductio argument has taken on different formulations. Here we will concentrate on a version that was offered by John Post in a tightly argued paper, which is in fact an improved version of arguments that had been put forward by John Pollock and James Cornman.[19]

Post starts his argument by defining an infinite justificational regress as a "non-circular, justification-saturated regress", by which he means that "every statement in the regress is justified by an earlier statement, and none is justified by any set of later statements".[20] As we have seen in Chapter 2, Post sees the justification relation as entailment, or better, 'proper entailment': "if anything counts as an inferential justification relation, proper entailment does ... If A_n properly entails A_{n-1}, then A_{n-1} is justified".[21] Now consider again the infinite chain

$$A_0 \longleftarrow A_1 \longleftarrow A_2 \longleftarrow A_3 \longleftarrow A_4 \longleftarrow \ldots \tag{6.4}$$

[19] Post 1980; Pollock 1974, 28-29; Cornman 1977.

[20] Post 1980, 3.

[21] Ibid. Post has X and Y where we write A_n and A_{n-1}.

where it is assumed that the propositions are connected by proper entailment relations in the sense of Post, and where again A_0 does duty for the target q. According to Post, chain (6.4) is a non-circular, justification-saturated regress if and only if the following three conditions are satisfied:

a. A_n entails A_{n-1} $(n > 0)$;
b. A_n is not entailed by any $A_{m<n}$;
c. A_n is not justified on the basis of any set of $A_{m<n}$.

The first condition captures the idea that justification is a relation of entailment. The second condition is meant to ensure non-circularity. The third condition is added in order to block the possibility that a set of propositions might in some way or other together conspire to justify a proposition higher in the chain, which would make the regress circular after all. In the following we will always assume non-circularity in the background.

The construction of (6.4) as a non-circular, justification-saturated regress presupposes that at every step of the regress there indeed exists some proposition, A_n, which satisfies conditions a, b and c. Are there any examples of (6.4) that do the job? According to Post there are many, since there are many forms of proper entailment which meet the three conditions above. One of them is obtained by using *modus ponens* to interpret the links in the chain as follows:

$$\begin{aligned}
A_0 &= B_0 \\
A_1 &= B_1 \wedge (B_1 \rightarrow B_0) \\
A_2 &= B_2 \wedge (B_2 \rightarrow (B_1 \wedge (B_1 \rightarrow B_0))) \\
A_3 &= B_3 \wedge (B_3 \rightarrow (B_2 \wedge (B_2 \rightarrow (B_1 \wedge (B_1 \rightarrow B_0))))) \,,
\end{aligned} \tag{6.5}$$

and by adding the restriction that B_1 is some proposition not entailed by A_0, that B_2 is some proposition not entailed by A_1, and so on. Under these restrictions it is the case that A_1 entails A_0, A_2 entails A_1, and so on; but A_0 does not entail A_1, A_1 does not entail A_2, and so on. Moreover, there is no set of propositions that together justify a proposition higher in the chain, so the conditions a, b and c are fulfilled.

Since $B \wedge (B \rightarrow A)$ is formally equivalent to $B \wedge A$, (6.5) can also be written as

$$\begin{aligned}
A_0 &= B_0 \\
A_1 &= B_1 \wedge B_0 \\
A_2 &= B_2 \wedge B_1 \wedge B_0 \\
A_3 &= B_3 \wedge B_2 \wedge B_1 \wedge B_0 \,,
\end{aligned} \tag{6.6}$$

and so on, so that the chain (6.4) amounts to

$$B_0 \leftarrow (B_1 \wedge B_0) \leftarrow (B_2 \wedge B_1 \wedge B_0) \leftarrow (B_3 \wedge B_2 \wedge B_1 \wedge B_0) \leftarrow \ldots \tag{6.7}$$

Each link in (6.7) justifies its neighbour to the left, with the exception of B_0, which has no left-hand neighbour.[22]

Does it make sense to say that (6.7) justifies A_0? Post rightly claims that it does not. For in this manner a regress of propositions can be constructed for *any* target proposition, in particular for the negation of A_0. We only need to construct the infinite chain:

$$A'_0 \leftarrow A'_1 \leftarrow A'_2 \leftarrow A'_3 \leftarrow \ldots \tag{6.8}$$

where the A'_n are interpreted as

$$\begin{aligned} A'_0 &= \neg B_0 \\ A'_1 &= B'_1 \wedge (B'_1 \rightarrow \neg B_0) \\ A'_2 &= B'_2 \wedge (B'_2 \rightarrow (B'_1 \wedge (B'_1 \rightarrow \neg B_0))) \\ A'_3 &= B'_3 \wedge (B'_3 \rightarrow (B'_2 \wedge (B'_2 \rightarrow (B'_1 \wedge (B'_1 \rightarrow \neg B_0))))) \,. \end{aligned} \tag{6.9}$$

Chain (6.8) reduces to

$$\neg B_0 \leftarrow (B'_1 \wedge \neg B_0) \leftarrow (B'_2 \wedge B'_1 \wedge \neg B_0) \leftarrow (B'_3 \wedge B'_2 \wedge B'_1 \wedge \neg B_0) \leftarrow \ldots \tag{6.10}$$

So if an infinite regress could justify a target proposition A_0, then another could justify $\neg A_0$, which is of course absurd. Hence the reductio argument, which shows that an infinite regress of proper entailments cannot justify a proposition.

Both Peter Klein and Scott Aikin made an attempt to ward off the reductio. Klein's idea is that an infinite chain of proper entailments as set up by Post is necessary, but not sufficient for the justification of a target: in order to be sufficient, the propositions in the chain should also be "available" as reasons.[23] Aikin has argued that the only way to repel the reductio argument is by taking a mixed view: infinitism and foundationalism do not exclude one another, for a proposition can be both inferentially and noninferentially

[22] Eq.(6.6) is used in Oakley's second argument against justification by infinite regress (Oakley 1976, 227-228). Aikin calls (6.6) "the simplification reductio" (Aikin 2011, 58.)

[23] Klein 1999, 312; Klein 2003, 722.

justified.[24] Aikin here takes up an idea by Jay Harker, namely that not all regresses of entailment make sense as justificatory chains, but that some do. According to Harker, a regress merely of beliefs is insufficient; a justificatory chain must contain relations to facts as well, although it may still be infinite.[25]

Thus Klein, Aikin and Harker all endorse the intuition that more is needed for justification than an infinite, unanchored chain of proper entailments; something has to be added to this chain in order to make it a *justificatory* chain. We fully share this intuition, but we think that a chain of entailments does not lend itself so easily to such an add-on — it is somehow too self-contained for that. What helps to prevent the reductio is to abandon the idea that the links in the chain are connected via proper entailment and to adopt connections through probabilistic support. Holding on to the assumption of entailment means strenghtening the reductio argument; the argument is better combated by assuming regresses to be probabilistic, as we will explain in the following section.

6.4 How the Probabilistic Regress Avoids the Reductio

In a standard finite chain such as (6.1), where the arrow represents entailment, the ground A_{m+1} is all-important: the truth value of the target A_0 is a function of the truth value of A_{m+1} and of nothing else. The story is basically the same in the infinite case. However, there is then no ground, which is precisely the reason why it does not make sense to say that the target is justified. The concept of entailment is the culprit here, for it forces us to accept two things that are hard to combine, namely that the ground is all-important and non-existent at the same time. Exactly this combination precipitates the reductio argument. Nothing now restricts us in gratuituously constructing a rivalling regress that 'justifies' the target's negation, since the only restriction that matters, to wit the truth value of the ground, is conspicuous by its very absence.

The situation is entirely different in an infinite probabilistic regress. True, there too a ground is lacking. But this is irrelevant, for the probability value of the target is a function of the conditional probabilities alone. So it all de-

[24] Aikin 2011, 59-60 and Chapter 3.

[25] Harker 1984. Selim Berker takes a comparable route, offering the infinitist a way to avoid a fundamentalist regression stopper without running the gauntlet of the reductio argument — in Section 8.6 we will briefly come back to Berker.

pends on the question: What, in a justificatory chain, determines the value of the target? In a standard chain of entailments, the truth value of the target is determined by that of the ground, independently of the length of the chain. In a probabilistic chain, however, the length of the chain is relevant. If the probabilistic chain is finite, then the target's probability value is a function of both the unconditional probability of the ground and the conditional probabilities. As the chain gets longer, the influence of the ground decreases while the influence of the combined conditional probabilities increases. In the limit that the chain goes to infinity, only the conditional probabilities matter, and the rôle of the ground has died out (in the usual class). In this regard the difference between a non-probabilistic and a probabilistic regress could not be greater: in the former, the only variable that counts is a function of the ground, whereas in the latter the ground is of no significance whatsoever.[26]

We may conclude that the reductio argument misfires when the regress is a probabilistic one. The argument hinges on the assumption that the only variable which is responsible for the truth value of the target, namely the truth value of the ground, is non-existent. This absence of a ground allows us to concoct as many free-floating regresses as we wish, since the only variable that would determine the truth value of the target, viz. the truth value of the ground, is forever postponed and never actualized. In a probabilistic regress, on the other hand, the non-existent ground is not pertinent to the probability value of the target.

However, one could argue that this is too easy. For is it not possible to construct a rivalling probabilistic regress, i.e. a regress that supports the negation of our target? The only thing we would have to do is to come up with a set of conditional probabilities that numerically, and thus purely formally, bestow upon the target a probability value that for example exceeds the chosen threshold. If these conditional probabilities are not in any way connected to the world, we can cook them up *ad libitum*. We could then well end up with two rivalling probabilistic regresses, one probabilistically justifying A_0, and the other one probabilistically justifying $\neg A_0$.

Although the above argument is formally valid, it is not applicable to the issue that we are talking about. For it only works if the conditional probabilities are regarded as free variables, whose values may be chosen at will. We are however interested in epistemic justification, i.e. in the justification of propositions about our knowledge of how the world actually is, and this means that the conditional probabilities are not freely chosen. On the contrary, as we explained in Section 4.4, in a probabilistic regress the conditional

[26] See also Peijnenburg and Atkinson 2014a.

probabilities carry all the empirical thrust. Once we admit empirically determined conditional probabilities, we are not free to invent other conditional probabilities in a competing regress for the negation of our target proposition: the conditional probabilities are determined too, and they yield a probability for the negation of the target that is one minus the probability of the target. If the target probability clears a threshold of acceptance greater than one half, the probability of the negation of the target will not do so.

Our opponent might not be satisfied, and complain that it remains unclear *how* conditional probabilities can carry empirical information; after all, the interface between our propositions and the world is fraught with difficulty. To this we would reply that, of course, such difficulties exist, and they are well documented; the problem of finding a transducer between our propositions and the world cuts deep and might even turn out to be insoluble. But as we made clear in Section 4.4, our aim is not to say something about that problem: we are not trying to formulate an answer to the sceptic. Rather our aim is to draw attention to probabilistic regresses and to phenomena such as those of fading foundations and of the emergence of justification, and to point out that these phenomena have consequences for the age-old objections to infinite regresses.

Andrew Cling has argued that an infinite regress can only justify a proposition if a certain condition is satisfied, notably that the regress is not "pure fiction" but has "grounding in how things are, are likely to be, or are reasonably believed to be".[27] The trouble with infinitism, says Cling, is that this condition can only be satisfied if simultaneously the very idea of justification by an infinite regress is undermined. Our analysis indicates that Cling is correct if the justificatory regress is a regress of entailments, not if it is probabilistic. For a probabilistic regress, as we have seen, can probabilistically justify a proposition while still having entry points for the world in the form of the conditional probabilities.

We have provisionally argued that these conditional probabilities arise from experiments, but of course they are not indubitable, and they can be questioned in turn. In that case they become the targets of new probabilistic chains. As we will explain in Section 8.5, this takes us from one-dimensional chains to multi-dimensional networks, where the effect of fading foundations still obtains.[28]

---

[27] Cling 2004, 111; see also Moser 1985, who makes a point similar to that of Cling.

[28] William Roche doubts whether a probabilistic regress can take away Cling's worry (Roche 2016). We think that it can indeed, for the reasons explained here and in Sections 4.4 and 8.5.

6.5 Threshold and Closure Constraints

It would be foolhardy to claim that probabilistic support along a chain of propositions or beliefs is sufficient for their justification. An obvious objection to such a claim would be that, after all the contributions from the conditional probabilities have been summed, the resulting probability of the target might turn out to be less than a half, which means that, relative to this particular chain, the target would be more likely false than true. Under these circumstances one would not say that the chain justifies the target. Indeed, as we have stressed, something must be added to probabilistic support to achieve a sufficient condition for justification.

Although it is certainly not our ambition to answer the difficult question of sufficiency, we shall in this section discuss two additional candidate desiderata for justification. The first is simply a threshold constraint on the target probability; the second is a modified threshold requirement for a measure of justification that has been proposed by Tomoji Shogenji. We first look at the simple threshold constraint, using the tables in Chapter 4 as illustration. We recall the well-known fact that this constraint falls foul of the intuition that justification should be closed under conjunction. But should unrestricted closure be a desideratum for justification? We argue that it should not: closure should be required only for *independent* propositions. The simple threshold constraint does not respect this modified closure requirement, and so it should be rejected. Shogenji's threshold condition, however, does respect this modified closure requirement. What makes Shogenji's condition especially interesting for us, moreover, is that it sails between entailment and probabilistic support: it is stronger than mere probabilistic support, but weaker than entailment. It is therefore a refined desideratum for justification; but we are not so incautious as to claim that it is a sufficient condition.

The simple threshold constraint amounts to the introduction of a context-dependent threshold of acceptance, say t, that is greater than one-half, but less than one.[29] As a first attempt, we might propose that if q is justified to degree t by a single proposition, or by a finite or infinite chain of propositions, then there must be probabilistic support along the chain, *and* $P(q)$ must be not less than t. Here is an example. Suppose that we take $t = \frac{3}{4}$ and refer to the tables in Chapter 4. We see from Table 4.1 that $P(q)$ does *not* clear $\frac{3}{4}$ with a chain of ten or fewer intermediate A's, but that it does so with a chain of twenty-five or more intermediate A's.

[29] Carnap 1980, 43, 70, 107; Fitelson 2013.

For a second example, look at Table 4.2, and again let $t = \frac{3}{4}$. Now we see that $P(q)$ clears the threshold in all cases, even when there is only one intermediate A. The reason for this is simply that the ground p has a high probability; and in connection with the chosen values of the conditional probabilities α and β (0.99 and 0.04) this means that $P(q)$ already exceeds the threshold of $\frac{3}{4}$ after one step. Had α and β both been small, then the situation would have been very different; for then no number of steps would have been enough to reach the threshold, no matter how large the probability of p was. It can also happen that the value of $P(q)$ is larger than the threshold after a few steps, but sinks below the threshold if the chain gets longer. This can be illustrated by appealing to Table 4.2 again, and adopting the more demanding threshold of $t = 0.85$ instead of 0.75. With ten or fewer steps this more stringent threshold is exceeded, but with twenty-five or more steps we see that $P(q)$ has sunk below the new threshold. In such a case q might appear to be justified (to degree 0.85), but later, as the chain lengthens, we discover that this is not so.

Now consider still another example. Let the conditional probabilities both be very large, for example 0.99 and 0.96. Here again the target proposition, q, will have a probability well in excess of the threshold of $\frac{3}{4}$, even when there is only one intermediate A. And this is so irrespective of what the probability of p might be. Here the joint conditional probabilities are already doing all the work. On the other hand, if both conditional probabilities are very small, then the probability of q will be very small, again irrespective of $P(p)$. This is because the rule of total probability shows that $P(q)$ is an interpolation between the two conditional probabilities, $P(q|A_1)$ and $P(q|\neg A_1)$. In such a case the target could not be justified by the regress.

What these examples show is that the conditional probabilities, together with the unconditional probability of p, determine how long it takes before $P(q)$ reaches the threshold, if indeed it does so. Sometimes the unconditional probability of p has considerable influence, sometimes its influence is smaller: it is all contingent on the particular values. In the case of an infinite regress in the usual class, if the probability clears the threshold, this is achieved by the infinite set of conditional probabilities alone, without any contribution from p.

However, requiring that justification implies that the target probability meet a threshold of acceptance runs into difficulties, as we have intimated. For if target propositions q and q' are each supported by A_1, and if each meets some threshold, t, which is strictly less than one, it does not follow that the probability of the conjunction of q and q' meets t. Should we require that, if propositions q and q' are each separately *justified* by the same evidence A_1, then the proposition 'q and q'' is justified by the same evidence A_1? That is, should we require that justification is closed under conjunction? To see that an unqualified 'yes' would be too quick an answer, let us look at a simple example. Suppose that a fair die is tossed, but not yet inspected. Let q be the proposition 'the die shows 5', and q' be the proposition 'the die shows 6'. Let A_1 be the proposition 'the die shows more than 4'. Then $P(q) = P(q') = \frac{1}{6}$, and $P(q|A_1) = P(q'|A_1) = \frac{1}{2}$, so both q and q' are probabilistically supported by A_1. However, q and q' are incompatible with one another, so $P(q \wedge q') = 0$; and of course we would not want to claim that A_1 justifies the impossibility $q \wedge q'$. The conclusion is that we should not allow unlimited closure of justification under conjunction. This is of course the lesson that many people have drawn from the lottery paradox and similar quandaries concerning unrestricted closure of justification under conjunction. If one is justified in believing that ticket t_i in a fair lottery will lose, and that ticket t_j will also lose, is one justified in believing to the same extent that both t_i and t_j will lose? Evidently not, for the two failures to win are not independent of one another: if t_i loses, the chance that t_j will lose is reduced.

If unrestricted closure is forbidden, what would be a reasonable requirement concerning closure? Look at another example: suppose now that two coloured dice are tossed, but not yet inspected. Let q be the proposition 'the red die shows 5', and let q' be the proposition 'the blue die shows 6', and let A_1 be the proposition 'each die shows more than 4'. Once more $P(q) = P(q') = \frac{1}{6}$, and $P(q|A_1) = P(q'|A_1) = \frac{1}{2}$, so again both q and q' are probabilistically supported by A_1 to the same degree. Now q and q' are compatible, moreover they are independent of one another, both unconditionally and conditionally:

$$P(q \wedge q') = P(q)P(q') = \tfrac{1}{36}$$
$$P(q \wedge q'|A_1) = P(q|A_1)P(q'|A_1) = \tfrac{1}{4}\,.$$

Again A_1 supports q and q' probabilistically, but it also supports the conjunction, $q \wedge q'$, for $P(q \wedge q'|A_1) \geq P(q \wedge q')$. Note that the degree of probabilistic support that A_1 gives to the conjunction $q \wedge q'$ is not the same as the degree of support it gives to the conjuncts. However, if A_1 *justifies* $q \wedge q'$, then it *is* reasonable to require that A_1 justifies the conjunction to the same degree as it justifies the conjuncts. After all, if one is justified (to some extent) in expecting the red die to show 5, and also in expecting the blue die to show 6, on the basis of knowledge that each of the dice shows either 5 or 6, then one should be justified, to the same extent, in expecting that the red die shows 5 and the blue die shows 6, on the same knowledge basis. That the red die shows 5 does not influence whether the blue die shows 6. Evidently the requirement that the probability clear a threshold of acceptance is not an adequate criterion; and it must be rejected as a desideratum for justification. The problem now is to find a measure of justification that clears a threshold *and* respects the findings of the above dice scenarios, and others like it.

Tomoji Shogenji has constructed just such a measure of justification.[30] Suppose that q and q' are independent, both unconditionally and also when conditioned by A_1. Suppose further that both q and q' have measures of justification greater than some threshold of acceptance, s. Then Shogenji requires that their conjunction $q \wedge q'$ also has a measure of justification greater than s. Thus his measure $J(q,A_1)$, the justification that A_1 bestows on q, respects closure in the restricted sense.

Measure $J(q,A_1)$ is a function of the various probabilities associated with q and A_1. But which function should it be? There are three independent candidates for the arguments of the function, for example $P(q)$, $P(A_1)$ and $\alpha_0 = P(q|A_1)$. Shogenji's first step is to strike out $P(A_1)$, on the grounds that, if one were to conjoin to A_1 some independent and irrelevant proposition, I, the justification that $A_1 \wedge I$ gives to q should be the same as that given by A_1. But $P(A_1 \wedge I) = P(A_1)P(I)$, and so the degree of justification *would* be changed by the conjunction if the measure were to depend on $P(A_1)$. So the required measure of justification must be a function, f, of $P(q)$ and $P(q|A_1)$ alone:[31]

$$J(q,A_1) = f[P(q),P(q|A_1)].$$

This immediately rules out the confirmation measure

$$S(q,A_1) = P(q|A_1) - P(q|\neg A_1),$$

as a candidate for a measure of justification, since that may be rewritten as

$$S(q,A_1) = \frac{P(q|A_1) - P(q)}{1 - P(A_1)},$$

which is manifestly a function of $P(A_1)$, as well as $P(q)$ and $P(q|A_1)$.[32]

Evidently the standard measure of confirmation, D,

$$D(q,A_1) = P(q|A_1) - P(q),$$

---

[30] Shogenji 2012.

[31] Note that $P(q|A_1 \wedge I) = P(q|A_1)$, if I is independent of A_1 and of $q \wedge A_1$.

[32] $S(q,A_1)$ is of course the same as γ_0.

does satisfy Shogenji's first desideratum for J. As we remarked in Chapter 2, Carnap called this an "increase in firmness", the extent to which the probability of q is increased by conditioning it on A_1. Shogenji requires that $J(q,A_1)$ should increase if $P(q|A_1)$ increases while $P(q)$ is held fixed, and decrease if $P(q)$ increases while $P(q|A_1)$ is held fixed. It is clear that the measure D does these things.

Could D be the required measure of justification, J? Not so, as we can see from the example of the coloured dice, since

$$D(q,A_1) = D(q',A_1) = \tfrac{1}{2} - \tfrac{1}{6} = \tfrac{1}{3}$$
$$D(q \wedge q',A_1) = \tfrac{1}{4} - \tfrac{1}{36} = \tfrac{2}{9}\,,$$

which are different, whereas the degree of justification of the conjunction of the independent propositions q and q' should be the same as that for q and q' separately. But not only does D not satisfy this closure requirement, none of the many other measures of confirmation do so either![33]

Shogenji shows that the following new measure does satisfy the requirement of closure:

$$J(q,A_1) = 1 - \frac{\log P(q|A_1)}{\log P(q)}\,. \tag{6.11}$$

Although this is not the only function that satisfies Shogenji's desiderata for a measure of justification, it has been proved that all functions that do so are ordinally equivalent to Shogenji's J function.[34] That is to say, if A_1 gives a higher degree of justification to one proposition than it does to another, according to the measure (6.11), then this ordering of justificatory degrees will be the same for any other measure that satisfies Shogenji's conditions. We may say that the measure (6.11) is the unique solution of the problem, up to ordinal equivalence. A proof of the above is given in Appendix B; but here we shall simply check that the Shogenji measure works properly for our coloured dice. From (6.11) we calculate

$$J(q,A_1) = J(q',A_1) = 1 - \frac{\log \frac{1}{2}}{\log \frac{1}{6}}$$
$$J(q \wedge q',A_1) = 1 - \frac{\log \frac{1}{4}}{\log \frac{1}{36}} = 1 - \frac{2\log \frac{1}{2}}{2\log \frac{1}{6}} = 1 - \frac{\log \frac{1}{2}}{\log \frac{1}{6}}\,.$$

[33] See Atkinson, Peijnenburg and Kuipers 2009 for a list of ten measures of confirmation. A seminal paper on different measures of confirmation is Fitelson 1999.
[34] Atkinson 2012.

Thus $J(q,A_1) = J(q',A_1) = J(q \wedge q',A_1)$, so if $J(q,A_1) \geq s$ and $J(q',A_1) \geq s$, for some s, it is trivially the case that $J(q \wedge q',A_1) \geq s$. In words, if q and q' are Shogenji-justified to the same degree, their conjunction is also Shogenji-justified to that degree, as should be the case.

If the degree of Shogenji justification that A_1 gives to q is not less than s, i.e. $J(q,A_1) \geq s$, then

$$1 - \frac{\log P(q|A_1)}{\log P(q)} \geq s\,,$$

and this can be recast in the form (see Appendix B)

$$P(q|A_1) \geq [P(q)]^{1-s}\,. \tag{6.12}$$

Note that when $s = 0$ — so there is effectively no threshold — this inequality reduces to

$$P(q|A_1) \geq P(q)$$

which is equivalent to our condition of probabilistic support (or neutrality, in the case of the equals sign). On the other hand, when the threshold is at its maximum, so that $s = 1$, the relation becomes

$$P(q|A_1) \geq 1\,, \quad \text{which of course implies} \quad P(q|A_1) = 1\,,$$

since no probability can be greater than one. This is the probabilistic condition that corresponds to entailment.

For non-extremal values of the degree s, the measure J interpolates between probabilistic support and entailment. Since entailment is too strong a requirement for a viable understanding of justification, and probabilistic support is too weak, it is very suggestive that this measure of Shogenji may be a step in the right direction in the search for the holy grail of a sufficient condition for justification.

6.6 Symmetry and Nontransitivity

In this chapter we have discussed the two conceptual objections to infinite epistemic chains that occur most frequently in the literature, the no starting point objection and the reductio argument, and we argued that they lose their bite when justification is seen as something that involves probabilistic support rather than entailment. Since probabilistic support is not enough for justification, we looked in the previous section at two candidates for add-ons.

One could however raise objections to the very concept of probabilistic support itself. It is after all the child of a theory that is beset by a number of serious pitfalls: the problem of old evidence, the problem of spurious relations, of irrelevant conjunctions, of randomness, and more.

Whenever a theory encounters problems, either we reject it because the problems are too serious, or we continue to use it, trying in the meantime to put things right. In the case of Kolmogorovian probability theory the choice seems clear. Aside from exotics such as quantum probability and Robinsonian nonstandard analysis, Kolmogorov's calculus is very much the only game in probability town. When in epistemology we say that one proposition 'probabilifies' another, it would be wise to take Kolmogorov's system seriously, at least until we have found a better interpretation of 'probabilifies'.

This book is not the place to dwell on all the snags and hitches of Kolmogorovian probability. Yet there are two properties of the concept of probabilistic support that require some further consideration, since epistemologists may find them troublesome in the context of epistemic justification. The first is the fact that probabilistic support is *not transitive* and the second one is that it is *symmetric*.

Many epistemologists have explicitly or implicitly expressed the view that epistemic justification is transitive: if A_n is justified by A_{n+1} and A_{n+1} is justified by A_{n+2}, then A_n is justified by A_{n+2}. Such a view is of course apposite if justification is perceived as entailment or implication, for then justification is transmitted unchanged from one proposition to another. But if justification is understood as involving probabilistic support, then transitivity may be violated. It all depends on what must be added to the relation of probabilistic support to yield that of justification. For example, if justification were equivalent to probabilistic support plus the Markov condition, then justification would be transitive, since transitivity is a property of probabilistic support when the Markov restriction is in place. If however justification were equivalent to probabilistic support plus a threshold condition, then it would not be transitive. As we have made clear, we refrain from making any claims about what has to be added to probabilistic support in order to yield justification. The point to make here is just that probabilistic support as a necessary condition for justification entails nothing about the transitivity of justification.

A similar argument applies to the required asymmetry of justification. When considered qualitatively, probabilistic support is symmetrical: if A_{n+1} supports A_n, then A_n supports A_{n+1}. However, from the fact that probabilistic support is (qualitatively) symmetric, it does not follow that *justification* is qualitatively symmetric as well. An argument parallel to the one just given about transitivity shows that the symmetry of probabilistic support entails nothing about the symmetry of justification. In fact the example of the Markov condition fits the bill here, too. For if A_{n+1} supports A_n, and A_{n+1} screens off A_n from all 'ancestor' propositions in the chain, i.e. A_m where $m > n+1$, then A_n will in general not screen off A_{n+1} from all 'descendent' propositions, i.e. A_m where $m < n$. Thus if justification were equivalent to probabilistic support plus the Markov condition, it would not be qualitatively symmetric. As we stressed above, the Markov model is not meant to be taken as a serious candidate as to how justification should be defined: it merely shows that justification can be asymmetric, even though probabilistic support is symmetric. A formal demonstration of this fact is as follows. Consider these three statements:

(1) if A_{n+1} justifies A_n, then A_{n+1} probabilistically supports A_n
(2) if A_{n+1} probabilistically supports A_n, then A_n probabilistically supports A_{n+1}
(3) if A_{n+1} justifies A_n, then A_n justifies A_{n+1}.

The point is that (3) does not follow from (1) and (2). What does follow from the latter two statements is:

(3′) if A_{n+1} justifies A_n, then A_{n+1} probabilistically supports A_n and A_n probabilistically supports A_{n+1}.

The consequent of (3′) expresses the fact that probabilistic support is symmetric. But this does not mean that justification is symmetric; it does not follow from this that A_n justifies A_{n+1}.

It is important to note that the matter is quite different with respect to fading foundations. The effect of fading foundations is not a property like transitivity or symmetry. As a result, it does follow that justification implies the existence of fading foundations (within the usual class). In detail:

(1″) if A_{n+1} justifies A_n, then A_{n+1} probabilistically supports A_n
(2″) if A_{n+1} probabilistically supports A_n, and the conditional probabilities belong to the usual class, then fading foundations ensue
(3″) if A_{n+1} justifies A_n, and the conditional probabilities belong to the usual class, then fading foundations ensue.

In this case (3″) does follow from (1″) and (2″). Irrespective of whether we are talking about probabilistic support or about epistemic justification, the phenomenon of fading foundations is the same, the reason being that the latter does not have a meaning independent of probability theory, which we take to be necessary for justification: if there is no probabilistic support, then there is no justification. The properties of transitivity and symmetry, on the other hand, do not need to refer to probability theory in order to have the meanings that they have. Thus under justification the influence of the probability of the ground on the probability of the target decreases as the number of links in the chain increases. And in the limit that the number of links goes to infinity, this probabilistic influence vanishes completely, leaving the probability of the target fully independent of the probability of the ground.