VIGOR: Virtual Interaction with Gravitational Waves to Observe Relativity

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Abstract. In 2015, a century after Albert Einstein published his theory of general relativity, the Laser Interferometer Gravitational-wave Observatory (LIGO) detected gravitational waves from binary black holes fully consistent with this theory. Our goal for VIGOR (Virtual-reality Interaction with Gravitational waves to Observe Relativity) is to communicate this revolutionary discovery to the public by visualizing the gravitational waves emitted by binary black holes. VIGOR has been developed using the Unity game engine and VR headsets (Oculus Rift DK2 and Samsung Gear VR). Wearing a VR headset, VIGOR users control an avatar to "fly" around binary black holes, experiment on the black holes by manipulating their total mass, mass ratio, and orbital separation, and witness how gravitational waves emitted by the black holes stretch and squeeze the avatar. We evaluated our prototype of VIGOR with high school students in 2016 and are further improving VIGOR based on our findings.

Keywords: Virtual reality \cdot Physics education \cdot Head-Mounted display \cdot Binary black holes \cdot Gravitational waves

1 Introduction

In 1915, Albert Einstein proposed his theory of general relativity as a replacement for Isaac Newton's earlier theory of gravity [1, 2]. General relativity predicts the existence of black holes, objects whose gravity is so intense that they possess event horizons from within which nothing can escape. Although astronomers were initially hesitant to accept the existence of such exotic objects, starting in the 1960's evidence began to mount supporting the existence of stellar-mass black holes formed in the collapse of massive stars and supermassive black holes (billions of times the mass of the Sun) in galactic centers. These black holes often occur in binaries in which two black holes orbit each other. General relativity predicted that binary black holes would emit gravitational waves which are 'ripples' in the fabric of space-time caused only by some of the most violent and energetic processes in the Universe, such as colliding black holes and collapsing stellar cores. Those waves would disrupt space-time in such a way that 'waves' of distorted space would radiate from the source like the movement of waves away from a stone thrown into a pond. Furthermore, these ripples would travel

at the speed of light through the Universe, carrying information about their cataclysmic origins, as well as invaluable clues to the nature of gravity itself.

The National Science Foundation has funded an experiment called the Laser Interferometer Gravitational-wave Observatory (LIGO) designed to detect gravitational waves from binary stellar-mass black holes billions of light years away from Earth [3]. On September 14, 2015, a century after Einstein published his theory, LIGO discovered gravitational waves from the merger of 36 and 29 solar-mass black holes 1.3 billion light years away [4]. Subsequent observations during LIGO's "O1" observing run revealed gravitational waves from a second black-hole merger [5] and a third event of lower signal-to-noise ratio also consistent with a black-hole merger [6]. These observations represent the dawn of gravitational-wave astronomy, an entirely new window into our Universe beyond the electromagnetic spectrum of conventional astronomy. LIGO anticipates observing several additional gravitational-wave sources (including perhaps the first gravitational waves from a neutron-star merger) during its second "O2" observing run, which began in November 2016 and will continue through mid-2017. In addition, pulsar-timing arrays [7-9] and the space-based Laser Interferometer Space Antenna (LISA) [10] seek to observe longer-wavelength gravitational waves emitted by binary supermassive black holes. The future of gravitational-wave astronomy appears bright, but without a greater intuitive understanding of gravitational waves the public will be unprepared to appreciate progress in this field.

1.1 Vigor

Our goal for VIGOR (Virtual-reality Interaction with Gravitational waves to Observe Relativity) is to communicate this revolutionary discovery to the general public by visualizing the gravitational waves emitted by binary black holes. Since the gravitational waves travel hundreds of millions of light years from the merging black holes to Earth, they are extremely weak on Earth and induce only tiny fluctuations in the distances between LIGO's test masses. Although we cannot take students on field trips to where the gravitational waves are far stronger, virtual reality provides them with an immersive interactive experience as they fly through a virtual environment surrounding the two black holes. Our hypothesis is that the immersive and interactive experience that VR provides assists understanding gravitational waves to a greater degree than passive traditional presentations on a flat screen.

VIGOR has been developed using the Unity game engine and VR headsets (Oculus Rift DK2 and Samsung Gear VR). Wearing a VR headset, VIGOR users control an avatar to "fly" around binary black holes as they orbit each other. VIGOR users experiment on binary black holes by manipulating their total mass, mass ratio, and orbital separation. Changes to these parameters lead to changes in the amplitude, polarization, and frequency of the gravitational waves emitted by the black holes. VIGOR users experience gravitational waves as the avatar accompanying them on their journey is stretched and squeezed in response to the gravitational waves at their position, much like the LIGO detectors are affected by real gravitational waves. (The users can exaggerate the magnitude of gravitational waves so that their impacts are easily observable.) By witnessing how the changes to binary black holes'



Fig. 1. Binary black holes and an avatar in VIGOR

parameters (which result in changes in the gravitational waves) deform the avatar differently, the users gain an intuitive understanding of physical concepts like amplitude, frequency, and polarization that describe gravitational waves (Fig. 1).

More generally this research is situated in the larger field of visual mathematics where computer technology bridges art and science by enabling interactive experience of the behavior of mathematical systems [11].

2 Related Work

Over the past three decades, educators have developed a wide variety of computer applications for teaching physics concepts through simulations. Most of these are desktop-based applications and few are immersive VR systems.

2.1 Desktop-Based Instructional Physics Simulators

Notable examples of desktop-based instructional physics simulators include PhET interactive simulations [12], the web physics package called Physlets [13] and Open Source Physics [14]. PhET is a set of interactive web-based simulations of physical phenomena. PhET is fairly broad in scope as it includes simulations in physics, biology, chemistry, earth science, and mathematics. PhET's "force and motion" example allows the user to interactively position different virtual characters on either side of a "tug of war" application, thus learning about net forces when summed together. The Physlet example of a "central force" is one where simple orbital mechanics are simulated to study angular momentum and energy. The number of interactive options in Physlets is less than that of most PhET simulations. In Open Source Physics there are similar options for exploring mechanics and introductory physics.

2.2 Immersive VR Instructional Physics Simulators

The advantages that immersive VR instructional physics simulators have over desktop-based ones include: (a) large fields of regard (i.e., the total size of the visual field surrounding the user [15], (b) stereoscopy, and (c) head tracking for natural viewing. These advantages provide additional benefits, such as improved spatial

understanding [15] and a greater sense of presence (i.e., the sense of "being there" in a virtual environment [16]). Despite the advantages of immersive VR systems, there are a limited number of examples of immersive VR instructional physics simulators, such as [17, 18]. The closest to VIGOR we have found is Teaching Physics Using Virtual Reality which is a desk-top simulator specifically to teach Einstein's theory of special relativity [18]. We believe VIGOR is the first attempt to teach gravitational waves utilizing the advantages that immersive VR technology can offer.

3 Computing Gravitational Waves

Einstein's equation of general relativity, $G_{\mu\nu} = 8\pi T_{\mu\nu}$, relates the curvature of spacetime (described by the metric $g_{\mu\nu}$ from which the Einstein tensor $G_{\mu\nu}$ can be calculated) to the stress-energy tensor $T_{\mu\nu}$ sourced by the matter and energy in our Universe. It is a generalization of Newton's famous inverse-square law $a = -GM/r^2$ relating the gravitational acceleration a of an object to the mass M and distance r of a second object. Einstein's equation is actually a series of coupled nonlinear partial differential equations that in general can only be solved numerically [19-21]. However, far from black holes where the metric is nearly equal to the Minkowski metric $\eta_{\mu\nu}$ of flat spacetime (for which Euclidean geometry holds), Einstein's equation can be linearized in the metric perturbations $h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$. These metric perturbations satisfy the same wave equation obeyed by far more familiar sound and light waves, i.e. they can be decomposed into sinusoids with wavelength λ and frequency f related by $c = \lambda f$ where the wave speed c is equal to the speed of light. This is why we refer to them as gravitational waves. However, these waves are not density and pressure perturbations (like sound waves) or electromagnetic perturbations (like light), but perturbations to the metric that determines spacetime itself.

As gravitational waves pass through an object (like LIGO's 4 km long interferometers), they induce a tidal field transverse to the direction of their propagation that oscillates at the gravitational-wave frequency f. These tides are similar to the tides that the Moon exerts on the Earth's oceans in that they stretch the object in one direction (along the line of separation between the Earth and Moon in the case of lunar tides) while squeezing it in the perpendicular direction. In the post-Newtonian approximation valid when the relative velocity of the binary black holes is small compared to the speed of light, the gravitational waves are generated by the time-varying quadrupole moment of the spacetime itself rather than each individual black holes (nothing can be emitted classically from a black hole's event horizon). These gravitational waves travel radially outwards from the center of mass of the binary with frequency f twice the orbital frequency of the binary and amplitude inversely proportional to the distance from the binary (leading to an inverse-square law energy flux). Since the tidal fields induced by these gravitational waves are transverse to the radial direction, there are two distinct gravitational-wave polarizations, similar to the two polarizations of electromagnetic waves. The directions of the two linear gravitational-wave polarizations are rotated by 45° with respect to each other, unlike 90° for the two linear electromagnetic polarizations, because tidal stretching is unchanged by a 180° rotation, unlike an electric-field vector that is preserved by a full 360° rotation. The two gravitational-wave polarizations

are referred to as h_+ and h_{\times} because the "+" and " \times " symbols are similarly related by a 45° rotation.

More quantitatively, the gravitational waves emitted by binary black holes are given in the Newtonian, quadrupole-moment approximation by the Eqs. (1, 2) [22]

$$h_{+}(t) = -(2m_{1}m_{2}/rDM)[1 + (L \bullet N)^{2}]\cos 2\Phi(t)$$
(1)

$$h_{\times}(t) = it(4m_1m_2/rDM)(\boldsymbol{L} \bullet \boldsymbol{N})\sin 2\Phi(t)$$
⁽²⁾

where m_1 and m_2 are the masses of the two black holes, $M = m_1 + m_2$ is the total mass, r is the distance between the black holes, D is the distance from the center of mass of the black holes to the observer, L is a unit vector in the direction of the orbital angular momentum of the binary, N is a unit vector pointing from the observer to the center of mass, and $\Phi(t)$ is the orbital phase defined to be zero when the binary separation is parallel to the principal + direction $P = N \times L/[N \times L]$ with respect to which the + and × polarizations are defined. We define $E = P \times N$ to be a second unit vector in the transverse direction. An object in a gravitational-wave field will be stretched by a factor F = 1 + A in the direction S and squeezed by a factor 1/F in the orthogonal direction T, where the Eqs. (3, 4, 5, 6) give these quantities in terms of the previously defined gravitational waves and transverse basis vectors.

$$A = F - 1 = \sqrt{(h_{+}^{2} + h_{\times}^{2})}$$
(3)

$$S = \boldsymbol{E}\cos\theta - \boldsymbol{P}\sin\theta \tag{4}$$

$$T = \mathbf{P}\cos\theta + \mathbf{E}\sin\theta \tag{5}$$

$$\tan 2\theta = h_{\times}/h_{+} \tag{6}$$

These equations can be challenging to visualize even for a physicist, reinforcing the need for a project like VIGOR to help both the public and scientists build conceptual understanding of gravitational waves. As users of VIGOR and their accompanying avatar virtually "fly" around the black holes as shown in Fig. 1, their distance D from the center of mass of the black holes changes, as does the direction of the unit vector N pointing from their position to the center of mass. According to the above equations, this changes the gravitational waves h_{\pm} and h_{\times} at the avatar's position, the factor F by which the avatar is stretched and squeezed by the resulting tidal field, and the directions S and T along which this stretching and squeezing occurs. Users of VIGOR will experience the increased stretching of their avatars as they approach the black holes, and the change in gravitational wave polarization from linear (only h_{+}) to circular (equal magnitudes of h_{+} and h_{\times} out of phase by 90°) as they move from the orbital plane of the black holes to viewing them from above. Users of VIGOR can also change the masses m_1 and m_2 of the two black holes and the distance r between them. Shrinking r (which occurs naturally as gravitational waves extract orbital energy from the binary) will increase both the amplitude F = 1 + A and frequency f of the tidal stretching, since $A \propto 1/r$ and $f = 2d\Phi/dt = 2\sqrt{(GM/r^3)}$. An exaggeration factor can be

used to increase the stretch factor F to improve visualization. Our goal for VIGOR is to provide an interactive and immersive environment in which the user can develop an intuitive understanding of gravitational waves as they watch the tidal distortion of their avatars change as they navigate the 3D environment and adjust the binary black-hole parameters.

4 System Requirements and Design

To implement the virtual environment of VIGOR including the orbiting black holes, the gravitational waves they emit, and the user avatar, we chose the following software and hardware systems.

4.1 Software: Unity 5 with Oculus Mobile Utilities

Unity 5 is one of the most popular game engines and many games for entertainment and education have been developed and played on Unity 5. It offers a wide variety of ready-to-use functions, components (e.g., geometries, lights, and materials), API¹ for multiple VR devices and other input/output devices, and example codes for game creations. Unity 5 allows users to create highly realistic 3D games and simulations with which they can interact and modify. The most significant advantages that Unity 5 provides us for VIGOR are its VR supports and cross-platform nature: With only minor changes in the sources code, using Unity 5 we can utilize VR headsets without cumbersome low level coding and deploy VIGOR on many platforms, such as table-top/laptop computers, tablets and mart phones with or without no VR-devices. By taking advantages of Unity 5's API and the software developer kits for the VR headsets, Oculus Rift DK2 and Samsung Gear VR, we integrated Unity with Oculus Rift DK2 and Samsung Gear VR as described below.

4.2 Hardware: Samsung Gear VR

We decided to develop VIGOR as an immersive VR application because immersive VR applications have been shown to provide greater spatial understanding than non-immersive applications [15]. There were several VR hardware products to consider. Initially we developed VIGOR for Oculus Rift DK2. Overtime we realized Samsung Gear VR's advantages over Oculus Rift. With a 101° field of view (FOV), 360° field of regard (FOR [23]), a resolution of 1280×1440 per eye, and an internal tracker for rotational head tracking, Samsung Gear VR provides an immersive user experience. Unlike most other VR headsets, Samsung Gear VR provides a focal adjustment that accommodates nearsightedness and farsightedness. The device weighs

¹ Application Program Interface (APS) is a set of subroutine definitions, protocols, and tools for building application software.

only 318 g and could use alone with a Samsung phone without any complicated cables and systems.

Thus, the device is highly mobile that can deploy anywhere without wiring. Moreover, Samsung Gear VR has a built-in touch panel and button on a side of the headset, while Oculus Rift DK2 requires an additional input device, such as a keyboard, mouse, or joystick. Although Samsung Gear VR's touchpad and back button offer easy-to-use interactions without a need for additional devices, the functionalities that can be assigned to them are limited. Given the scale of VIGOR, we assumed that Samsung Gear VR's interface capabilities were enough for our interaction and navigation methods and developed our VIGOR further for Samsung Gear VR.

4.3 Parameter Input and Navigational Method

For parameter input, we used a floating menu and ray-casting to select and change items in the menu. Ray-casting is a method that casts an infinite ray to represent a forward vector from the user's point of view towards the target (Fig. 2). By looking around, the user can target the ray at any objects in the scene. Tapping on back button of a Samsung Gear VR headset brings up the floating menu. The menu has the options to increase or decrease the exaggeration factor, total mass, mass ratio and binary separation. The user can increase or decrease one of the options by tapping on the touch panel until a desired parameter value is obtained. The method allows the user to interact with the floating menu in a 3D space in a manner similar to the use of mouse buttons with a 2D interface.



Fig. 2. Floating menu and ray-casting

In the experiment (detailed in the next section), we used a modified human joystick method for navigation. The method allows the user to travel in the 3D space forward or backward to the direction which the user is looking at by moving the torso forward or backward physically. Unlike a traditional human joystick method [23], the modified one allows the user to move around without leaving a seat. It is intuitive and easy to navigate with method; however, four of the seventeen participants reported that they felt dizzy while navigating in VIGOR. We suspected that the light motion sickness that they experienced was caused by the navigation method and after the experiment we

switched from the modified human joystick method to a gaze-directed steering method. Gaze-directed steering makes use of ray casting technique where the ray is cast from the user's point of view towards the target. With a gaze-directed steering method the user looks at the direction that she/he wants to go and touches on the touch pad. Then, the avatar travels to that direction until the user lifts the finger off the touch pad. Because this method and ray-casting use the same forward vector from the user's point of view, we implemented a simple mechanism to separate those two: While in menu mode, traveling is locked; to travel again, the user needs to close the menu. We have not tested the new version of VIGOR with the gaze-directed steering method yet and have not proved that gaze-directed steering reduces the occurrences of motion sickness in users.

5 User Experience Experiment

In summer 2016, 17 female participants of ages 14 to 16 in the Women in Physics Advanced Camp [24] at the University of Texas at Dallas were provided with the opportunity to test a prototype of VIGOR using Samsung Gear VRs (Fig. 3). The campers interacted with VIGOR team members, watched a five-minute YouTube video entitled, "LIGO Detects Gravitational Waves" [25], and experienced VIGOR for approximately 10 min each. A 2-D VIGOR simulation was also displaced on a screen for passive participant viewing. At the end of their experience, each participant was asked to fill out a Likert-scale questionnaire created in-house to evaluate both the VIGOR experience and impacts of the experience on participant attitudes towards science and technology (Fig. 4). We followed the headset manufacturer safety recommendations and University of Texas at Dallas's Institutional Review Board protocols and therefore asked the participants to be seated while wearing the VR headset for their safety.

On the questionnaire 87% of the participants reported that "participating" in the VR experience with VIGOR was interesting and engaging while 67% reported that



Fig. 3. VIGOR experiment

	Strongly Agree	Agree	Neither Agree nor Disagree	Disagree	Strongly Agree
I found the YouTube video(s) to					
be interesting and engaging.					
I found watching the interactive					
simulation to be interesting and					
engaging.		·			
I found participating in the virtual					
reality experience to be					
interesting and engaging.					
I found controlling the interactive					
simulation to be interesting and					
engaging.					
I found watching the interactive					
simulation to be more interesting					
and engaging than the YouTube					
video(s).			-		
I found participating in the virtual					
reality experience to be more					
interesting and engaging than the					
YouTube video(s). I found controlling the interactive		-			
simulation to be more interesting					
and engaging than the YouTube					
video(s).					
Video(3).					
I was aware of discovery of	· · · · ·	<u> </u>			
gravitational waves before this					
activity.					
I was interested in the science of					
gravitational waves and black					
holes before this activity.					
I am more interested in the					
science of gravitational waves					
and black holes after participation					
in this activity.					
I am more interested in science					
after participation in this activity.					
I am more interested in					
technology after participation in					
this activity.					

Fig. 4. Visualizations of binary black holes and gravitational waves survey

"watching" an interactive simulation was interesting and engaging, indicating the importance of the active participation that VR technology provides. It remains unclear which component of the interactive VR experience had the greater impact on the increase in participant engagement: the active participation or the VR itself. All participants reported that they were more interested in "the science of gravitational waves and black holes," "science," and "technology" after the VIGOR experiment. This first test of the VIGOR experience in an informal education setting did not assess participant understanding of the science of gravitational waves.

One notable concern with the experiment was that 4 out of the 17 participants reported that they felt dizzy while they were navigating in VIGOR. We speculate that the participants' dizziness was caused by the navigational method (Sect. 4.3) used in the version of VIGOR we tested. Samsung Gear VR and Oculus Rift DK2 offer different advantages for the user of VIGOR. We selected Samsung Gear VR over Oculus Rift DK2 for the experiment mainly because Samsung Gear VR is wireless and easy to set up. The wireless input of Samsung Gear VR was convenient but limiting:

Samsung Gear VR's single button alone did not allow users to navigate through the virtual environment as freely as we wanted them to be able to do so. Instead, the seated users used their upper torsos to move in space, turning them into human joysticks. As the users moved through the space, the view rapidly changed. We suspect the rapidly changing view was the responsible for the participants' dizziness. To determine the cause and a solution for it, we need to develop VIGOR further and conduct more experiments.

6 Conclusions and Future Work

In 2015 gravitational waves were directly detected by LIGO confirming Einstein's general theory of relativity and news media covered the discovery in early 2016. Although the discovery of gravitational waves may be one of the most significant scientific breakthroughs of this century and received a great deal of media attention, the public has limited insight into this exciting new phenomenon. We have developed a VR system for facilitating the general public's understanding of gravitational waves that we call VIGOR. VIGOR uses a Samsung Gear VR headset to let the user fly around binary black holes accompanied by an avatar, manipulate the black holes' parameters, and view how the gravitational waves emitted by the black holes stretch and squeeze the avatar in order to gain an intuitive understanding of physical concepts like amplitude, frequency, and polarization that describe gravitational waves. The result of our experiment with a VIGOR prototype and high school students was summarized in this paper.

For the next version of VIGOR we plan to (1) optimize the gaze-directed steering navigation method, (2) optimize VIGOR's scene by reducing draw calls, using low poly models and baking light maps to achieve a better frame rate, and (3) add a handheld wireless controller for better interactions. We believe that the improved navigational method, a better frame rate, and an introduction of a handheld wireless controller can reduce the risk of dizziness for future VIGOR users. Moreover, the current VIGOR supports two avatars which are a human figure and a small planet like the Earth. We plan to make a wide variety of avatars available to appeal to all demographic groups.

As this study is about teaching abstract areas of physics using simulations, we also plan to research if a bimodal virtual environment which takes advantages of visual and auditory stimuli can enhance learning. It has been reported that students were more engaged in highly visual phenomena when they were allowed to change variables in an interactive simulation of "real time relativity" [26]. We believe that the use of sound sources [27] in a virtual environment to support the visual models will help lower the attentional load² on users. This is because humans have a better sense of detecting the direction of a sound source in a horizontal plane [28] and the use of sound sources in

² Attention load – all the information that can be perceived voluntarily within the brain's capacity [35]. According to load theory, attentional selection of information occurs during early and late stages of processing.

virtual environments where users can orient themselves will reduce the user's information processing time in the environment.

Future versions of VIGOR will also incorporate additional aspects of the physics that describes binary black holes. We will include radiation reaction which naturally decreases the distance r between the black holes as gravitational waves extract energy from the binary. This radiation reaction causes the black holes to eventual merge into a single larger black hole that rings down to rest like a bell struck by a clapper. The distinctive shape of the inspiral-merger-ringdown signal which increases in frequency with time is what allows LIGO to distinguish faint events from much larger background noise. Real binary black holes are also described by spins S_1 and S_2 in addition to their masses m_1 and m_2 [29, 30]. These spins can be misaligned with the orbital angular momentum of the binary, much like the 23° inclination of the Earth's axis with respect to the ecliptic responsible for the seasons. Coupling between misaligned black-hole spins and the orbital angular momentum causes the direction of the angular momentum L to vary with time, making the orbital plane of the black holes precess and changing the gravitational-wave emission through the dependence of h_+ and h_{\times} on L given by the equations of Sect. 3. We will allow users of VIGOR to manipulate the black-hole spins and experience the resulting spin-modulated gravitational waves using new solutions for black-hole spin precession derived by one of the authors [31, 32]. Double-spin precession is not trivial to visualize and we believe that even physicists would appreciate being able to explore the spin parameter space and see the effects on orbital-plane precession and gravitational waves using VIGOR. The ability to manipulate the black-hole spins will help VIGOR users appreciate the gravitational-wave signatures that allow LIGO to measures black-hole spins. These spin measurements will provide insight into the stellar evolution responsible for astrophysical binary black-hole formation [33, 34]. Once the next version of VIGOR is completed, we plan to conduct a larger-scale usability test before exploring additional opportunities for informal education and outreach.

Moreover, we are developing, mVIGOR, a mobile version of VIGOR for tablet computers. mVIGOR will not require peripherals or provide users with immersive experience (Sect. 4.2), while it is being developed using Unity 5 like VIGOR. One goal we have for mVIGOR is to reach those who have no virtual reality equipment. The use of mVIGOR will also provide a way to measure the relative impact of user engagement on the interactive VIGOR experience with and without the VR component. Another goal is to compare the user's experience and understanding of gravitational waves between VIGOR and mVIGOR to test our hypothesis that gravitational waves are understood better in an immersive environment than in a non-immersive environment.

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