11 River Basin Modeling

Multipurpose river basin development typically involves the identification and use of both structural and nonstructural measures designed to increase the reliability and decrease the cost of municipal, industrial, and agriculture water supplies, to protect against droughts and floods, to improve quality, to provide for commercial navigation and recreation, to enhance aquatic ecosystems, and to produce hydropower, as appropriate for the particular river basin. Structural measures may include diversion canals, reservoirs, hydropower plants, levees, flood proofing, irrigation delivery and drainage systems, navigation locks, recreational facilities, groundwater wells, and water and treatment treatment plants along with their distribution and collection systems. Nonstructural measures may include land-use controls and zoning, flood warning and evacuation measures, and economic incentives that affect human behavior with regard to water and watershed use. Planning the development and management of water resource systems involves identifying just what and when and where structural or nonstructural measures are needed, the extent to which they are needed, and their combined economic, environmental, ecological, and social impacts. This chapter introduces some modeling approaches for doing this. Having just reviewed some water quality modeling approaches in the previous chapter, this chapter focuses on quantity management.

11.1 Introduction

Various types of models can be used to assist those responsible for planning and managing various components of river systems. These components include streams, rivers, lakes, reservoirs, and wetlands, and diversions to demand sites that could be within or outside the basin boundaries. Each of these components can be impacted by water management policies and practices. The management of any single component can impact the performance of other components in the basin. Hence, for the overall management of the water in river basin systems, a systems view is usually taken. This systems view requires the modeling of multiple interacting and interdependent components. These multicomponent models are useful for analyzing alternative designs and management policies for improving the performance of integrated river basin systems.

The discussion in this chapter is limited to water quantity management. Clearly the regimes of flows, velocities, volumes, and other properties of water quantity will impact the quality of that water as well. However, unless water allocations allocations and uses are based on requirements for water quality, such as for the dilution of pollution, water quality does not normally affect water quantity. For this reason among others, it is common to separate discussions of water quantity

D.P. Loucks and E. van Beek, *Water Resource Systems Planning and Management*,
DOI 10.1007/978-3-319-44234-1_11

management from water quality management (Chap. 10). However, when attempting to predict the impacts of any management policy on both water quantity and quality, both water quantity and quality models are needed.

This chapter begins with a discussion of selecting appropriate model time periods that will depend on the issues being addressed as well as on the variability of the water supplies and demands. Discussed next are methods for estimating streamflows at various sites of interest throughout a basin based on gage (measured) flows at other sites. Following these discussions several methods are reviewed and compared for estimating reservoir storage requirements for water supplies. Model components are defined for withdrawals and diversions, and for reservoir storage. Reservoir storage can serve the needs over time for water supply, flood control, recreation, and hydroelectric power generation. Next flood control structures, such as levees and channel flow capacity improvements at potential flood damage in a river basin are introduced. These components are then combined into a multiple purpose multi-objective planning model for a hypothetical river basin. The chapter concludes with an introduction to some dynamic models for assisting in the scheduling and time sequencing of multiple projects within a river basin.

11.2 Model Time Periods

When analyzing and evaluating various water management plans designed to distribute the natural unregulated flows over time and space, it is usually sufficient to consider average conditions within discrete time periods. In optimization models, weekly, monthly, or seasonal flows are commonly used as opposed to daily flows. The shortest time period duration usually considered in optimization models developed for identifying and evaluating alternative water management plans and operating policies is one that is no less than the time water takes to flow from the upper end of the applicable river basin to the lower end of the basin. In this case stream and river flows can be defined by simple mass balance or continuity equations. For shorter duration time periods flow routing may be required.

The actual length or duration of each within-year period defined in a model may vary from period to period. Modeled time period time periods need not be equal. Generally what is important is to capture in the model the needed capacities of infrastructure that are determined in large measure by the variation in supplies and demands. These variations should be captured in the model by appropriately selecting the number and duration of time periods. If say over a three-month period there is little variation in both water supplies and demands or for the purposes water serves, such as flood control, hydropower, or recreation, there is no need to divide that three-month period into multiple time periods.

Another important factor to consider in making a decision regarding the number and duration of time periods to include in any model is the purpose for which the model is to be used. Some analyses are concerned only with identifying designs designs and operating policies of various engineering projects for managing water resources at some fixed time (say a typical year) in the future. Multiple years of hydrological records are used, usually in simulation models, to obtain an estimate of just how well a system might perform, at least in a statistical sense, in that future time period. The within-year period durations can have an impact on those performance indicator values as well as on the estimate of over-year as well as within-year storage that may be needed to meet various goals. These static analyses are not concerned with investment project scheduling or sequencing.

Dynamic planning models are used to estimate the impacts of changing conditions over time. These changes could include hydrological inputs, economic, environmental and other objectives, water demands, and design and operating parameters. As a result, dynamic models generally span many more years than do static models, but they may have fewer within-year periods.

11.3 Streamflow Estimation

Water resource managers need estimates of streamflows at each site, where management decisions are being considered. These streamflow estimates can be based on the results of rainfall-runoff models or on measured historical flows at gage sites. For modeling alternative management policies, these gage-based flows at the sites of interest should be those that would have occurred under natural conditions. These are called naturalized flows that have been derived from measured flows or rainfall–runoff models and then adjusted to take into account any upstream regulation and diversions. Many gage flow values reflect actions such as diversions and reservoir releases that occurred upstream that altered the downstream flows, unless such upstream water management and use policies are to continue, these measured gage flows should be converted to unregulated or natural flows prior to their use in management models.

Assuming that unregulated streamflow data are available at gage sites, these data can be used to estimate the unregulated flows at sites where they are needed. These sites would include any place where diversions might occur or where reservoirs for regulating flows might be built.

Consider, for example, the simple river basin illustrated in Fig. 11.1. Assume streamflows have been recorded over a number of years at gage sites 1 and 9. Knowledge of the flows Q_t^s in each period t at gage sites 1 and 9 permits the estimation of flows at any other site in the basin as well as the incremental flows between those sites in each period t.

The method used to estimate flows at ungaged sites will depend on the characteristics of the watershed or river basin. In humid regions where streamflows increase in the downstream direction due to rainfall runoff, and the spatial distribution of average monthly or seasonal rainfall is more or less the same from one part of the river basin to another, the runoff per unit land area is typically assumed constant. In these situations, estimated flows, q_t^s, at any site s can be based on the watershed areas, A^s, contributing flow to those sites, and the corresponding streamflows and watershed areas above the nearest or most representative gage sites.

For each gage site, the runoff per unit land area can be calculated by dividing the gage flow Q_t^s by the upstream drainage area, A^s. This can be done for each gage site in the basin. Thus for any gage site g, the runoff per unit drainage area in month or season t is Q_t^g divided by A^g. This runoff per unit land area times the drainage area upstream of any site s of interest will be the estimated streamflow in that period at that site s. If there are multiple gage sites, such as illustrated in Fig. 11.1, the estimated streamflow at some ungaged site s can be a weighted combination of those unit area runoffs times the area contributing to the flow at site s. The nonnegative weights, w_g, that sum to 1, reflect the relative significance of each gage site with respect to site s. Their values

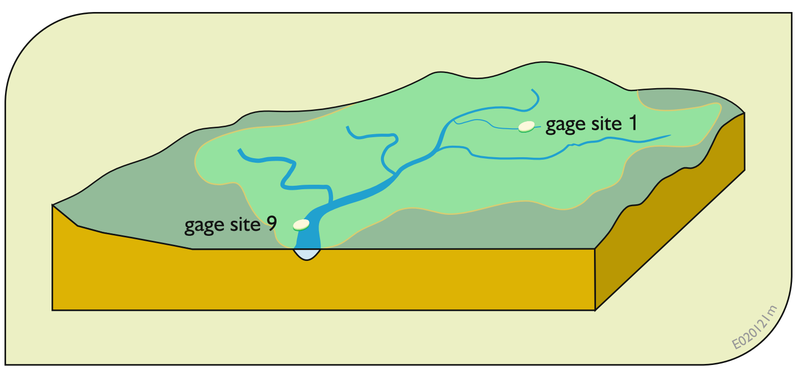


Fig. 11.1 River basin gage sites where streamflows are measured and recorded

will be based on the judgments of those who are familiar with the basin's hydrology.

$$Q_t^s = \left\{ \sum_g w_g Q_t^g / A^g \right\} A^s \qquad (11.1)$$

In all the models developed and discussed below, the variable Q_t^s will refer to the mean natural (unregulated) flow (L^3/T) at a site s in a period t.

The difference between the natural streamflows at any two sites is called the *incremental flow*. Using Eq. 11.1 to estimate streamflows will result in positive incremental flows. The downstream flow will be greater than the upstream flow. In arid regions runoff is not constant over the region. Incremental flows may not exist and hence due to losses, the flows may be decreasing in the downstream direction. In these cases there is a net loss in flow in the downstream direction. This might be the case when a stream originates in a wet area and flows into a region that receives less rainfall. In such arid areas the runoff is often less than the evapotranspiration and infiltration into the ground along the stream channel.

For stream channels where there exist relatively uniform conditions affecting loss loss, where there are no known sites where the stream abruptly enters or exits the ground, as can occur in karst conditions, the average streamflow for a particular period t at site s can be based on the nearest or most representative gage flow, Q_t^g, and a loss rate per unit length of the stream or river, L^{gs} between gage site g and an ungaged site s. If there are at least two gage sites along the portion of the stream or river that is in the dry region, one can compute the loss of flow per unit stream length, and apply this loss rate to various sites along the stream or river. This loss rate per unit length may not be constant over the entire length between the gage stations, or even for all flow rates, however. Losses will likely increase with increasing flows simply because more water surface is exposed to evaporation and seepage. In these cases one can define a loss rate per unit length of stream or river as a function of the magnitude of flow.

In watersheds characterized by significant elevation changes and consequently varying rainfall and runoff runoff, other methods may be required for estimating average streamflows at ungaged sites. The selection of the most appropriate method to use, as well as the most appropriate gage sites to use for estimating the streamflow, Q_t^s, at a particular site s can be a matter of judgment. The best gage site need not necessarily be the nearest gage to site s, but rather the site most similar with respect to important hydrologic variables.

The natural incremental flow between any two sites is simply the difference between their respective natural flows.

11.4 Streamflow Routing

If the duration of a within-year period is less than the time of flow throughout the stream or river system being modeled, and the flows vary within the system, some type of streamflow routing must be used to keep track of where the varying amounts of water are in each time period. There are many proposed routing methods (as described in any hydrology text or handbook, e.g., Maidment 1993). Many of these more traditional methods can be approximated with sufficient accuracy using relatively simple methods. Two such methods are described in the following paragraphs.

The outflow, O_t, from a reach of stream or river during a time period t is a function of the amount of water in that reach, i.e., its initial storage, S_t, and its inflow, I_t. Because of bank storage, that outflow is often dependent on whether the quantity of water in the reach is increasing or decreasing. If bank inflows and outflows are explicitly modeled, or if bank storage is not that significant, the outflow from a reach in any period t can be expressed as a simple two-parameter power function of the form $a(S_t + I_t)^b$. Mass balance equations, that may take into account losses, update the initial storage volumes in each succeeding time period. The reach–dependent parameters a and b can be determined through calibration procedures such as genetic algorithms (Chap. 5) using a time series of reach inflows and outflows. The resulting outflow function is typically concave (the parameter b will be less than 1), and thus the minimum value of $S_t + I_t$ must be at least 1. If due to evaporation or other losses the reach volume drops below this or any preselected higher amount, the outflow can be assumed to be 0.

Alternatively one can adopt a three- or four-parameter routing approach that fits a wider range of conditions. Each stream or river reach can be divided into a number of segments. That number n is one of the parameters to be determined. Each segment s can be modeled as a storage unit, having an initial storage volume, S_{st}, and an inflow, I_{st}. The three-parameter approach assumes the outflow, O_{st}, is a linear function of the initial storage volume and inflow:

$$O_{st} = \alpha\, S_{st} + \beta I_{st} \qquad (11.2)$$

Equation 11.2 applies for all time periods t and for all reach segments s in a particular reach. Different reaches will likely have different values of the parameters n, α, and β. The calibrated values of α and β are nonnegative and no greater than 1. Again a mass balance equation updates each segment's initial storage volume in the following time period. The outflow from each reach segment is the inflow into the succeeding reach segment.

The four-parameter approach assumes that the outflow, O_{st}, is a nonlinear function of the initial storage volume and inflow

$$O_{st} = (\alpha\, S_{st} + \beta I_{st})^{\gamma} \qquad (11.3)$$

The parameter γ is greater than 0 and no greater than 1. In practice γ is very close to 1. Again the values of these parameters, including the number of reaches n, can be found using nonlinear optimization methods, such as genetic algorithms, together with a time series of observed reach inflows and outflows.

Note the flexibility available when using the three- or four-parameter routing approach. Even blocks of flow can be routed a specified distance downstream over a specified time, regardless of the actual flow. This can be done by setting α and γ to 1, β to 0, and the number of segments n to the number of time periods it takes to travel that distance. This may not be very realistic, but there exist some river basin reaches where managers believe this particular routing applies.

11.5 Lakes and Reservoirs

Lakes and reservoirs are sites in a basin where surface water storage needs to be modeled. Thus, variables defining the water volumes at those sites must be defined. Let S_t^s be the initial storage volume of a lake or reservoir at site s in period t. Omitting the site index s for the moment, the final storage volume in period t, S_{t+1}, (which is the same as the initial storage in the following period $t + 1$) will equal the initial volume, S_t, plus the net surface and groundwater inflows, Q_t, less the release or discharge, R_t, and evaporation and seepage losses, L_t. All models of lakes and reservoirs include this mass balance equation for each period t being modeled.

$$S_t + Q_t - R_t - L_t = S_{t+1} \qquad (11.4)$$

The release from a natural lake is a function of its surrounding topography and its water surface elevation. It is determined by nature, and unless it is made into a reservoir its discharge or release is not controlled or managed. The release from a reservoir is controllable, and, as discussed in Chaps. 4 and 8, is usually a function of the reservoir storage volume and time of year. Reservoirs also have fixed storage capacities, K. In each period t, reservoir storage volumes, S_t, cannot exceed their storage capacities, K.

$$S_t \le K \quad \text{for each period } t. \qquad (11.5)$$

Equations 11.4 and 11.5 are the two fundamental equations required when modeling water supply reservoirs. They apply for each period t.

The primary purpose of all reservoirs is to provide a means of regulating downstream surface water flows over time and space. Other purposes may include storage volume management for recreation and flood control, and storage and release management for hydropower production. Reservoirs are built to alter the natural spatial and temporal distribution of the streamflows. The capacity of a reservoir together with its release (or operating) policy determine the extent to which surface water flows can be stored for later release.

The use of reservoirs for temporarily storing streamflows often results in a net loss of total streamflow due to increased evaporation and seepage. Reservoirs also bring with them changes in the ecology of a watershed and river system. They may also displace humans and human settlements. When considering new reservoirs, any benefits derived from regulation of water supplies, from flood damage reduction, from hydroelectric power, and from any navigational and recreational activities should be compared to any ecological and social losses and costs. The benefits of reservoirs can be substantial, but so may the costs. Such comparisons of benefits and costs are always challenging because of the difficulty of expressing all such benefits and costs in a common metric (Chap. 9).

Reservoir storage capacity can be divided among the three major uses: (1) the *active storage* used for downstream flow regulation and for water supply, recreational development or hydropower production; (2) the the *dead storage* used for sediment collection; and (3) the *flood storage* capacity reserved to reduce potential downstream flood damage during flood events. These separate storage capacities are illustrated in Fig. 11.2. The distribution of active and flood control storage capacities may change over the year. For example there is no need for flood control storage in seasons that are not likely to experience floods.

The next several sections of this chapter address how these capacities may be determined.

11.5.1 Estimating Active Storage Capacity

11.5.1.1 Mass diagram Analyses

Perhaps one of the earliest methods used to calculate the active storage capacity required to meet a specified reservoir release, R_t, in a sequence of periods t, was developed by Rippl (1883). His *mass diagram analysis* is still used today by many planners. It involves finding the maximum positive cumulative difference between a sequence of prespecified (desired) reservoir releases R_t and known inflows Q_t. One can visualize this as starting with a full reservoir, and going through a sequence of simulations in which the inflows and releases are added and subtracted from that initial storage volume value.

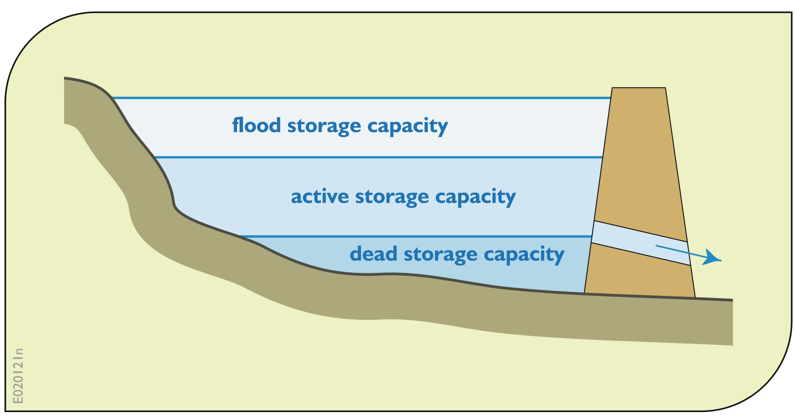


Fig. 11.2 Total reservoir storage volume capacity consisting of the sum of dead storage, active storage, and flood control storage capacities

Doing this over two cycles of the record of inflows will identify the maximum deficit volume associated with that release. This is the required reservoir storage. Having this initial storage volume, the reservoir would always have enough water to meet the desired releases. However, this only works if the sum of all the desired releases does not exceed the sum of all the inflows over the same sequence of time periods. Reservoirs cannot make water.

Equation 11.6 represents this process. The active storage capacity, K_a, will equal the maximum accumulated storage deficit one can find over some interval of time within two successive record periods, T.

$$K_a = \text{maximum}\left[\sum_{t=i}^{j}(R_t - Q_t)\right], \qquad (11.6)$$

where $1 \leq i \leq j \leq 2T$.

Equation 11.6 is the analytical equivalent of graphical procedures proposed by Rippl for finding the active storage requirements. Two of these graphical procedures are illustrated in Figs. 11.3 and 11.4 for a 9-period inflow record of 1, 3, 3, 5, 8, 6, 7, 2, and 1. Rippl's original

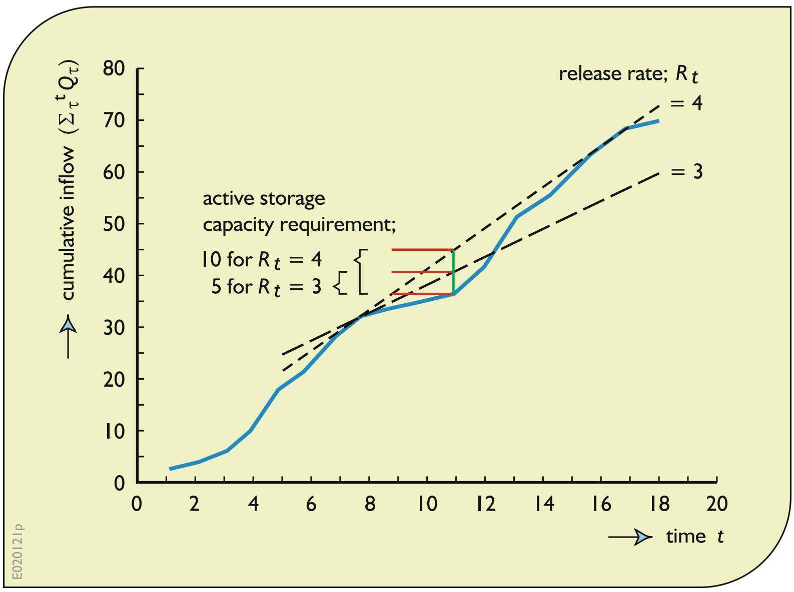


Fig. 11.3 The Rippl or mass diagram method for identifying reservoir active storage capacity requirements. The releases R_t are assumed constant for each period t

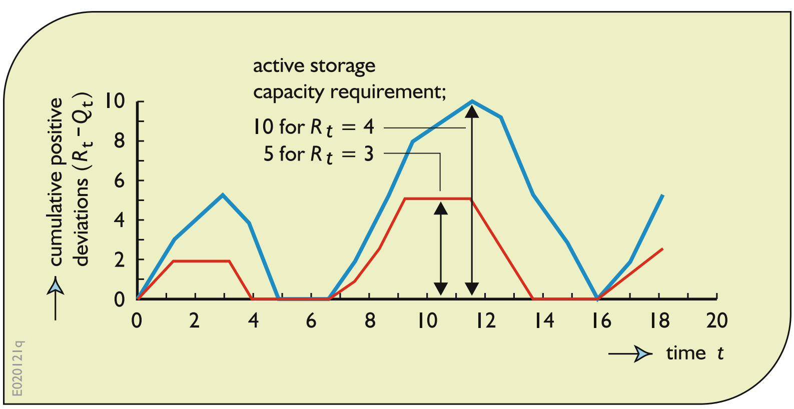


Fig. 11.4 Alternative plot for identifying reservoir active storage capacity requirements

approach, shown in Fig. 11.3, involves plotting the cumulative inflow $\sum_{t=1}^{t} Q_{\tau}$ versus time t. Assuming a constant reservoir release, R, in each period t, a line with slope R is placed so that it is tangent to the cumulative inflow curve. To the right of these points of tangency the release R exceeds the inflow Q_t. The maximum vertical distance between the cumulative inflow curve and the release line of slope R equals the maximum water deficit, and hence the required active storage capacity. Clearly, if the average release R is greater than the mean inflow, a reservoir will not be able to meet the demand no matter what its active storage capacity.

An alternative way to identify the required reservoir storage capacity is to plot the cumulative nonnegative deviations, $\sum_{\tau}^{t}(R_{\tau} - Q_{\tau})$, and note the biggest total deviation, as shown in Fig. 11.4.

These graphical approaches do not account for losses. Furthermore, the method shown in Fig. 11.3 is awkward if the desired releases in each period t are not the same. The equivalent method shown in Fig. 11.4 is called the sequent peak method. If the sum of the desired releases does not exceed the sum of the inflows, calculations over at most two successive hydrologic records of flows are needed to identify the largest cumulative deficit inflow. After that the procedure will produce repetitive results. It is much easier to consider changing release values when determining the maximum deficit by the sequent peak method.

11.5.1.2 Sequent peak analyses

The sequent peak procedure is illustrated in Table 11.1. Let K_t be the maximum total storage requirement needed for periods 1 up through period t. As before, let R_t be the required release in period t, and Q_t be the inflow in that period. Setting K_0 equal to 0, the procedure involves calculating K_t using Eq. 11.7 consecutively for up to twice the total length of record. This assumes that the record repeats itself to take care of the case when the critical sequence of flows occurs at the end of the streamflow record, as indeed it does in the example 9-period record of 1, 3, 3, 5, 8, 6, 7, 2, and 1.

$$\begin{aligned} K_t &= R_t - Q_t + K_{t-1} \quad \text{if positive,} \\ &= 0 \quad \text{otherwise} \end{aligned} \tag{11.7}$$

The maximum of all K_t is the required storage capacity for the specified releases R_t and inflows, Q_t. Table 11.1 illustrates this sequent peak procedure for computing the active capacity $K_{a,}$ i.e., the maximum of all K_t, required to achieve a release R_t = 3.5 in each period given the series of 9 streamflows. Note this method does not require all releases to be the same.

11.5.2 Reservoir Storage-Yield Functions

Reservoir storage-yield functions define the minimum active active storage capacity required to insure a given constant release rate for a specified sequence of reservoir inflows. Mass diagrams, sequent peak analyses, and linear optimization (Chap. 4) are three methods that can be used to define these functions. Given the same sequence of known inflows and specified releases, each method will provide identical results. Using optimization models, it is possible to obtain such functions from multiple reservoirs and to account for losses based on storage volume surface areas, as will be discussed later.

There are two ways of defining a linear optimization (linear programming) model to estimate the active storage capacity requirements. One approach is to minimize the active storage capacity, K_a, subject to minimum required constant releases, Y, the yield. This minimum active storage capacity is the maximum storage volume, S_t, required given the sequence of known inflows Q_t, and the specified yield, Y, in each period t. The problem is to find the storage volumes, S_t, and releases, R_t that

$$\text{Minimize } K_a \tag{11.8}$$

Table 11.1 Illustration of the sequent peak procedure for computing active storage requirements

time t	$(R_t - Q_t + K_{t-1})^+ = K_t$	
1	3.5 − 1.0 + 0.0 = 2.5	
2	3.5 − 3.0 + 2.5 = 3.0	
3	3.5 − 3.0 + 3.0 = 3.5	
4	3.5 − 5.0 + 3.5 = 2.0	
5	3.5 − 8.0 + 2.0 = 0.0	
6	3.5 − 6.0 + 0.0 = 0.0	
7	3.5 − 7.0 + 0.0 = 0.0	
8	3.5 − 2.0 + 0.0 = 1.5	
9	3.5 − 1.0 + 1.5 = 4.0	
1	3.5 − 1.0 + 4.0 = 6.5	
2	3.5 − 3.0 + 6.5 = 7.0	
3	3.5 − 3.0 + 7.0 = 7.5	K_a
4	3.5 − 5.0 + 7.5 = 6.0	
5	3.5 − 8.0 + 6.0 = 1.5	
6	3.5 − 6.0 + 1.5 = 0.0	repetition begins
7	3.5 − 7.0 + 0.0 = 0.0	
8	3.5 − 2.0 + 0.0 = 1.5	
9	3.5 − 1.0 + 1.5 = 4.0	

E020121r

subject to
mass balance constraints

$$S_t + Q_t - R_t = S_{t+1} \quad t = 1, 2, \ldots, T; \; T + 1 = 1 \tag{11.9}$$

capacity constraints

$$S_t \le K_a \quad t = 1.2, \ldots, T \tag{11.10}$$

minimum release constraints

$$R_t \ge Y \quad t = 1.2, \ldots, T \tag{11.11}$$

for various values of the yield, Y.

Alternatively one can maximize the constant release yield, Y, for various values of active storage capacity, K_a, subject to the same constraint Eqs. 11.9–11.11.

$$\text{Maximize } Y \tag{11.12}$$

Constraints 11.9 and 11.11 can be combined to reduce the model size by T constraints.

$$S_t + Q_t - Y \ge S_{t+1} \quad t = 1,\ 2, \ldots, T; \; T + 1 = 1 \tag{11.13}$$

The solutions of these two linear programming models, using the 9-period flow sequence referred

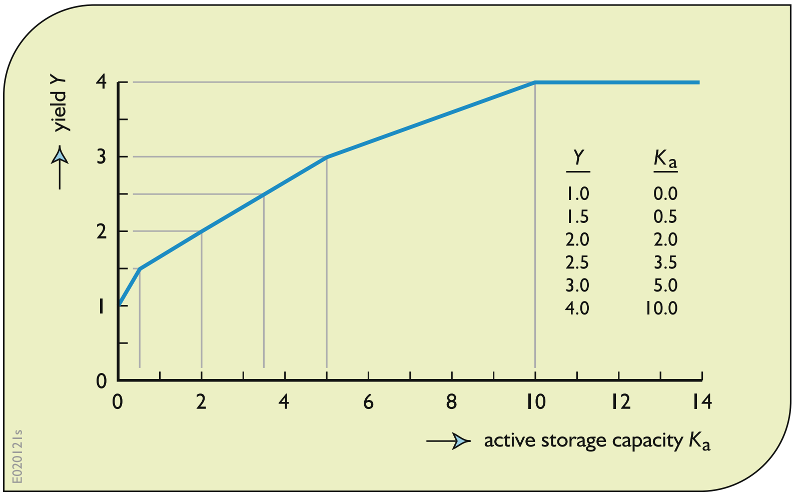


Fig. 11.5 Storage-yield for the sequence of flows 1, 3, 3, 5, 8, 6, 7, 2, and 1

to above and solved for various values of yield or capacity, respectively, are plotted in Fig. 11.5. The results are the same as could be found using the mass diagram or sequent peak methods.

There is a probability that the storage-yield function just defined will fail. A record of only 9 flows, for example, is not very long and hence will not give one much confidence that they will define the critical low-flow period of the future. One rough way to estimate the reliability of a storage-yield function is to rearrange and rank the inflows in order of their magnitudes. If there are n ranked inflows there will be $n + 1$ intervals separating them. Assuming there is an equal probability that any future flow could occur in any interval between these ranked flows, there is a probability of $1/(n + 1)$ that a future flow will be less than the lowest recorded flow. If that record low flow occurs during a critical low-flow period, more storage may be required than indicated in the function.

Hence for a record of only 9 flows that are considered representative of the future, one can be only about 90% confident that the resulting storage-yield function will apply in the future. One can be only 90% sure of the predicted yield Y associated with any storage volume K. A much more confident estimate of the reliability of any derived sstorage-yield function can be obtained by synthetic flows to supplement any measured streamflow record, taking parameter uncertainty into account (as discussed in Chaps. 6 and 8). This will provide alternative sequences as well as more intervals between ranked flows.

While the mass diagram and sequent peak procedures are relatively simple, they are not readily adaptable to reservoirs where evaporation losses and/or lake level regulation are important considerations, or to problems involving more than one reservoir. Mathematical programming (optimization) methods provide this capability. These optimization methods are based on mass balance equations for routing flows through each reservoir. The mass balance or continuity equations explicitly define storage volumes (and hence storage areas from which evaporation occurs) at the beginning of each period t.

11.5.3 Evaporation Losses

Evaporation losses, L_t, from lakes and reservoirs, if any, take place on their surface areas. Hence to compute these losses their surface areas must be estimated in each period t. Storage surface areas are functions of the storage volumes, S_t. These functions are typically concave, as shown in Fig. 11.6.

In addition to the storage area-volume function, seasonal surface water evaporation loss depths, $E_t^{\max}$, must be assumed, perhaps based on measured evaporation losses over time.

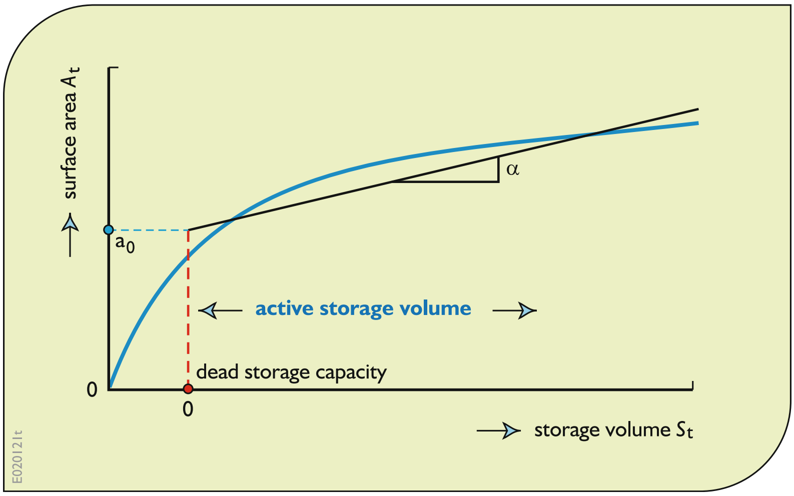


Fig. 11.6 Storage surface area as a function of reservoir storage volume along with its linear approximation. The slope parameter a is the assumed increase in surface area associated with a unit increase in the storage volume

Multiplying the average surface area, A_t, based on the initial and final storage volumes, S_t and S_{t+1}, by the loss depth, $E_t^{\max}$, yields the volume of evaporation loss, L_t, in the period t. The linear approximation of that loss is

$$L_t = [a_o + a(S_t + S_{t+1})/2]E_t^{\max} \qquad (11.14)$$

Letting

$$a_t = 0.5\ a\ E_t^{\max} \qquad (11.15)$$

the mass balance equation for storage volumes that include evaporation losses in each period t can be approximated as

$$(1 - a_t)S_t + Q_t - R_t - a_o E_t^{\max} = (1 + a_t)S_{t+1} \qquad (11.16)$$

If Eq. 11.16 are used in optimization models for identifying preliminary designsof a proposed reservoir and if the active storage storage capacity turns out to be essentially zero, or just that required to provide for the fixed evaporation loss, $a_o\ E_t^{\max}$, then clearly any reservoir at the site is not justified. These mass balance equations together with any reservoir storage capacity constraints should be removed from the model before resolving it again. This procedure is simpler than introducing 0,1 integer variables that will remove the terms $a_o\ E_t^{\max}$ in Eq. 11.16 if the active storage volume is 0 (using methods discussed in Chap. 4).

An alternative way to estimate evaporation loss that does not require a surface area—storage volume relationship, such as shown in Fig. 11.6, is to define the storage elevation-storage volume function. Subtracting the evaporation loss depth from the initial surface elevation associated with the initial storage volume will result in an adjusted storage elevation which in turn defines the initial storage volume after evaporation losses have been deducted. This adjusted initial volume can be used in continuity Eqs. 11.9 or 11.13. This procedure assumes that evaporation is only a function of the initial storage volume in each time period t. For relatively large volumes and short time periods such an assumption is usually satisfactory.

11.5.4 Over- and Within-Year Reservoir Storage and Yields

An alternative approach to modeling reservoirs is to separate out over-year storage and within-year storage, and to focus not on total reservoir releases, but on parts of the total releases that can be assigned specific reliabilities. These release components we call yields. To define these yields and the corresponding reservoir rules, we divide this section into four parts. The first outlines a method for estimating the reliabilities of various constant annual minimum flows or yields. The second discusses a modeling approach for estimating over-year and within-year active storage requirements to deliver a specified annual and within-year period yields having a specified reliability. The third and fourth parts expand this modeling approach to include multiple yields having different reliabilities, evaporation losses, and the construction of reservoir operation rule curves using these flow release yields.

It will be convenient to illustrate the yield models and their solutions using a simple example consisting of a single reservoir and two within-year periods per year. This example will be sufficient to illustrate the method that can be applied to models having more within-year periods. Table 11.2 lists the nine years of available streamflow data for each within-year season at a potential reservoir site. These streamflows are used to solve and compare the solutions of various yield models as well as to illustrate the concept of yield reliability.

11.5.4.1 Reliability of Annual yields

The maximum flow that can be made available at a specific site by the regulation of the historic streamflows from a reservoir of a given size is often referred to as the "firm yield" or "safe yield." These terms imply that the firm or safe yield is that yield which the reservoir will always be able to provide. Of course, this may not be true. If historical flows are used to determine this yield, then the resulting yield may be better

Table 11.2 Recorded unregulated historical streamflows at a reservoir site

time year y	within-year period Q_{ty} Q_{1y}	Q_{2y}	annual Q_y flow
1	1.0	3.0	4.0
2	0.5	2.5	3.0
3	1.0	2.0	3.0
4	0.5	1.5	2.0
5	0.5	0.5	1.0
6	0.5	2.5	3.0
7	1.0	5.0	6.0
8	2.5	5.5	8.0
9	1.5	4.5	6.0
total	**9.0**	**27.0**	**36.0**
average flow	1.0	3.0	4.0

E020130m

called an "historical yield." Historical and firm yield are often used synonymously.

A minimum flow yield is 100% reliable only if the sequence of flows in future years will never sum to a smaller amount than those that have occurred in the historic record. Usually one cannot guarantee this condition. Hence associated with any historic yield is the uncertainty, i.e., a probability, that it might not always be available in the future. There are some ways of estimating this probability.

Referring to the nine-year streamflow record listed in Table 11.2, if no reservoir is built to increase the yields downstream of the reservoir site, the historic firm yield is the lowest flow on record, namely 1.0 that occurred in year 5. The reliability of this annual yield is the probability that the streamflow in any year is greater than or equal to this value. In other words, it is the probability that this flow will be equaled or exceeded. The expected value of the exceedance probability of the lowest flow in an n-year record is approximately $n/(n + 1)$, which for the $n = 9$ year flow record is $9/(9 + 1)$, or 0.90. This is based on the assumption that any future flow has an equal probability of being in any of the intervals formed by ordering the record of 9 flows from the lowest to the highest value, and that the lowest value has a rank of 9.

Ranking the n flows of record from the highest to the lowest and assigning the rank m of 1 to the highest flow, and n to the lowest flow, the expected probability p that any flow of rank m will be equaled or exceeded in any year is approximately $m/(n + 1)$. An annual yield having a probability p of exceedance will be denoted as Y_p.

For independent events, the expected number of years until a flow of rank m is equaled or exceeded is the reciprocal of its probability of exceedance p, namely $1/p = (n + 1)/m$. The recurrence time or expected time until a failure (a flow less than that of rank m) is the reciprocal of the probability of failure in any year. Thus, the expected recurrence time T_p associated with a flow having an expected probability p of exceedance is $1/(1 - p)$.

11.5.4.2 Estimation of Active Reservoir Storage Capacities for Specified Yields

A reservoir with active over-year storage capacity provides a means of increasing the magnitude and/or the reliabilities of various annual yields. For example, the sequent peak algorithm defined by Eq. 11.7 provides a means of estimating the reservoir storage volume capacity required to meet various "firm" yields $Y_{0.9}$, associated with the nine annual flows presented in Table 11.1. The same yields can be obtained from a linear optimization model that minimizes active over-year storage capacity, K_a^o,

$$\text{Minimize } K_a^o \tag{11.17}$$

This active over-year storage capacity must satisfy the following storage continuity and capacity constraint equations involving only annual storage volumes, S_y, inflows, Q_y, yields, Y_p, and excess releases, R_y. For each year y:

$$S_y + Q_y - Y_p - R_y = S_{y+1} \tag{11.18}$$

$$S_y \le K_a^o \tag{11.19}$$

Once again, if the year index $y = n$, the last year of record, then year $y + 1$ is assumed to equal 1. For annual yieldsof 3 and 4, the over-year storage requirements are 3 and 8, respectively, as can be determined just by examining the right-hand column of annual flows in Table 11.2.

The over-year model, Eqs. 11.17–11.19, identifies only annual or over-year storage requirements based on specified (known) annual flows, Q_y, and specified constant annual yields, Y_p. Within-year periods t requiring constant yields y_{pt} that sum to the annual yield Y_p may also be considered in the estimation of the required over-year and within-year or total active storage capacity, K_a. Any distribution of the over-year yield within the year that differs from the distribution of the within-year inflows may require additional active reservoir storage capacity. This additional capacity is called the within-year storage capacity.

The sequent peak method, Eq. 11.7, can be used to obtain the total over-year and within-year active storage capacity requirements for specified within-year period yields, y_{pt}. Alternatively a linear programming model can be developed to obtain the same information along with associated reservoir storage volumes. The objective is to find the minimum total active storage capacity, K_a, subject to storage volume continuity and capacity constraints for every within-year period of every year. This model is defined as

$$\text{minimize } K_a \tag{11.20}$$

subject to

$$S_{ty} + Q_{ty} - y_{pt} - R_{ty} = S_{t+1,y} \quad \forall t, y \tag{11.21}$$

$$S_{ty} \le K_a \quad \forall t, y \tag{11.22}$$

In Eq. 11.21, if t is the final period T in year y, the next period $T + 1 = 1$ in year $y + 1$, or year 1 if y is the last year of record, n.

The within-year storage requirement, K_a^w, is the difference in the active capacities resulting from these two models, Eqs. 11.17–11.19, and Eqs. 11.20–11.22.

Table 11.3 shows some results from solving both of the above models. The over-year storage capacity requirements, K_a^o, are obtained from Eqs. 11.17–11.19. The combined over-year and within-year capacities, K_a, are obtained from solving Eqs. 11.20–11.22. The difference between the over-year storage capacity, K_a^o, required to meet only the annual yields and the total capacity, K_a, required to meet each specified within-year yield distribution of those annual yields is the within-year active storage capacity K_a^w.

Clearly, the number of continuity and reservoir capacity constraints in the combined over-year and within-year model (Eqs. 11.20–11.22) can become very large when the number of years n and within-year periods T are large. Each reservoir site in the river system will require $2nT$ continuity and capacity constraints. Not all these constraints are necessary, however. It is only a subset of the sequence of flows within the total record of flows that generally determines the required active storage capacity K_a of a reservoir. This is called the critical period. This critical period is often used in engineering studies to estimate the historical yield of any particular reservoir or system of reservoirs.

Even though the severity of future droughts is unknown, many planners accept the traditional practice of using the historical critical drought period for reservoir design and operation studies on the assumption that having observed such an event in the past, it is certainly possible to experience similar conditions in the future. In some parts of the world, notably those countries in the lower portions of the southern hemisphere, historical records are continually proven to be unreliable indicators of future hydrological conditions. In these regions especially, synthetically generated flows based on statistical methods (Chap. 6) are more acceptable as a basis for yield estimation.

Over and within-year storage Capacity

To begin the development of a smaller, but more approximate, model, consider each combined over-year and within-year storage reservoir to consist of two separate reservoirs in series (Fig. 11.7). The upper reservoir is the over-year storage reservoir, whose capacity required for an annual yield is determined by an over-year model, e.g., Eqs. 11.17–11.19. The purpose of the "downstream" within-year reservoir is to distribute as desired in each within-year period t a portion of the annual yield produced by the "upstream" over-year reservoir. Within-year storage capacity would not be needed if the distribution of the average inflows into the upper over-year reservoir exactly coincided with the desired distribution of within-year yields. Otherwise within-year storage may be required. The two separate reservoir capacities summed together will be an approximation of the total active reservoir storage requirement needed to provide those desired within-year period yields.

Table 11.3 Active Storagerequirements for various within-year yields

annual yield	within-year yields		required active storage volume capacity		
			within-year	over-year	total
$y_{0.9}$	$t = 1$	$t = 2$	K_a^w	K_a^o	K_a
3	0	3	1.0	3.0	4.0
	1	2	0.5	3.0	3.5
	2	1	1.5	3.0	4.5
	3	0	2.5	3.0	5.5
4	0	4	1.0	8.0	9.0
	1	3	0.0	8.0	8.0
	2	2	1.0	8.0	9.0
	3	1	2.0	8.0	10.0
	4	0	3.0	8.0	11.0

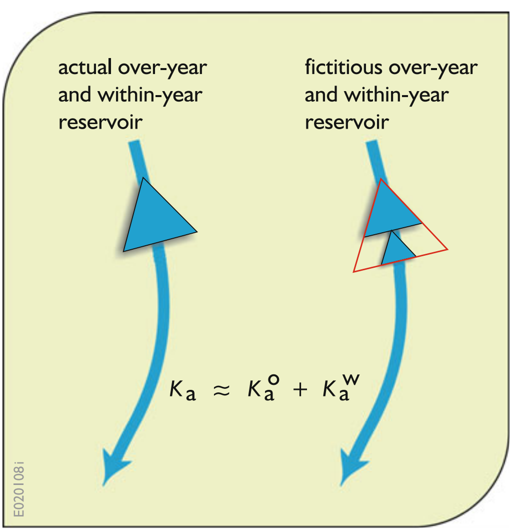


Fig. 11.7 Approximating a combined over-year and within-year reservoir as two separate reservoirs, one for creating annual yields, the other for distributing them as desired in the within-year periods

Assume the annual yield produced and released by the over-year reservoir is distributed in each of the within-year periods in the same ratio as the average within-year inflows divided by the total average annual inflow. Let the ratio of the average period *t* inflow divided by the total annual inflow be β_t. The general within-year model is to find the minimum within-year storage capacity, K_a^w, subject to within-year storage volume continuity and capacity constraints.

$$\text{Minimize } K_a^w \tag{11.23}$$

subject to

$$s_t + \beta_t Y_p - y_{pt} = s_{t+1} \quad \forall t \quad T + 1 = 1 \tag{11.24}$$

$$s_t \leq K_a^w \quad \forall t \tag{11.25}$$

Since the sum of β_t over all within-year periods t is 1, the model guarantees that the sum of the unknown within-year yields, y_{pt}, equals the annual yield, Y_p.

The over-year model, Eqs. 11.17–11.19, and within-year model, Eqs. 11.23–11.25, can be combined into a single model for an n-year sequence of flows

$$\text{Minimize } K_a \tag{11.26}$$

subject to

$$\begin{aligned} &S_y + Q_y - Y_p - R_y = S_{y+1} \quad \forall y \\ &\text{if} \quad y = n, \quad y+1 = 1 \end{aligned} \tag{11.27}$$

$$S_y \leq K_a^o \quad \forall y \tag{11.28}$$

$$\begin{aligned} &s_t + \beta_t Y_p - y_{pt} = s_{t+1} \quad \forall t \\ &\text{if} \quad t = T, \quad T+1 = 1 \end{aligned} \tag{11.29}$$

$$s_t \leq K_a^w \quad \forall t \tag{11.30}$$

$$\sum_t y_{pt} = Y_p \tag{11.31}$$

$$K_a \geq K_a^o + K_a^w \tag{11.32}$$

Constraint 11.31 is not required due to Eq. 11.29, but is included here to make it clear that the sum of within-year yields will equal the over-year yield. Such a constraint will be required for each yield of reliability p if multiple yields of different reliabilities are included in the model. In addition, constraint Eq. 11.30 can be combined with Eq. 11.32, saving a constraint. If this is done, the combined model contains $2n + 2T + 1$ constraints, compared to the more accurate model, Eqs. 11.20–11.22, that contains $2nT$ constraints.

If the fractions β_t are based on the ratios of the average within-year inflow divided by average annual inflow in the two within-year periods shown in Table 11.2, 0.25 of the total annual yield flows into the fictitious within-year reservoir in period $t = 1$, and 0.75 of the total annual yield flows into the reservoir in period $t = 2$. Suppose the two desired within-year yields are to be 3 and 0 for periods 1 and 2, respectively. The total annual yield, $Y_{0.9}$, is 3. Assuming the natural distribution of this annual yield of 3 in period 1 is 0.25 $Y_{0.9}$ = 0.75, and in period 2 it is 0.75 $Y_{0.9}$ = 2.25, the within-year storage required to redistribute these yields of 0.75 and 2.25 to become 3 and 0, respectively, is $K_a^w = 2.25$. From Tables 11.2 or 11.3 we can see that an annual yield of 3 requires an over-year storage capacity of 3. Thus, the estimated total storage capacity required to provide yields of 3 and 0 in periods 1 and 2 is the over-year capacity of 3 plus the within-year capacity of 2.25 equaling 5.25. This compares with 3 plus 2.5 of actual within-year capacity required for a total of 5.50, as indicated in Table 11.3.

There are ways to reduce the number of over-year constraints without changing the solution of the over-year model. Sequences of years whose annual inflow values equal or exceed the desired annual yield can be combined into one constraint. If the yield is an unknown variable then the mean annual inflow can be used as the desired annual yield since it is the upper limit of the annual yield. For example in Table 11.2 note that the last three years and the first year have flows equal or greater than 4, the mean annual inflow. Thus, these four successive years can be combined into a single continuity equation

$$S_7 + Q_7 + Q_8 + Q_9 + Q_1 - 4Y_p - R_7 = S_2 \tag{11.33}$$

This saves a total of 3 over-year continuity constraints and 3 over-year capacity constraints. Note that the excess release, R_7, represents the excess release in all four periods. Furthermore, not all reservoir capacity constraint Eq. 11.28 are needed, since the initial storage volumes in the years following low flows will probably be less than the over-year capacity.

There are many ways to modify and extend this yield model to include other objectives, fixed ratios of the unknown annual yield for each within-year period, and even multiple yields having different exceedance probabilities p.

The number of over-year periods being modeled compared to the number of years of flow records determines the highest exceedance probability or reliability a yield can have; e.g., 9/10 or 0.9 in the 9-year example used here. If yields having lower reliabilities are desired, such as a yield with a reliability of 0.80, then the yield variable Y_P can be omitted from Eq. 11.27 in that critical year that determines the required over-year capacity for a 0.90 reliable yield. (Since some outflow might be expected, even if it is less than the 0.90 reliable yield, the outflow could be forced to equal the inflow for that year.) If a 0.70 reliable yield is desired, then the yield variables in the two most critical years can be omitted from Eq. 11.27, and so on.

The number of years of yield failure determines the estimated reliability of each yield. An annual yield that fails in f years has an estimated probability $(n - f)/(n + 1)$ of being equaled or exceeded in any future year. Once the desired reliability of a yield is known, the problem is to select the appropriate failure years and to specify the permissible extent of failure in those f failure years.

To consider different yield reliabilities p let the parameter α_y^p be a specified value between 0 and 1 that indicates the extent of a failure in year y associated with an annual yield having a reliability of p. When α_y^p is 1 there is no failure, and when it is less than 1 there is a failure, but a proportion of the yield Y_p equal to α_y^p is released. Its value is in part dependent on the consequences of failure and on the ability to forecast when a failure may occur and to adjust the reservoir operating policy accordingly.

Theover-year storage continuity constraints for n years can now be written in a form appropriate for identifying any single annual yield Y_p having an exceedance probability p.

$$S_y + Q_y - \alpha_y^p Y_p - R_y = S_{y+1} \quad \forall y \quad \text{if} \quad y = n, \quad y+1 = 1 \tag{11.34}$$

When writing Eq. 11.34, the failure year or years should be selected from among those in which permitting a failure decreases the required reservoir capacity K_a. If a failure year is selected that has an excess release, no reduction in the required active storage capacity will result, and the reliability of the yield may be higher than intended.

The critical year or years that determine the required active storage volume capacity may be dependent on the yield itself. Consider, for example, the 7-year sequence of annual flows (4, 3, 3, 2, 8, 1, 7) whose mean is 4. If a yield of 2 is desired in each of the 7 years, the critical year requiring reservoir capacity is year 6. If a yield of 4 is desired (again assuming no losses), the critical years are years 2–4. The streamflows and yields in these critical years determine the required over-year storage capacity. The failure years, if any, must be selected from within the critical low-flow periods for the desired yield.

When the magnitudes of the yields are unknown, some trial and error solutions may be necessary to ensure that any failure years are within the critical period of years for the associated yields. To ensure a wider range of applicable yield magnitudes, the year having the lowest flow within the critical period should be selected as the failure year if only one failure year is selected. Even though the actual failure year may follow that low-flow year, the resulting required reservoir storage volume capacity will be the same.

Multiple Yields and Evaporation Losses

The yield models developed so far define only single annual and within-year yields. Incremental secondary yields having lower reliabilities can also be included in the model. Referring to the 9-year streamflow record in Table 11.3, assume that two yields are desired, one 90% reliable and the other 70% reliable. Let $Y_{0.9}$ and $Y_{0.7}$ represent those annual yields having reliabilities of 0.9 and

0.7, respectively. The incremental secondary yield $Y_{0.7}$ represents the amount in addition to $Y_{0.9}$ that is only 70% reliable. Assume that the problem is one of estimating the appropriate values of $Y_{0.9}$ and $Y_{0.7}$, their respective within-year allocations y_{pt} and the total active reservoir capacity K_a that maximizes some function of these yield and capacity variables.

In this case the over-year andwithin-year continuity constraints can be written

$$S_y + Q_y - Y_{0.9} - \alpha_y^{0.7} Y_{0.7} - R_y = S_{y+1} \quad \forall y \quad \text{if} \quad y = n, \; y+1 = 1 \tag{11.35}$$

$$s_t + \beta_t (Y_{0.9} + Y_{0.7}) - y_{0.9,t} - y_{0.7,t} = s_{t+1} \quad \forall t \quad \text{if} \quad t = T, \; T+1 = 1 \tag{11.36}$$

Now an additional constraint is needed to insure that each within-year yield of a reliability p adds up to the annual yield of the same reliability. Selecting the 90% reliable yield,

$$\sum_t y_{0.9,t} = Y_{0.9} \tag{11.37}$$

Evaporation losses must be based on an expected storage volume in each period and year since the actual storage volumes are not identified using these yield models. The approximate storage volume in any period t in year y can be defined as the initial over-year volume S_y, plus the estimated average within-year volume $(s_t + s_{t+1})/2$. Substituting this storage volume into Eq. 11.14 (see also Fig. 11.6) results in an estimated evaporation loss L_{yt}.

$$L_{yt} = \left[a_o + a(S_y + (s_t + s_{t+1})/2)\right] E_t^{\max} \tag{11.38}$$

Summing L_{yt} over all within-year periods t defines the estimated annual evaporation loss, E_y.

$$E_y = \sum_t \left[a_o + a(S_y + (s_t + s_{t+1})/2)\right] E_t^{\max} \tag{11.39}$$

This annual evaporation loss applies, of course, only when there is a nonzero active storage capacity requirement. These annual evaporation losses can be included in the over-year continuity constraints, such as Eq. 11.35. If they are, the assumption is being made that their within-year distribution will be defined by the fractions β_t. This may not be realistic. If it is not, an alternative would be to include the average within-year period losses, L_t, in the within-year constraints.

The average within-year period loss, L_t, can be defined as the sum of each loss L_{yt} defined by Eq. 11.38 over all years y divided by the total number of years, n.

$$L_t = \sum_y^n \left[a_o + a(S_y + (s_t + s_{t+1})/2)\right] E_t^{\max} / n \tag{11.40}$$

This average within-year period loss, L_t, can be added to the within-year's highest reliability yield, y_{pt}, forcing greater total annual yields of all reliabilities to meet corresponding total within-year yield values. Hence, combining Eq. 11.37 and 11.38, for p equal to 0.9 in the example,

$$Y_p = \sum_t \left\{ y_{pt} + \sum_y^n [a_o + a(S_y + (s_{t+1})/2)] E_t^{\max} / n \right\} \tag{11.41}$$

Since actual reservoir storage volumes in each period t of each year y are not identified in this model, system performance measures that are functions of those storage volumes, such as hydroelectric energy or reservoir recreation, are only approximate. Thus, as with any of these screening models, any set of solutions should be evaluated and further improved using more precise simulation methods.

Simulation methods require reservoir operating rules. The information provided by the solution of the yield model can aid in defining a reservoir operating policy for such simulation studies.

Reservoir Operation Rules

Reservoir operation rules are guides for those responsible for reservoir operation. They apply to reservoirs being operated in a steady-state condition (i.e., not filling up immediately after construction or being operated to meet a set of new and temporary objectives). There are several types of rules but each indicates the desired or required reservoir release or storage volumes at any particular time of year. Some rules identify storage volume targets (rule curves) that the operator is to maintain, if possible, and others identify storage zones, each associated with a particular release policy. This latter type of rule can be developed from the solution of the yield model.

To construct an operation rule that identifies storage zones, each having a specific release policy, the values of the dead and flood storage capacities, K_D and K_f are needed together with the over-year storage capacity, K_a^o, and within-year storage volumes s_t in each period t. Since both K_a^o and all s_t derived from the yield model are for all yields, Y_p, being considered, it is necessary to determine the over-year capacities and within-year storage volumes required to provide each separate within-year yield y_{pt}. Plotting the curves defined by the respective over-year capacity plus the within-year storage volume ($K_a^o + s_t$) in each within-year period t will define a zone of storage whose yield releases y_{pt} from that zone should have a reliability of at least p.

For example, assume again a 9-year flow record and 10 within-year periods. Of interest are the within-year yields, $y_{0.9,t}$ and $y_{0.7,t}$, having reliabilities of 0.9 and 0.7. The first step is to compute the over-year storage capacity requirement, K_a^o, and the within-year storage volumes, s_t, for just the yields $y_{0.9,t}$. The sum of these values, $K_a^o + s_t$, in each period t can be plotted as illustrated in Fig. 11.8.

The sum of the over-year capacity and within-year volume $K_a^o + s_t$ in each period t defines the zone of active storage volumes for each period t required to supply the within-year yields $y_{0.9,t}$. If the storage volume is in this shaded zone shown in Fig. 11.8, only the yields $y_{0.9,t}$ should be released. The reliability of these yields, when simulated, should be about 0.9. If at any time t the actual reservoir storage volume is within this zone, then reservoir releases should not exceed those required to meet the yield $y_{0.9,t}$ if the reliability of this yield is to be maintained.

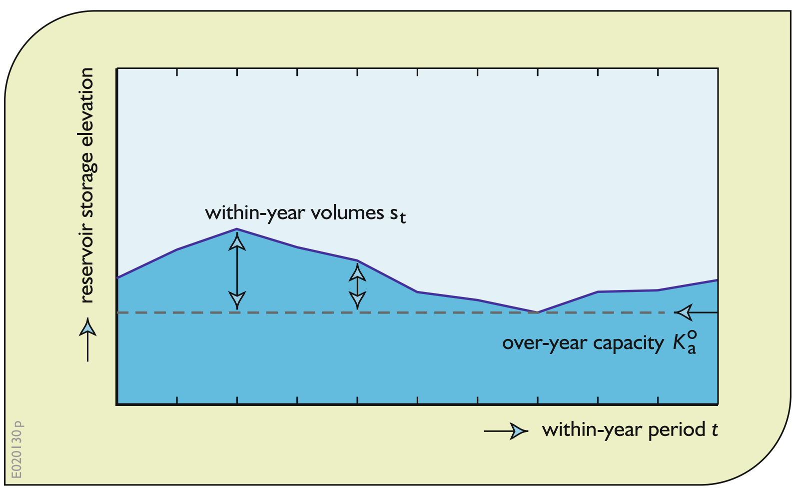


Fig. 11.8 Reservoir release rule showing the identification of the most reliable release zone associated with the within-year yields $y_{0.9,t}$

The next step is to solve the yield model for both yields $Y_{0.9}$ and $Y_{0.7}$. The resulting sum of over-year storage capacity and within-year storage volumes can be plotted over the first zone, as shown in Fig. 11.9.

If at any time t the actual storage volume is in the second lighter shaded zone in Fig. 11.9, both the release should be the sum of the most reliable yield, $y_{0.9,t}$ and the incremental secondary yield $y_{0.7,t}$. If only these releases are made, the probability of being in that zone, when simulated, should be about 0.7. If the actual storage volume is greater than the total required over-year storage capacity K_a^o plus the within-year volume s_t, the non-shaded zone in Fig. 11.23, then a release can be made to satisfy any downstream demand. Converting storage volume to elevation, this release policy is summarized in Fig. 11.10.

These yield models focus only on the active storage capacity requirements. They can be a part of a model that includes flood storage requirements as well (as previously discussed in this chapter). If the actual storage volume is within the flood control zone in the flood season, releases should be made to reduce the actual storage to a volume no greater than the total capacity less the flood storage capacity.

Once again, reservoir rules developed from simplified models such as this yield model are only guides, and once developed these rules

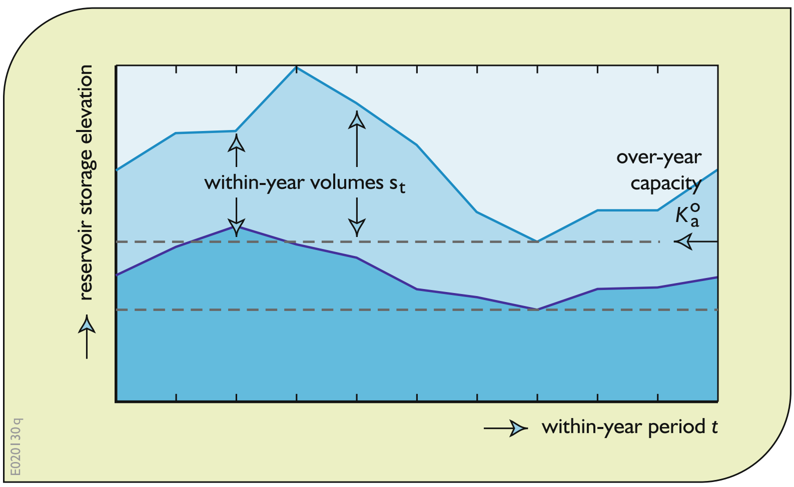


Fig. 11.9 Reservoir release rule showing the identification of the second most reliable release zone associated with the total within-year yields $y_{0.9,t} + y_{0.7,t}$

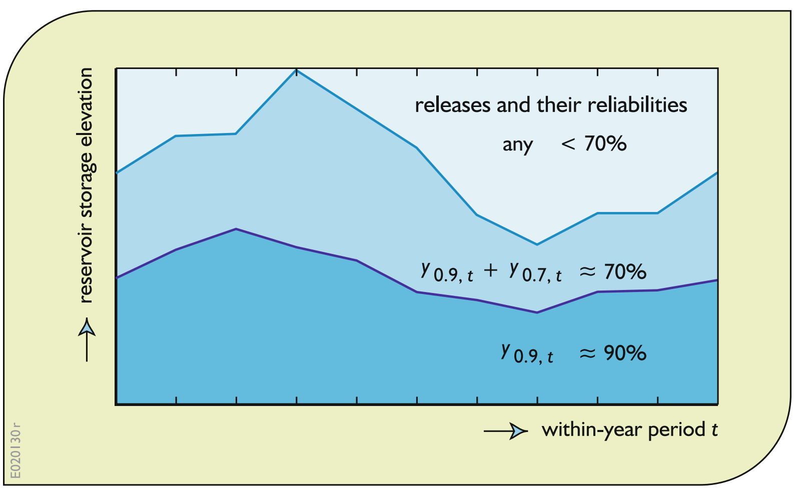


Fig. 11.10 Reservoir release rule defined by the yield model

should be simulated, evaluated, and refined prior to their actual adoption.

11.6 Drought and Flood Risk Reduction

11.6.1 Drought Planning and Management

Droughts are natural hazards that unlike floods, tornadoes, and hurricanes, occur slowly and gradually over a period of time. The absence of a precise drought threshold introduces some uncertainty about whether a drought exists and, if it does, its degree of severity. The impacts of drought are nonstructural and typically spread over a larger geographical area than are damages resulting from other natural hazards. All of these drought characteristics have impacted the development of effective drought preparedness plans.

Droughts result from a deficiency of precipitation compared to normal (long-term average) amounts that, when extended over a season or especially over a longer period of time, is insufficient to meet the demands of human activities and the environment. All types of drought results in water shortages for one or more water-using activities.

Droughts differ from one another in three essential characteristics: intensity, duration, and spatial coverage. Moreover, many disciplinary perspectives of drought exist. Because of these numerous and diverse disciplinary views, confusion often exists over exactly what constitutes a drought. Regardless of such disparate views, the overriding feature of drought is its negative impacts on people and the environment.

11.6.1.1 Drought Types

Droughts are normally distinguished by type: meteorological, hydrological, agricultural, and socioeconomic. Meteorological drought is expressed solely on the basis of the degree of dryness in comparison to some normal or average amount and the duration of the dry period. Drought intensity and duration are the key descriptors of this type of drought. Agricultural drought links various characteristics of meteorological drought to agricultural impacts, focusing on precipitation shortages, differences between actual and potential evapotranspiration, and soil water deficits.

Hydrological droughts are described based on the effects of low precipitation on surface or subsurface water supply (e.g., streamflow, reservoir storage, lake levels, and groundwater) rather than with precipitation shortfalls. Hydrological droughts usually lag the occurrence of meteorological and agricultural droughts because more time elapses before precipitation deficiencies are detected in rivers, reservoirs, groundwater aquifers, and other components of the hydrologic system. As a result, hydrological droughts are typically detected later than other drought types. Water uses affected by drought can include multiple purposes such as power generation, flood control, irrigation, domestic drinking water, industry, recreation, and ecosystem preservation.

Socioeconomic droughts are linked directly to the supply of some economic good. Increases in population can alter substantially the demand for these economic goods over time. The incidence of socioeconomic drought can increase because of a change in the frequency of meteorological drought, a change in societal vulnerability to water shortages, or both. For example, poor land use practices such as overgrazing can decrease animal carrying capacity and increase soil erosion, which exacerbates the impacts of, and vulnerability to, future droughts.

11.6.1.2 Drought Impacts

The impacts of drought are often widespread through the economy. They can be direct and indirect. Restrictions in water use resulting from drought is a direct or first-order impact of drought. However, the consequences of such restrictions could result in loss of income, farm and business foreclosures, and government relief programs) are possible indirect second- or third-order impacts.

The impacts of drought appear to be increasing in both developing and developed countries, which in many cases reflects the persistence of non-sustainable development and population growth. Lessening the impacts of future drought events typically requires the development of drought risk policies that emphasize a wide range of water conservation and early warning measures. Drought management techniques are often conditional on the severity of the drought. Identifying the actions to take and the thresholds indicating when to take them are best accomplished prior to a drought, as agreements among stakeholders are easier to obtain when individuals are not having to deal with the impacts of an ongoing drought.

Drought impacts can be economic, environmental, and social.

Economic impacts can include direct losses to agricultural and industrial users, losses in recreation, transportation, and energy sectors. Other indirect economic impacts can include resulting unemployment and loss of tax revenue to local, state, and federal governments.

Environmental losses include damages to plant and animal species in natural habitats, and reduced air and water quality; an increase in forest and range fires; the degradation of landscape quality; and possible soil erosion. These losses are difficult to quantify, but growing public awareness and concern for environmental quality has forced public officials to focus greater attention on them.

Social impacts can involve public safety, health, conflicts among water users, and inequities in the distribution of impacts and disaster relief programs. As with all natural hazards, the economic impacts of drought are highly variable within and among economic sectors and geographic regions, producing a complex assortment of winners and losers with the occurrence of each disaster.

11.6.1.3 Drought Preparedness and Mitigation

Droughts happen, and it makes no sense to wait until realizing a drought is happening before preparing plans and policies to mitigate the adverse impacts from a drought. As evidenced by the ongoing drought (at this writing) in California, and the even more severe drought those in southeastern Australia recently witnessed, drought management has to involve the institutions that not only manage water supply systems, but all those who use water, and all those who make land-use decisions that impact water runoff. It can involve hydrologic modeling methods discussed in Chap. 6, and reservoir modeling as discussed in Chaps. 4 and 8. Appendix C of this book (contained on a disk or downloadable from the web) discusses drought management modeling methods and options in more detail.

11.6.2 Flood Protection and Damage Reduction

Next consider the other extreme—floods. Two types of structural alternatives are often used for flood risk reduction. One is the provision of flood storage capacity in reservoirs designed to reduce downstream peak flood flows. The other is channel enhancement and/or flood-proofing structures that are designed to contain peak flood flows and reduce damage. This section introduces methods of modeling both of these alternatives for inclusion in either benefit–cost or cost-effectiveness analyses. The latter analyses apply to situations in which a significant portion of the flood control benefits cannot be expressed in monetary terms and the aim is to provide a specified level of flood protection at minimum cost.

The discussion will first focus on the estimation of flood storage capacity in a single reservoir upstream of a potential flood damage site. This analysis will then be expanded to include downstream channel capacity improvements. Each of the modeling methods discussed will be appropriate for inclusion in multipurpose river basin planning (optimization) models having longer time step durations than those required to predict flood peak flows.

11.6.2.1 Reservoir Flood Storage Capacity

In addition to the active storage capacities in a reservoir, some capacity may be allocated for the temporary storage of flood flows during certain periods in the flood season of the year, as shown in Fig. 11.2. Flood flows usually occur over time intervals lasting from a few hours up to a few days or weeks. Computational limitations make it impractical to include such short time durations in many of our multipurpose planning models that typically include time periods of a week, or 10 days, or months or seasons spanning several months. If we modeled these short daily or hourly durations, flood routing equations would have to be included in the model; a simple mass balance would not be sufficient. Nevertheless there are ways of including unknown flood storage variables within longer period optimization models .

Consider a potential flood damage site along a river. A flood control reservoir can be built upstream of that potential damage site. The question is how much flood storage capacity, if any, should the reservoir contain. For various assumed capacities and operating policies, simulation models can be used to predict the impact on the downstream flood peaks. These hydraulic simulation models must include flood routing procedures from the reservoir to the downstream potential damage site and the flood control operating policy at the reservoir. For various downstream flood peaks, water elevations and associated economic flood damages on the floodplain can be estimated. To calculate the expected annual damages associated with any upstream reservoir capacity , the probability of various damage levels being exceeded in any year needs to be calculated.

The likelihood of a flood peak of a given magnitude or greater is often described by its expected return period. How many years would one expect to wait, on average, to observe another flood of equal or greater than a flood of some specified magnitude? This is the reciprocal of the probability of observing such a flood or greater in any given year. A T-year flood has a probability of being equaled or exceeded in any year of $1/T$. This is the probability that could be calculated by adding up the number of years an annual flood of a given or greater magnitude is observed, say in 1000 or 10,000 years, divided by 1000 or 10,000, respectively. A one-hundred-year flood or greater has a probability of 1/100 or 0.01 of occurring in any given year. Assuming annual floods are independent, if a 100-year flood occurs this year, the probability that a flood of that magnitude or greater occurring next year remains 1/100 or 0.01.

If PQ is the random annual peak flood flow and PQ_T is a particular peak flood flow having a return period of T years, then by definition the probability of an actual flood of PQ equaling or exceeding PQ_T is $1/T$.

$$\Pr[PQ \geq PQ_T = 1/T] \tag{11.42}$$

The higher the return period, i.e., the more severe the flood, the lower the probability that a flood of that magnitude or greater will occur. Equation 11.42 is plotted in Fig. 11.11.

The exceedance probability distribution shown in Fig. 11.11 is simply 1 minus the cumulative distribution function $F_{PQ}(\cdot)$ of annual peak flood flows. The area under the function is the mean annual peak flood flow, $E[PQ]$.

The expected annual flood damage at a potential flood damage site can be estimated from an exceedance probability distribution of peak flood flows at that potential damage site together with a flow or stage damage function. The peak flow exceedance distribution at any potential damage site will be a function of the

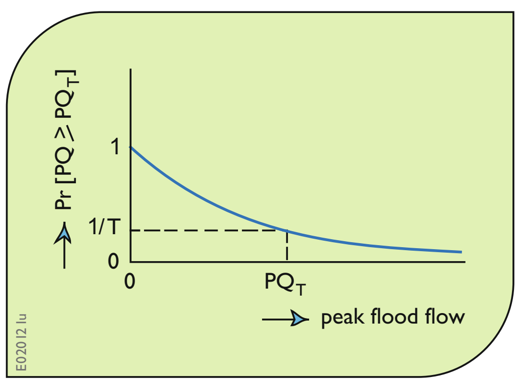


Fig. 11.11 Probability of annual peak flood flows being exceeded

upstream reservoir flood storage capacity K_f and the reservoir operating policy.

The probability that flood damage of FD_T associated with a flood of return period T will be exceeded is precisely the same as the probability that the peak flow PQ_T that causes the damage will be exceeded. Letting FD be a random flood damage variable, its probability of exceedance is

$$\Pr[\mathrm{FD} \geq \mathrm{FD}_T] = 1/T \qquad (11.43)$$

The area under this exceedance probability distribution is the expected annual flood damage , E[FD].

$$E[\mathrm{FD}] = \int_0^{\infty} \Pr[\mathrm{FD} \geq \mathrm{FD}_T] \mathrm{dFD}_T \qquad (11.44)$$

This computational process is illustrated graphically in Fig. 11.12. The analysis requires three input functions that are shown in quadrants (a), (b), and (c). The dashed-line rectangles define point values on the three input functions in quadrants (a), (b), and (c) and the corresponding probabilities of exceeding a given level of damages in the lower right quadrant (d). The distribution in quadrant (d) is defined by the intersections of these dashed-line rectangles. This distribution defines the probability of equaling or exceeding a specified damage in any given year. The (shaded) area under the derived function is the annual expected damage, $E[FD]$.

The relationships between flood stage and damage, and flood stage and peak flow, defined in quadrants (a) and (b) of Fig. 11.12, must be known. These do not depend on the flood storage

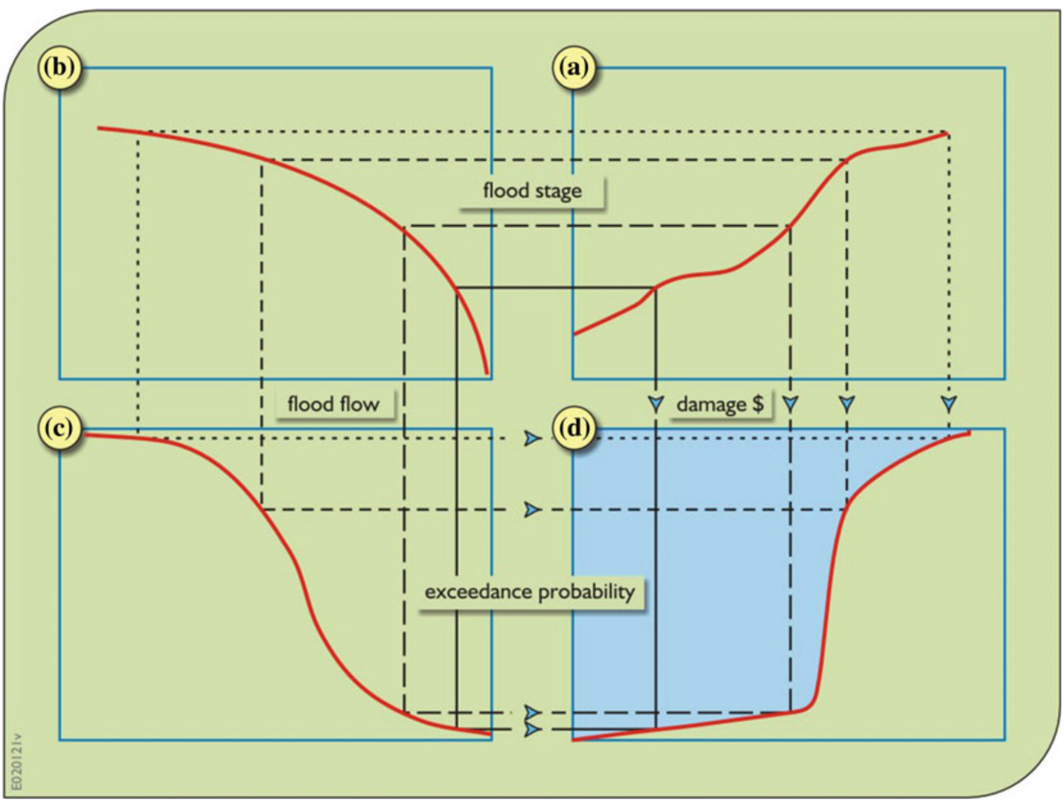


Fig. 11.12 Calculation of the expected annual flood damage shown as the shaded area in quadrant (**d**) derived from the expected stage damage function (**a**), the expected stage-flow relation (**b**), and the expected probability of exceeding an annual peak flow (**c**)

capacity in an upstream reservoir. The information in quadrant (c) is similar to that shown in Fig. 11.11 defining the exceedance probabilities of each peak flow. Unlike the other three functions, this distribution depends on the upstream flood storage capacity and flood flow release policy. This peak flow probability of exceedance distribution is determined by simulating the annual floods entering the upstream reservoir in the years of record.

The difference between the expected annual flood damage without any upstream flood storage capacity and the expected annual flood damage associated with a flood storage capacity of K_f is the expected annual flood damage reduction. This is illustrated in Fig. 11.13. Knowing the expected annual damage reduction associated with various flood storage capacities, K_f, permits the definition of a flood damage reduction function, $B_f(K_f)$.

If the reservoir is a single purpose flood control reservoir , the eventual tradeoff is between the expected flood reduction benefits, $B_f(K_f)$, and the annual costs, $C(K_f)$, of that upstream reservoir capacity. The particular reservoir flood storage capacity that maximizes the net benefits, $B_f(K_f) - C(K_f)$, may be appropriate from a national economic efficiency perspective but it may not be best from a local perspective. Those occupying the potential damage site may prefer a specified level of protection from that reservoir storage capacity, rather than the protection that maximizes expected annual net benefits, $B_f(K_f) - C(K_f)$.

If the upstream reservoir is to serve multiple purposes, say for water supply, hydropower, and recreation, as well as for flood control, the expected flood reduction benefit function just derived could be a component in the overall objective function for that reservoir.

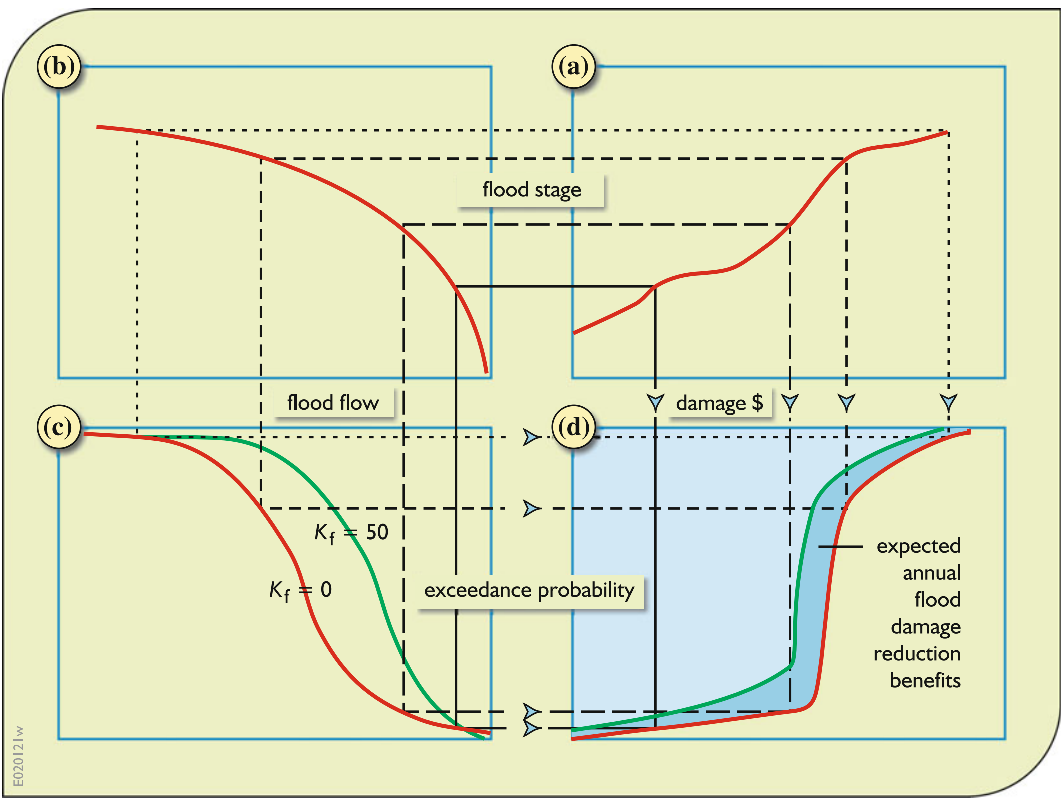


Fig. 11.13 Calculation of expected annual flood damage reduction benefits, shown as the darkened portion of quadrant (**d**), associated with a specified reservoir flood storagecapacity

Total reservoir capacity K will equal the sum of dead storage capacity K_d, active storage capacity K_a, and flood storage capacity K_f, assuming they are the same in each period t. In some cases they may vary over the year. If the required active storage capacity can occupy the flood storage zone when flood protection is not needed, the total reservoir capacity K will be the dead storage, K_d, plus the maximum of either (1) the actual storage volume and flood storage capacity in the flood season or (2) the actual storage volume in non-flood season.

$$K \geq K_d + S_t + K_f \quad \text{for all periods } t \text{ in flood season plus the following period that represents the end of the flood season} \tag{11.45}$$

$$K \geq K_d + S_t \quad \text{for all remaining periods } t \tag{11.46}$$

In the above equations the dead storage capacity, K_d, is assumed known. It is included in the capacity Eqs. 11.45 and 11.46 assuming that the active storage capacity is greater than zero. Clearly, if the active storage capacity were zero, there would be no need for dead storage.

11.6.2.2 Channel Capacity

The unregulated natural peak flow of a particular design flood at a potential flood damage site can be reduced by upstream reservoir flood storage capacity or it can be contained within the channel at the potential damage site by levees and other channel-capacity improvements. In this section, the possibility of levees or dikes and other channel capacity or flood-proofing improvements at a downstream potential damage site will be considered. The approach used will provide a means of estimating combinations of flood control storage capacity in upstream reservoirs and downstream channel capacity improvements that together will provide a prespecified level of flood protection at the downstream potential damage site.

Let QN_T be the unregulated natural peak flow in the flood season having a return period of T years. Assume that this peak flood flow is the design flood for which protection is desired. To protect from this design peak flow, a portion QS of the peak flow may be reduced by upstream flood storage capacity. The remainder of the peak flow QR must be contained within the channel. Hence if the potential damage site s is to be protected from a peak flow of QN_T, the peak flow reductions due to upstream storage, QS, and channel improvements, QR, must at least equal to that peak flow.

$$QN_T \leq QS + QR \tag{11.47}$$

The extent to which a specified upstream reservoir flood storage capacity reduces the design peak flow at the downstream potential damage site can be obtained by routing the design flood through the reservoir and the channel between the reservoir and the downstream site. Doing this for a number of reservoir flood storage capacities permits the definition of a peak flow reduction function, $f_T(K_f)$.

$$QS = f_T(K_f) \tag{11.48}$$

This function is dependent on the relative locations of the reservoir and the downstream potential damage site, on the characteristics and length of the channel between the reservoir and downstream site, on the reservoir flood control operating policy, and on the magnitude of the peak flood flow.

An objective function for evaluating these two structural flood control measures should include the cost of reservoir flood storage capacity, $\text{Cost}_K(K_f)$, and the cost of channel capacity improvements, $\text{Cost}_R(QR)$, required to contain a flood flow of QR. For a single purpose, single damage site, single reservoir flood control problem, the minimum total cost required to protect the potential damage site from a design flood peak of QN_T, may be obtained by solving the model:

$$\text{minimize } \text{Cost}_K\left(K_f\right) + \text{Cost}_R(QR) \tag{11.49}$$

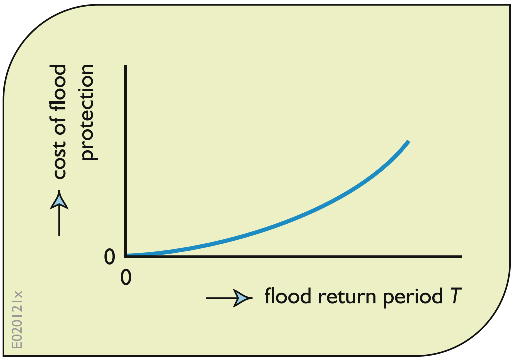


Fig. 11.14 Tradeoff between minimum cost of flood protection and flood risk, as expressed by the expected return period

subject to

$$QN_T \leq f_T(K_f) + QR \qquad (11.50)$$

Equations 11.49 and 11.50 assume that a decision will be made to provide protection from a design flood QN_T of return period T; it is only a question of how to provide the required protection, i.e., how much flood storage capacity and how much levee protection.

Solving Eqs. 11.47 and 11.48 for peak flows QN_T of various return periods T will identify the risk-cost tradeoff. This tradeoff function might look like what is shown in Fig. 11.14.

11.6.2.3 Estimating Risk of Levee Failures

Levees are built to reduce the likelihood of flooding on the flood plain. Flood flows prevented from flowing over a floodplain due to a levee will have relatively little effect on users of the flood plain, unless of course the levee fails to contain the flow. Levee failure can result from flood events that exceed (overtop) the design capacity of the levee. Failure can also result from various types of geostructural weaknesses. If any of the flow in the stream or river channel passes over, through or under the levee and onto the flood plain, the levee is said to fail. The probability of levee failure along a river reach is in part a function of the levee height, the probability distribution of flood flows in the stream or river channel, and the probability of geo-structural failure. The latter depends in part on how well the levee and its foundation is constructed. Some levees are purposely designed to "fail" at certain sites at certain flood stages to reduce the likelihood of more substantial failures and flood damages further downstream.

The probability of levee failure given the flood stage (height) in the stream or river channel is often modeled using two flood stages. The US Army Corps of Engineers calls the lower stage the probable non-failure point, *PNP*, and the higher stage is called the probable failure point, *PFP* (USACE 1991). At the *PNP*, the probability of failure is assumed to be 15%. Similarly, the probability of failure at the *PFP* is assumed to be 85%. A straight-line distribution between these two points is also assumed, as shown in Fig. 11.15. Of course these points and distributions are at best only guesses, as not many, if any, data will exist to base them on at any given site.

To estimate the risk of a flood in the floodplain protected by a levee due to overtopping or geo-structural levee failure, the relationships between flood flows and flood stages in the channel and on the floodplain must be defined, just as it had to be to carry out the analyses shown in Figs. 11.12 and 11.13.

Assuming no geo-structural levee failure, the flood stage in the floodplain protected by a levee is a function of the flow in the stream or river channel, the cross sectional area of the channel between the levees on either side, the channel slope and roughness, and the levee height. If floodwaters enter the floodplain, the resulting water level or stage in the floodplain will depend on the topological characteristics of the flood plain. Figure 11.16 illustrates the relationship between the flood stage in the channel and the flood stage in the flood plain, assuming no geo-structural failure of the levee. Obviously once the flood begins overtopping the levee, the flood stage in the flood plain begins to increase. Once the flood flow is of sufficient magnitude that its stage without the levee is the same as that with the levee, the existence of a levee has only a negligible impact on the flood stage.

Figure 11.17 illustrates the relationship between flood flow and flood stage in a

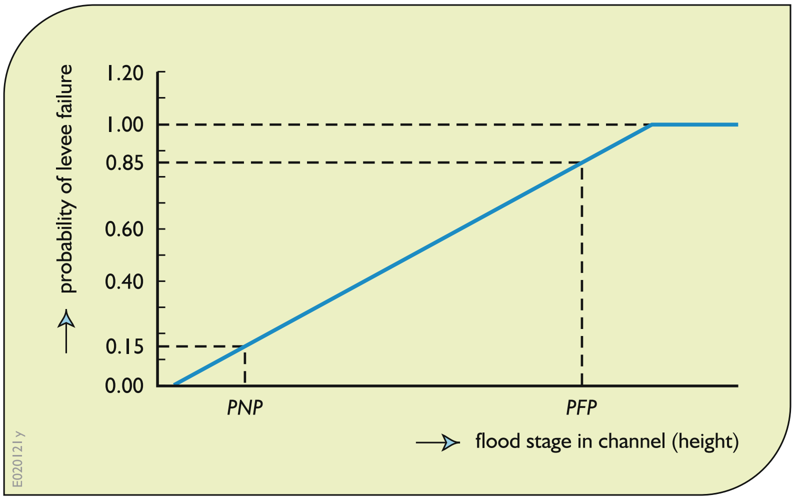


Fig. 11.15 Assumed cumulative probability distribution of levee failure expressed as function of flood stage in river channel

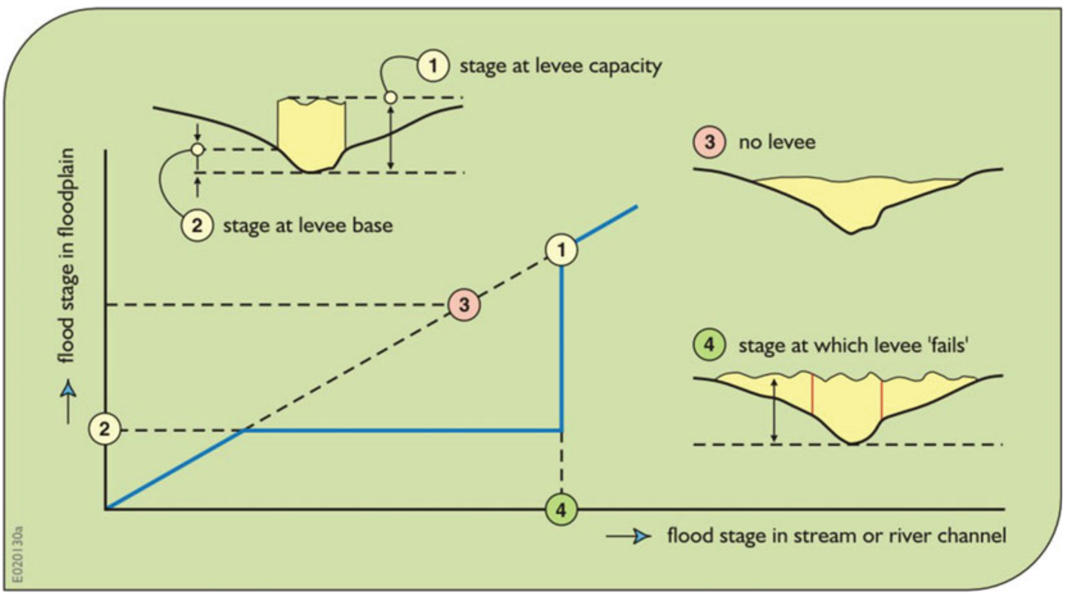


Fig. 11.16 Influence of a levee on the flood stage in floodplain compared to stream or river flood stage. The channel flood stage where the curve is vertical is the stage at which the levee fails due to overtopping or from geo-structural causes

floodplain with and without flood levees, again assuming no geo-structural levee failure.

Combining Figs. 11.16 and 11.17 defines the relationship between reach flow and channel stage. This is illustrated in the upper left quadrant of Fig. 11.18.

Combining the relationship between flood flow and flood stage in the channel (upper left quadrant of Fig. 11.18) with the probability distribution of levee failure (Fig. 11.15) and the probability distribution of annual peak flows being equaled or exceeded (Fig. 11.11), provides

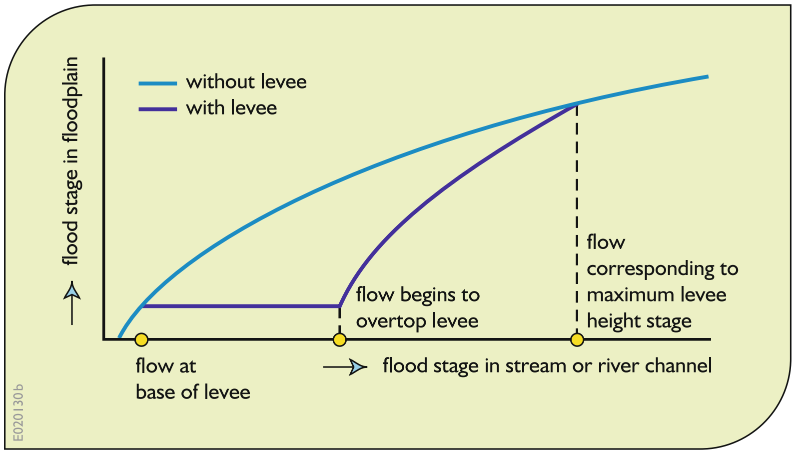


Fig. 11.17 Relationship between flood flow and flood stage in a floodplain with and without flood levees, again assuming no geo-structural levee failure

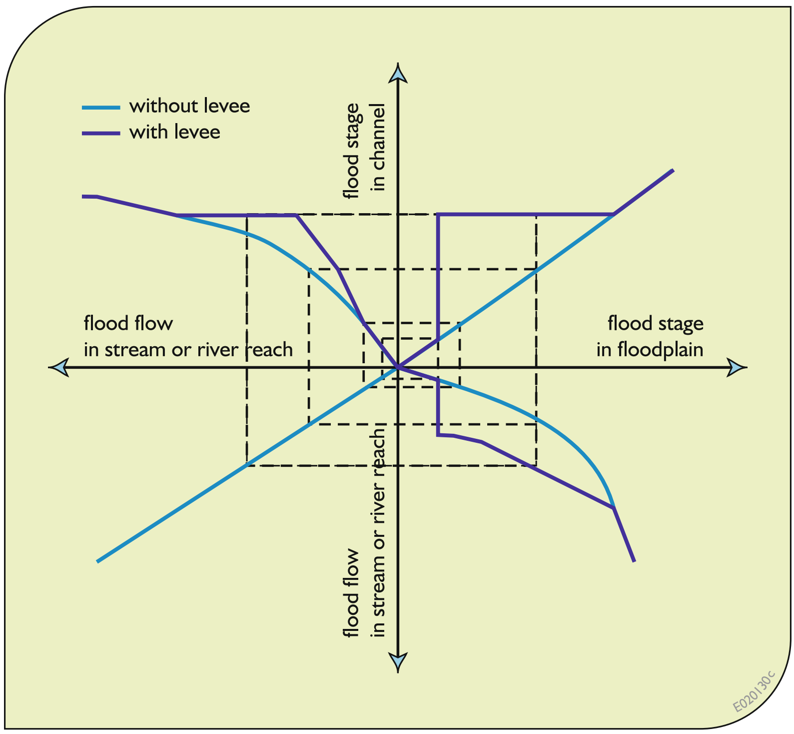


Fig. 11.18 Deriving the relation (shown in the *upper left quadrant*) between flood flow and flood stage in the channel

an estimate of the expected probability of levee failure. Figure 11.19 illustrates this process of finding, in the lower right quadrant, the shaded area that equals the expected annual probability of levee failure from overtopping and/or geo-structural failure.

The channel flood-stage function, $S(q)$, of peak flow q shown in the upper left quadrant of Fig. 11.19 is obtained from the upper left quadrant of Fig. 11.18. The probability of levee failure, PLF(S), a function of flood stage, $S(q)$, shown in the upper right quadrant is the same as in Fig. 11.15. The annual peak flow exceedance probability distribution, $F_Q(q)$, (or its inverse $Q(p)$) in the lower left quadrant is the same as Fig. 11.12 or that in the lower left quadrant (c) of Fig. 11.13. The exceedance probability function in the lower right quadrant of Fig. 11.19 is

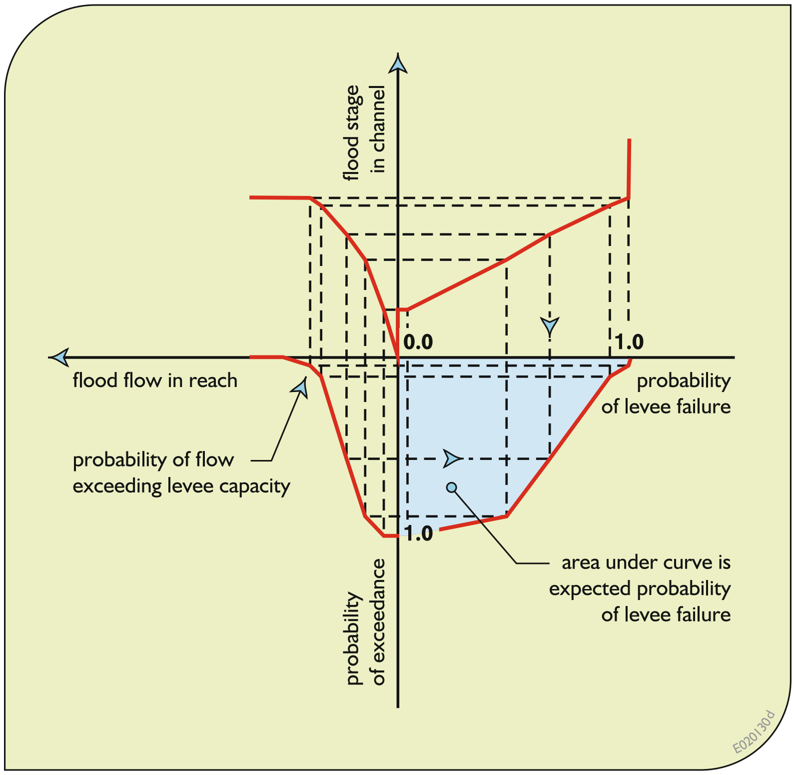


Fig. 11.19 Derivation of the probability of exceeding a given probability of levee failure, shown in lower right quadrant. The shaded area enclosed by this probability distribution is the annual expected probability of levee failure

derived from each of the other three functions, as indicated by the arrows, in the same manner as described in Fig. 11.12.

In mathematical terms, the annual expected probability of levee failure, E[PLF], found in the lower right quadrant of Fig. 11.19, equals

$$E[PLF] = \int_0^\infty PLF[S(q)]f(q)\mathrm{d}q = \int_0^1 PLF[S(Q(p))]\mathrm{d}p = \int_0^1 PLF'(p)\mathrm{d}p, \qquad (11.51)$$

where PLF′(p) is the probability of levee failure associated with a flood stage of $S(q)$ having an exceedance probability of p.

Note that if the failure of the levee was only due to channel flood stages exceeding the levee height (i.e., if the probability of geo-structural failure were zero) the expected probability of levee failure would be simply the probability of exceeding a channel flow whose stage equals the levee height, as defined in the lower left quadrant of Fig. 11.19. This is shown in Fig. 11.20.

Referring to Fig. 11.20, if the levee height is increased, the horizontal part of the curve in the upper right quadrant would rise, as would the horizontal part of the curve in the upper left quadrant as it shifts to the left. Hence given the same probability distribution as defined in the lower left quadrant, the expected probability of exceeding an increased levee capacity would decrease, as it should.

11.6.2.4 Annual Expected Damage from Levee Failure

A similar analysis can provide an estimate of the expected annual flood plain damage for a stream or river reach. Consider, for example, a parcel of land on a flood plain at some location i. If an economic efficiency objective were to guide the development and use of this parcel, the owner would want to maximize the net annual economic benefits derived from its use, B_i, less the

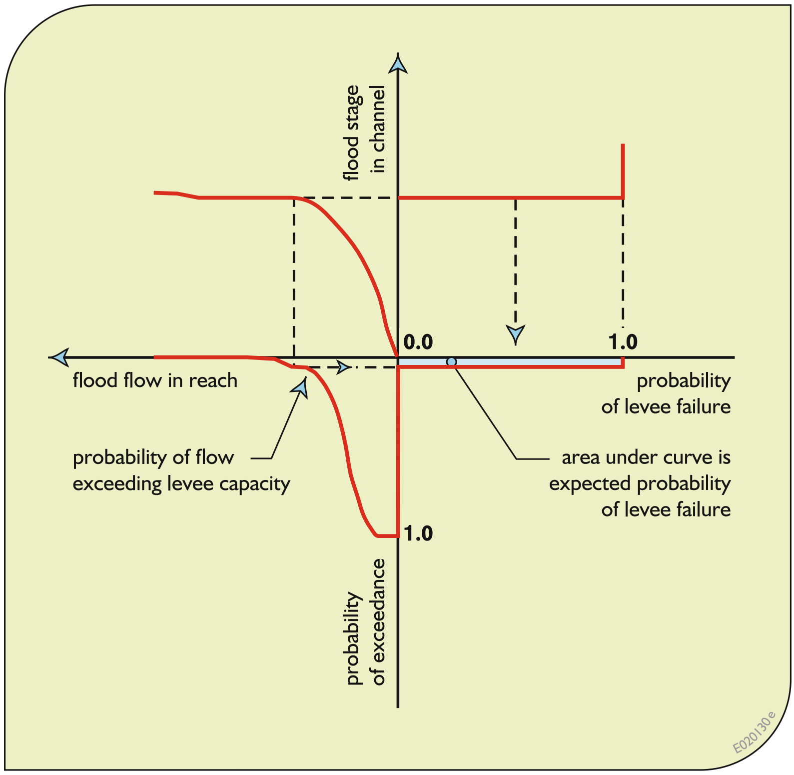


Fig. 11.20 Calculation of annual probability of equaling or exceeding any specified probability of levee overtopping, shown in the *lower right quadrant*. The shaded area in this quadrant is the expected probability of levee overtopping, assuming there is no geo-structural failure

annual (non-flood damage) costs, C_i, and the expected annual flood damages, EAD_i. The issue of concern here is the estimation of these expected annual flood damages.

Damages at location i resulting from a flood will depend in part on the depth of flooding at that location and a host of other factors (flood duration, velocity of and debris in flood flow, time of year, etc.). Assume that the flood damage at location i is a function of primarily the flood stage, S, at that location. Denote this potential damage function as $D_i(S)$. Such a function is illustrated in Fig. 11.21.

Integrating the product of the annual exceedance probability of flood stage, $F_s(S)$, and the potential damages, $D_i(S)$, over all stages S will yield the annual expected damages, $E[D_i]$, for land parcel i.

$$E[D_i] = \int D_i(S)F_s(S)\mathrm{d}S \qquad (11.52)$$

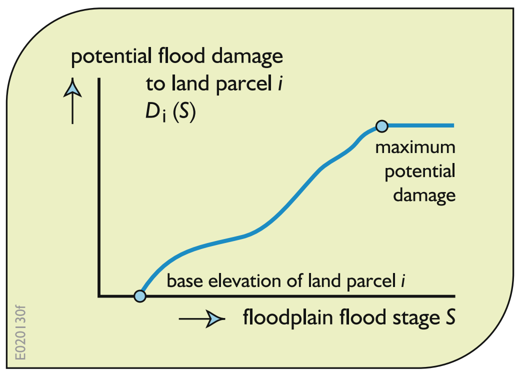


Fig. 11.21 An example function defining the damages that will occur given any flood stage S to a parcel of land i

The sum of these expected damage estimates over all the parcels of land i on the floodplain is the total expected damage that one can expect each year, on average, on the floodplain.

$$\mathrm{EAD} = \sum_i E[D_i] \qquad (11.53)$$

Alternatively the annual expected flood damage could be based on a calculated probability of exceeding a specified flood damage, as shown in Fig. 11.12. For this method the potential flood damages, $D_i(S)$, are determined for various stages S and then summed over all land parcels i for each of those stage values S to obtain the total potential damage function, $D(S)$, for the entire floodplain, defined as a function of flood stage S.

$$D(S) = \sum_i D_i(S) \qquad (11.54)$$

This is the function shown in quadrant (a) in Fig. 11.12.

Levee failure probabilities, PLF′(p), based on the exceedance probability p of peak flows, or stages, as defined in Fig. 11.31 and Eq. 11.49 can be included in calculations of expected annual damages. Expressing the damage function, $D(S)$, as a function, $D'(p)$, of the stage exceedance probability p and multiplying this flood damage function $D'(p)$ times the probability of levee failure, PLF′(p) defines the joint exceedance probability of expected annual damages. Integrating over all values of p yields the expected annual flood damage, EAD.

$$\text{EAD} = \int_0^1 D'(p)\, PLF'(p)\, \mathrm{d}p \qquad (11.55)$$

Note that the flood plain damages and probability of levee failure functions in Eq. 11.55 both increase with increasing flows or stages, but as peak flows or stages increase, their exceedance probabilities decrease. Hence with increasing p the damage and levee failure probability functions decrease. The effect of levees on the expected annual flood damage, EAD, is shown in Fig. 11.22. The "without levee" function in the lower right quadrant of Fig. 11.18, is $D'(p)$. The "with levee" function is the product of $D'(p)$ and PLF′(p). If the probability of levee failure, PLF′(p) function were as shown in Fig. 11.18, i.e., if it were 1.0 for values of p below some overtopping stage associated with an exceedance probability p^*, and 0 for values of p greater than p^*, then the function would appear as shown "with levee—overtop only" in Fig. 11.22.

11.6.2.5 Risk-Based Analyses

Risk-based analyses attempt to identify the uncertainty associated with each of the inputs used to define the appropriate capacities of various flood risk reduction measures. There are numerous sources of uncertainty associated with each of the functions shown in quadrants (a), (b), and (c) in Fig. 11.25. This uncertainty translates to uncertainty associated with estimates of flood risk probabilities and expected annual flood damage reductions obtained from reservoir flood storage capacities and channel improvements.

Going to the substantial effort and cost of quantifying these uncertainties, which themselves will be surely be uncertain, does however provide additional information. The design of any flood protection plan can be adjusted to reflect attitudes of stakeholders toward the uncertainty associated with specified flood peak return periods or equivalently their probabilities of occurring in any given year.

For example, assume a probability distribution capturing the uncertainty about the expected probability of exceedance of the peak flows at the potential damage site (as shown in Fig. 11.12) is defined from a risk-based-analysis. Figure 11.23 shows that exceedance function together with its 90% confidence bands near the higher flood peak return periods. To be, say, 90% sure that protection is provided for the T-year return period flow, PQ_T, one may have to for an equivalent expected $T + \Delta$ year return period flow, $PQ_{T+\Delta}$, i.e., the flow having a $(1/T) - \delta$ expected probability of being exceeded. Conversely, protection from the expected $T + \Delta$ year peak flood flow will provide 90% assurance of protection from flows that will occur less than once in T years on average.

If society wanted to eliminate flood damage it could do it, but at a high cost. This would require either costly flood control structures or eliminating economic activities on lands subject to possible flooding. Both reduce expected economic returns from the floodplain. Hence such actions are not likely to be taken. There will

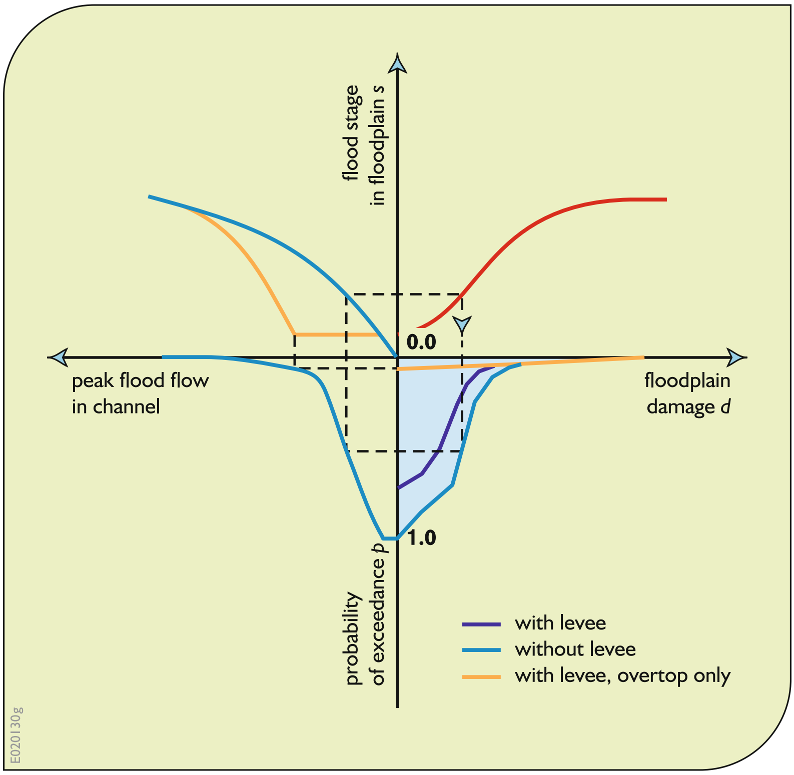


Fig. 11.22 Calculation of expected annual flood damage taking into account probability of levee failure

always be a risk of flood damage. Analyses such as those just presented help identify these risks. Risks can be reduced and managed but not eliminated. Finding the best levels of flood protection and flood risk, together with risk insurance or subsidies (illustrated in Fig. 11.24) is the challenge for public and private agencies alike. Floodplain management is as much concerned with good things not happening on them as with bad things—like floods—happening on them.

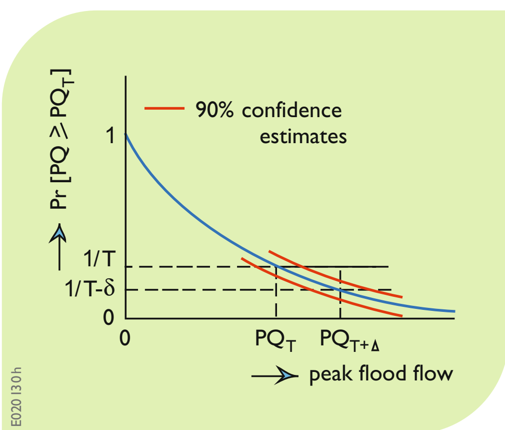


Fig. 11.23 Portion of peak flow probability of exceedance function showing contours containing 90% of the uncertainty associated with this distribution. To be 90% certain of protection from a peak flow of PQ_t, protection is needed from the higher peak flow, $PQ_{t+\Delta}$ expected once every $T + \Delta$ years, i.e., with an annual probability of $1/(T + \Delta)$ or $(1/T) - \delta$ of being equaled or exceeded

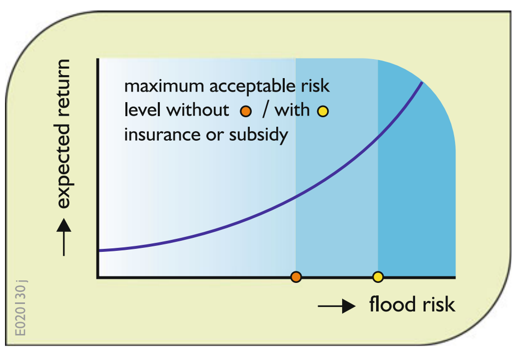


Fig. 11.24 Relationship between expected economic return from flood plain use and risk of flooding. The lowest flood risk does not always mean the best risk, and what risk is acceptable may depend on the amount of insurance or subsidy provided when flood damage occurs

11.7 Hydroelectric Power Production

Hydropower plants, Fig. 11.25, convert the energy from the flowing water to mechanical and then electrical energy. These plants containing turbines and generators are typically located either in or adjacent to dams. Pipelines (penstocks) carry water under pressure from the reservoir to the powerhouse. Power transmission systems transport the produced electrical energy from the powerhouse to where it is needed.

The principal advantages of using hydropower are the absence of polluting emissions during operation, its capability to respond relatively quickly to changing utility load demands, and its relatively low operating costs. Disadvantages can include high initial capital cost and potential site-specific and cumulative environmental impacts. Potential environmental impacts of hydropower projects include altered flow regimes below storage reservoirs or within diverted stream reaches, water quality degradation, mortality of fish that pass through turbines, blockage of fish migration, and flooding of terrestrial ecosystems by impoundments. However, in many cases, proper design and operation of hydropower projects can mitigate some of these impacts. Hydroelectric projects can also provide additional benefits such as from recreation in reservoirs or in tailwaters below dams.

Hydropower plants can be either *conventional* or *pumped storage*. Conventional hydropower plants use the available water from a river, stream, canal system, or reservoir to produce electrical energy. In conventional multipurpose reservoirs and run-of-river systems, hydropower production is just one of many competing purposes for which

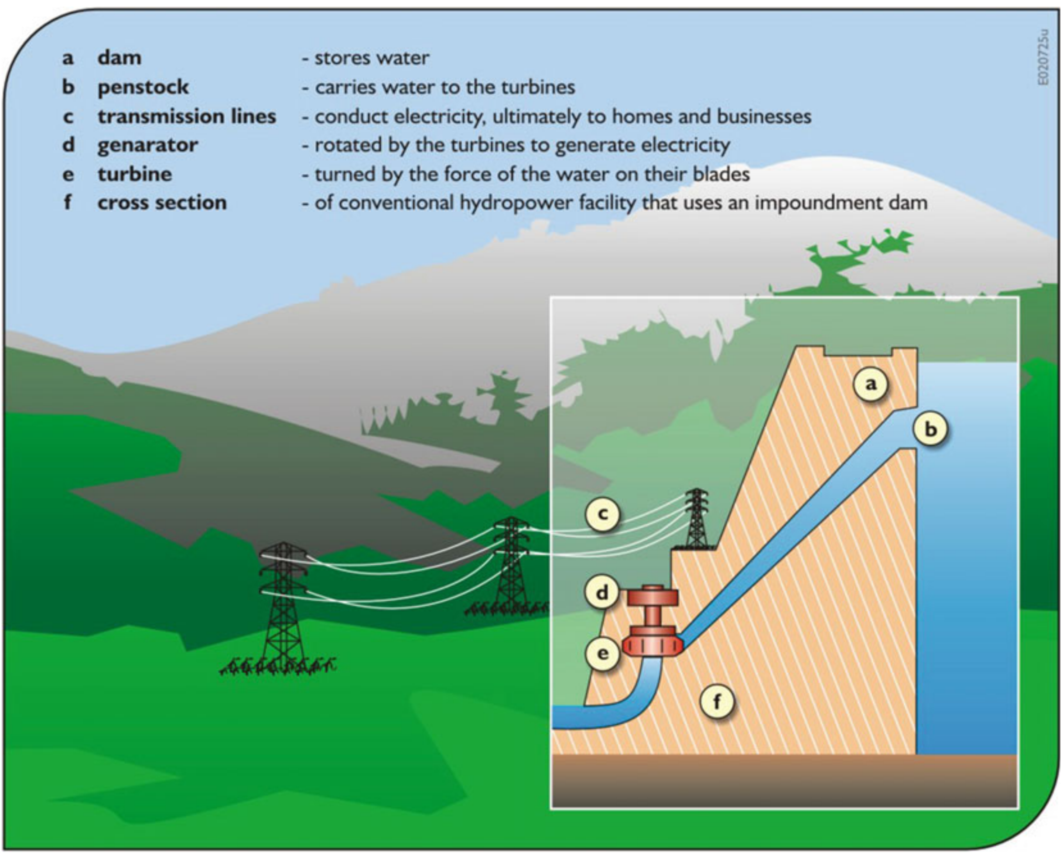


Fig. 11.25 Hydropower system components

water resources may be used. Competing water uses may include irrigation, flood control, navigation, downstream flow dilution for quality improvement, and municipal and industrial water supply. Pumped storage plants pump the water, usually through a reversible turbine that acts as a pump, from a lower supply source to an upper reservoir. While pumped storage facilities are net energy consumers, they are income producers. They are valued by a utility because they can be brought online rapidly to operate in a peak power production mode when energy prices are the highest. The pumping to replenish the upper reservoir is performed during off-peak hours when electricity costs are low. Then they are released through the power plant when the electricity prices are higher. The system makes money even though it consumes more energy. This process benefits the utility by increasing the load factor and reducing the cycling of its base load units. In most cases, pumped storage plants run a full cycle every 24 h (DOE 2002).

Run-of-river projects use the natural flow of the river and produce relatively little change in the stream channel and stream flow. A peaking project impounds and releases water when the energy is needed. A storage project extensively impounds and stores water during high-flow periods to augment the water available during low-flow periods, allowing the flow releases and power production to be more constant. Many projects combine the modes.

The power capacity of a hydropower plant is primarily the function of the flow rate through the turbines and the hydraulic head. The hydraulic head is the elevation difference the water falls (drops) in passing through the plant or to the tailwater, which ever elevation difference is less. Project design may concentrate on either of these flow and head variables or both, and on the hydropower plant installed designed capacity.

The production of hydroelectric energy during any period at any particular reservoir site is dependent on the installed plant capacity; the flow through the turbines; the average effective productive storage head; the duration of the period; the plant factor (the fraction of time energy is produced); and a constant for converting the product of flow, head, and plant efficiency to electrical energy. The kilowatt-hours of energy, KWH_t, produced in period t is proportional to the product of the plant efficiency, e, the productive storage head H_t, and the flow q_t through the turbines.

A cubic meter of water, weighing 10^3 kg, falling a distance of 1 m, acquires 9.81×10^3 J (nm) of kinetic energy. The energy generated in one second equals the watts (joules per second) of power produced. Hence an average flow of q_t cubic meters per second falling a height of H_t meters in period t yields $9.81 \times 10^3 q_t H_t$ watts or $9.81\, q_t H_t$ kilowatts of power. Multiplying by the number of hours in period t yields the kilowatt-hours of energy produced given a head of H_t and an average flow rate of q_t. The total kilowatt-hours of energy, KWH_t, produced in period t assuming 100% efficiency in conversion of potential to electrical energy is

$$\begin{aligned} KWH_t &= 9.81\, q_t H_t (\text{seconds in period } t)/ \\ &\qquad (\text{seconds per hour}) \\ &= 9.81\, q_t H_t (\text{seconds in period } t)/3600 \end{aligned} \tag{11.56}$$

Since the total flow, Q_t^T through the turbines in period t, equals the average flow rate q_t times the number of seconds in the period, the total kilowatt-hours of energy produced in period t given a plant efficiency (fraction) of e equals

$$\begin{aligned} KWH_t &= 9.81\ Q_t^T\ H_t\ e/3600 \\ &= 0.002725\ Q_t^T\ H_t\ e \end{aligned} \tag{11.57}$$

The energy required for pumped storage, where instead of producing energy the turbines are used to pump water up to a higher level, is

$$KWH_t = 0.002725\ Q_t^T\ H_t/e \tag{11.58}$$

For Eqs. 11.57 and 11.58, Q_t^T is expressed in cubic meters and H_t is in meters. The storage head, H_t, is the vertical distance between the water surface elevation in the lake or reservoir that is the source of the flow through the turbines and the maximum of either the turbine elevation or the downstream discharge elevation. In variable head reservoirs, storage heads are functions of storage volumes (and possibly the reservoir release if the tailwater elevation affects the head). In optimization models for capacity planning, these heads and the turbine flows are among the unknown variables. The energy produced is proportional to the product of these two unknown variables. This results in non-separable functions in model equations that must be written at each hydroelectric site for each time period t.

A number of ways have been developed to convert these non-separable energy production functions to separable ones for use in linear optimization models for estimating design and operating policy variable values. These methods inevitably increase the number of model variables and constraints. For a preliminary screening of hydropower capacities prior to a more detailed analysis (e.g., using simulation or other nonlinear or discrete dynamic programming methods) one can (1) solve the model using both optimistic and pessimistic assumed fixed head values, (2) compare the actual derived heads with the assumed ones and adjust the assumed heads, (3) resolve the model, and (4) compare the capacity values. From this iterative process, one should be able to identify the range of hydropower capacities that can then be further refined using simulation.

Alternatively average heads, H_t^o, and flows, Q_t^o, can be used in a linear approximation of the non-separable product terms, $Q_t^T H_t$.

$$Q_t^T H_t = H_t^o Q_t^T + Q_t^o H_t - Q_t^o H_t^o \qquad (11.59)$$

Again, the model may need to be solved several times in order to identify reasonably accurate average flow and head estimates, Q_t^o and H_t^o, in each period t.

The amount of electrical energy produced is limited by the installedkilowatts of plant capacity P as well as on the plant factor $p_{t.}$ The plant factor is a measure of hydroelectric power plant use in each time period. Its value depends on the characteristics of the power system and the demand pattern for hydroelectric energy. The plant factor is defined as the average power load on the plant for the period divided by the installed plant capacity. The plant factor accounts for the variability in the demand for hydropower during each period t. This factor is usually specified by those responsible for energy production and distribution. It may or may not vary for different time periods.

The total energy produced cannot exceed the product of the plant factor p_t, the number of hours, h_t, in the period, and the plant capacity P, measured in kilowatts.

$$\mathrm{KWH}_t \leq P\ h_t p_t \qquad (11.60)$$

11.8 Withdrawals and Diversions

Major demands for the withdrawal of water include those for domestic or municipal uses, industrial uses (including cooling water), and agricultural uses including iirrigation. These uses generally require the withdrawals of water from a river system, from reservoirs, or from other surface or groundwater bodies. The water withdrawn may be only partially consumed, and that which is not consumed may be returned, perhaps at a different site, at a later time period, and containing different concentrations of constituents.

Water can also be allocated to instream uses that alter the distribution of flows in time and space. Such uses include (1) reservoir storage, possibly for recreational use as well as for water supply; (2) for flow augmentation, possibly for water quality control or for navigation or for ecological benefits; and (3) for hydroelectric power production. The instream uses may complement or compete with each other or with various off-stream municipal, industrial, and agricultural demands. One purpose of developing management models of river basin systems is to help derive policies that will best serve these multiple uses, or at least identify the tradeoffs among the multiple purposes and objectives.

The allocated flow q_t^s to a particular use at site s in period t must be no greater than the total flow available, Q_t^s, that site and in that period.

$$q_t^s \leq Q_t^s \qquad (11.61)$$

The quantity of water that any particular user expects to receive in each particular period is termed the *target allocation.* Given a multi-period (e.g., annual) known or unknown target allocation T^s at site *s,* some (usually known) fraction, f_t^s, of that target allocation will be expected in period *t.* If the actual allocation, q_t^s, is less than the target allocation, $f_t^s T^s$, there will be a *deficit*, D_t^s. If the allocation is greater than the target allocation, there will be an excess, E_t^s. Hence, to define those unknown variables the following constraint equation can be written for each applicable period *t*.

$$q_t^s = f_t^s T^s - D_t^s + E_t^s \tag{11.62}$$

Even though allowed, one would not expect a solution to contain nonzero values for both D_t^s and E_t^s.

Whether or not any deficit or excess allocation should be allowed at any demand site *s* depends on the quantity of water available and the losses or penalties associated with deficit or excess allocations to that site. At sites where the benefits derived in each period are independent of the allocations in other periods, the losses associated with deficits and the losses or benefits associated with excesses can be defined in each period *t* (Chap. 9). For example, the benefits derived from the allocation of water for hydropower production in period *t* in some cases will be essentially independent of previous allocations.

For any use in which the benefits are dependent on a sequence of allocations, such as at irrigation sites, the benefits may be based on the annual (or growing season) target water allocations T^s and their within season distributions, $f_t^s T^s$. In these cases one can define the benefits from those water uses as functions of the unknown season or annual targets, T^s, where the allocated flows q_t^s must be no less than the specified fraction of that unknown target.

$$q_t^s \geq f_t^s T^s \quad \text{for all relevant } t \tag{11.63}$$

If, for any reason, an allocation variable value q_t^s must be low, or even zero, due to other more beneficial uses, then clearly from Eq. 11.63 the annual or growing season target allocation T^s would be low (or zero) and presumably so would be the benefits associated with that target value.

Water stored in reservoirs can often be used to augment downstream flows for instream uses such as recreation, navigation, and water quality control. During natural low-flow periods in the dry season, it is not only the increased volume but also the lower temperature of the augmented flows that may provide the only means of maintaining certain species of fish and other aquatic life. Dilution of wastewater or runoff from non-point sources may be another potential benefit from flow augmentation. These and other factors related to water quality management are discussed in greater detail in Chap. 10.

The benefits derived from navigation on a potentially navigable portion of a river system can usually be expressed as a function of the stage or depth of water in various periods. Assuming known stream or river flow-stage relationships at various sites in the river, a possible constraint might require at least a minimum acceptable depth, and hence flow, for those sites.

11.9 Lake-Based Recreation

Recreation benefits derived from natural lakes as well as reservoirs are usually dependent on their storage levels. Where recreational facilities have been built, recreational benefits will also be dependent on recreational target lake levels as well. If docks, boathouses, shelters, and other recreational facilities were installed based on some assumed (target) lake level, and the lake levels deviate from the target value, there can be reduced recreational benefits. These storage targets and any deviations can be modeled similar to Eq. 11.62. The actual storage volume at the beginning of a recreation period *t* equals the target storage volume less any deficit or plus any excess.

$$S_t^s = T^s - D_t^s + E_t^s \tag{11.64}$$

The recreational benefits in any recreational period t can be defined based on what they would be if the target were met less the average of any losses that may occur from initial and final storage volume deviations, D_t^s or E_t^s, from the target storage volume in each period of the recreation season (Chap. 9).

11.10 Model Synthesis

Each of the model components discussed above can be combined, as applicable, into a model of a river system. One such river system together with some of its interested stakeholders is shown in Fig. 11.26.

One of the first tasks in modeling this basin is to identify the actual and potential system components and their interdependencies. This is facilitated by drawing a schematic of the system at the level of detail that will address the issues being discussed and of concern to these stakeholders. This schematic can be drawn over the basin as in Fig. 11.27. The schematic without the basin is illustrated in Fig. 11.28.

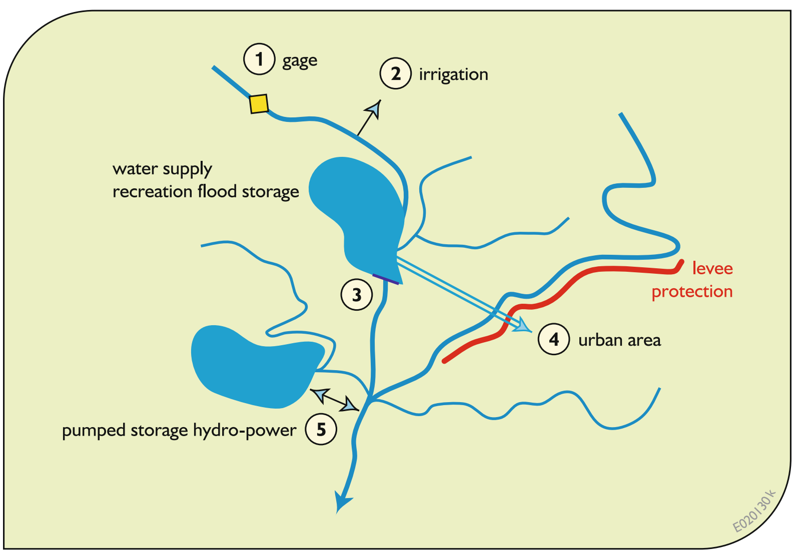
A site number must be assigned at each point of interest. These sites are usually where some decision must be made. Mass balance and other constraints will need to be defined at each of those sites.

As shown in the schematic in Fig. 11.28, this river has one streamflow gage site, site 1, two reservoirs, sites 3 and 5, two diversions, sites 2 and 3, one hydropower plant, site 5, and a levee desired at site 4 to help protect against floods in the urban area. The reservoir at site 5 is a pumped storage facility. The upstream reservoir at site 3 is used for recreation, water supply, and

Fig. 11.26 A multipurpose river system whose management is of concern to numerous stakeholders

Fig. 11.27 A schematic representation of the basin components and their interdependencies drawn over the map image of the basin

Fig. 11.28 Schematic of river system showing components of interest at designated sites



flood control. The downstream reservoir is strictly for hydropower production.

Before developing a model of this river system, the number of time periods t to include in the model and the length of each within-year time period should be determined. If a river system's reservoirs are to contain storage for the distribution of water among years, called over-year storage, then a number of periods encompassing multiple years of operation must be included in the model. This will allow an evaluation of the possible benefits of storing excess water in wet years for release in dry years.

Many reservoir systems completely fill almost every year, and in such cases one is concerned only with the within-year operation of the system. This is the problem addressed here. To model the within-year operation of the system, a year is divided into a number of within-year periods. The number of the periods and the duration of each period will depend on the variation in the hydrology, the demands, and on the particular objectives, as previously discussed.

Once the number and duration of the time steps to be modeled have been identified, the variables and functions used at each site must be named. It is convenient to use notation that can be remembered when examining the model solutions. The notation made up for this example is shown in Table 11.4.

For this example assume we are interested in maximizing a weighted combination of all the net economic benefits derived from all the designated uses of water. There could, and no doubt should, be other objective components defined as well, as discussed in Chap. 9. Nevertheless these economic objective components serve the purpose here of illustrating how a model of this river system can be constructed:

The overall objective might be a weighted combination of all net benefits, NB^s, obtained at each site s in the basin:

$$\text{Maximize} \sum_s w_s NB^s \qquad (11.65)$$

This objective function does not identify how much each stakeholder group would benefit and how much they would pay. Who benefits and who pays, and by how much, may matter. If it is known how much of each of the net benefits derived from each site are to be allocated to each stakeholder group i, then these allocated fraction, f_i, of the total net benefits, NB^s, can be included in the overall objective:

$$\text{Maximize} \sum_i w_i \sum_s f_i NB^s \qquad (11.66)$$

Using methods discussed in Chap. 9, solving the model for various assumed values of these weights can help identify the tradeoffs between different conflicting objectives, Eq. 11.65, or conflicting stakeholder interests, Eq. 11.66.

The next step in model development is to define the constraints applicable at each site. It is convenient to begin at the most upstream sites and work downstream. As additional variables or functions are needed, invent notation for them. These constraints tie the decision variables together and identify the interdependencies among system components. In this example the site index is shown as a superscript.

At site 1:

- No constraints are needed. It is the gage site.

At site 2:

- the diverted water, $X^2(t)$, cannot exceed the streamflow, $Q^2(t)$, at that site.

$$Q^2(t) \geq X^2(t) \quad \forall t \text{ in the irrigation season} \qquad (11.67)$$

- the diversion flow, $X^2(t)$, cannot exceed the diversion channel capacity, X^2.

$$X^2 \geq X^2(t) \quad \forall t \text{ in the irrigation season} \qquad (11.68)$$

Table 11.4 Names associated with required variables and functions at each site in Fig. 11.28

design and operating variables and parameters

all sites *s* and time periods t:
natural streamflows, based on gage flows at site 1, $Q^S(t)$
site 2
irrigation allocations, $X^2(t)$, for all periods t
annual irrigation target allocation, T^2
known fraction of annual irrigation target required for each period t, δ_t^2
irrigation diversion channel capacity, X^2
site 3
active initial storage volume. $S^3(t)$, in each period t
dead storage volume, Kd^3
reservoir release downstream, $R^3(t)$, in each period t
recreation storage volume target, T^3
deficit and excess storage volumes, $D^3(t)$ and $E^3(t)$, for each period t
flood storage volume capacity, $K_f{}^3$
total reservoir storage capacity, K^3
urban diversions, $X^3(t)$, in each period t
diversion capacity, X^3
site 4
channel flood flow capacity, Q^4
water supply target, T^4
known fraction of annual water supply target for each period t, δ_t^4
site 5
active initial storage volume, $S^5(t)$, in each period t
dead storage volume, KD^5
reservoir release through turbines, $QO^5(t)$, in each period t
quantity of water pumped back into reservoir, $QI^5(t)$, in each period t
energy produced, $EP^5(t)$, in each period t
energy consumed, $EC^5(t)$, in each period t
total storage capacity of reservoir, K^5
power plant/pump capacity, P^5
storage head function, $h(S^5(t))$, in each period t
average storage head, $H^5(t)$, in each period t

E020912a

The units of these variables and parameters, however defined, must be consistent

- the diversion flow, $X^2(t)$, must meet the irrigation target, $\delta_t^2 T^2$

$$X^2(t) \geq \delta_t^2 T^2 \quad \forall t \text{ in the irrigation season} \tag{11.69}$$

- NB^2 = benefit function associated with the annual target irrigation allocation, T^2, less the annual cost function associated with the diversion channel capacity, X^2.

At site 3:

- storage volume mass balances (continuity of storage), assuming no losses.

$$S^3(t+1) = S^3(t) + Q^3(t) - X^3(t) - R^3(t) \quad \forall t, \quad T+1=1 \tag{11.70}$$

- define storage deficits, $D^3(t)$, and excesses, $E^3(t)$, relative to recreation target, T^3.

$$S^3(t) = T^3 - D^3(t) + E^3(t) \quad \forall t \text{ in recreation season plus } \textit{following} \text{ period.} \tag{11.71}$$

- diverted water, $X^3(t)$, cannot exceed diversion channel capacity, X^3.

$$X^3(t) \leq X^3 \quad \forall t \tag{11.72}$$

- reservoir storage capacity constraints involving dead storage, K_D^3, and flood storage, K_F^3, capacities.

$$\begin{aligned} &S^3(t) \leq K^3 - K_D^3 - K_F^3 \\ &\quad \forall t \text{ in flood season plus following period.} \\ &S^3(t) \leq K^3 - K_D^3 \\ &\quad \text{for all other periods } t. \end{aligned} \tag{11.73}$$

- NB^3 = sum of annual benefit functions for T^3 and K_F^3 less annual costs of K^3 and X^3 less annual recreation losses associated with all $D^3(t)$ and $E^3(t)$.

At site 4:

- define deficit diversion, $D^4(t)$, from site 3, associated with target, $\delta_t^4 T^4$, if any.

$$X^3(t) = \delta_t^4 T^4 - D^4(t) \quad \forall t \tag{11.74}$$

- channel capacity, Q^4, must equal peak flood flow, PQ_t^4, associated with selected return period, T.

$$Q^4 = PQ_T^4 \tag{11.75}$$

- NB^4 = sum of annual benefit functions for T^4 less annual cost of Q^4 less annual losses associated with all $D^4(t)$.

At site 5:

- continuity of pumped storage volumes, involving inflows, $QI^5(t)$, and outflows, $QO^5(t)$, and assuming no losses.

$$S^5(t+1) = S^5(t) + QI^5(t) - QO^5(t) \quad \forall t \tag{11.76}$$

- active storage capacity involving dead storage, K_D^5.

$$S^5(t) \leq K^5 - K_D^5 \quad \forall t \tag{11.77}$$

- pumped inflows, $QI^5(t)$, cannot exceed the amounts of water available at the intake. This includes the release from the upstream reservoir, $R^3(t)$, and the incremental flow, $Q^5(t) - Q^3(t)$.

$$QI^5(t) \leq Q^5(t) - Q^3(t) + R^3(t) \quad \forall t \tag{11.78}$$

- define the energy produced, $EP^5(t)$, given the average storage head, $H(t)$, flow through the turbines, $QO^5(t)$, and efficiency, e.

$$EP^5(t) = (\text{const.})(H(t))(QO^5(t))e \quad \forall t \tag{11.79}$$

- define the energy consumed, $EC^5(t)$, from pumping given the amount pumped, $QI^5(t)$.

$$EC^5(t) = (\text{const.})(H(t))(QI^5(t))/e \quad \forall t \tag{11.80}$$

- Energy production, $EP^5(t)$, and consumption, $EC^5(t)$, constraints given power plant capacity, P^5.

$$EP^5(t) \leq P^5 \text{ (hours of energy production in } t) \quad \forall t \tag{11.81}$$

$$EC^5(t) \leq P^5 (\text{hours of pumping in } t) \quad \forall t \tag{11.82}$$

- Define the average storage head, $H(t)$, based on storage head functions, $h(S^5(t))$.

$$H(t) = \left[h(S^5(t+1)) + h(S^5(t))\right]/2 \quad \forall t \tag{11.83}$$

- NB^5 = Sum of benefits for the energy produced, $EP^5(t)$, less the costs of the energy consumed, $EC^5(t)$, less the annual costs of capacities K^5 and P^5.

Equations 11.67–11.83 together with objective Eq. 11.65 or 11.66 define the general structure of this river system model. Before the model can be solved, the actual functions must be defined. Then they may have to be made piecewise linear if linear programming is to be the optimization procedure used to solve the model. The process of defining functions may add variables and constraints to the model, as discussed in Chaps. 4 and 9.

For T within-year periods t, this static model of a single year includes between $14T + 8$ and $16T + 5$ constraints, depending on the number of periods in the irrigation and recreation seasons. This number does not include the additional constraints that surely will be needed to define the functions in the objective function components and constraints. Models of this size and complexity, even though this is a rather simple river system, are usually solved using linear programming algorithms simply because other nonlinear or dynamic programming (optimization) methods are more difficult to use.

The model just developed is for a typical single year. In some cases it may be more appropriate to incorporate over-year as well as within-year mass balance constraints, and yields with their respective reliabilities, within this modeling framework. This can be done as outlined in Sect. 5.4 of this chapter.

The information derived from optimization models of river systems such as this one should not be considered as a final answer. Rather it is an indication of the range of system design and operation policies that should be further analyzed using more detailed analyses. Optimization models of the type just developed serve as ways to eliminate inferior alternatives from further consideration more than as ways of finding a solution all stakeholders will accept as the best.

11.11 Project Scheduling

The river basin models discussed thus far in this chapter deal with static planning situations in that system components and their capacities once determined are not assumed to change over time. Project capacities, targets, and operating policies take on fixed values and one examines "snapshot steady-state" solutions for a particular time in the future. These "snapshots" only allow for fluctuations caused by the variability of supplies and demands. The non-hydrologic world is seldom static, however. Targets and goals and policies change in response to population growth, investment in agriculture and industry, and shifting priorities for water use. In addition, financial resources available for water resources investment are limited and may vary from year to year.

Dynamic planning models can aid those responsible for the long-run development and expansion of water resources systems. Although static models can identify target values and system configuration designs for a particular period in the future, they are not well adapted to long-run capacity expansion planning over a 10-, 20-, or 30-year period. But static models may identify projects for implementation in early years which in later year simulations do not appear in the solutions (Chap. 4 contains an example of this).

This is the common problem in capacity expansion, where each project has a fixed construction and implementation cost as well as variable operating, repair, and maintenance cost component. If there are two mutually exclusive competing projects, one may be preferred at a site when the demand at that site is low, but the other may be preferred if the demand is, as it is later projected to be, much higher. Which of the two projects should be selected now when the demand is low, given the uncertainty of the projected increase over time, especially assuming it makes no economic sense to destroy and replace a project already built?

Whereas static models consider how a water resources system operates under a single set of fixed conditions, dynamic expansion models must consider the sequence of changing conditions that might occur over the planning period. For this reason, dynamic expansion models are potentially more complex and larger than are their static counterparts. However, to keep the size and cost of dynamic models within the limitations of most studies, these models are generally restricted to very simple descriptions of the economic and hydrologic variables of concern. Most models use deterministic hydrology and are constrained either to stay within predetermined investment budget constraints or to meet predetermined future demand estimates.

Dynamic expansion models can be viewed as network models for solution by linear or dynamic programming methods. The challenge in river system capacity expansion or projectscheduling models is that each component's performance, or benefits, may depend on the design and operating characteristics of other components in the system. River basin project impacts tend to be dependent on what else is happening in the basin, i.e., what other projects are present and how they are designed and operated.

Consider a situation in which n fixed-scale discrete projects may be built during the planning period. The scheduling problem is to determine which of the projects to build and in what order. The solution of this problem generally requires a resolution of the timing problem. When should each project be built, if at all?

For example, assume there are $n = 3$ discrete projects that might be beneficial to implement sometime over the next 20 years. Let this 20-year period consists of four 5-year construction periods y. The actual benefits derived from any new project may depend on the projects that already exist. Let $\boldsymbol{S}$ be the set of projects existing at the beginning of any construction period. Finally let $Na_y(\boldsymbol{S})$ be the maximum present value of the total net benefits derived in construction period y associated with the projects in the set $\boldsymbol{S}$. Here "benefits" refer to any composite of system performance measures.

These benefit values for various combinations discrete projects could be obtained from static river system models, solved for all combinations of discrete projects for conditions existing at the end of each of these four 5-year periods. It might be possible to just do this for one or two of these four periods and apply applicable discount rates for the other periods. These static models can be similar to those discussed in the previous section of this chapter. Now the challenge is to find the sequencing of these three projects over the periods y that meet budget constraints and that maximize the total present value of benefits.

This problem can be visualized as a network. As shown in Fig. 11.29, the nodes of this network represent the sets $\boldsymbol{S}$ of projects that exist at the beginning of the construction period. For these sets $\boldsymbol{S}$ we have the present value of their benefits, $NB_y(\boldsymbol{S})$, in the next 5-years. The links represent the project or projects implemented in that construction period. Any set of new projects that exceeds the construction funds available for that period is not shown on the network. Those links are infeasible. For the purposes of this example, assume it is not financially feasible to

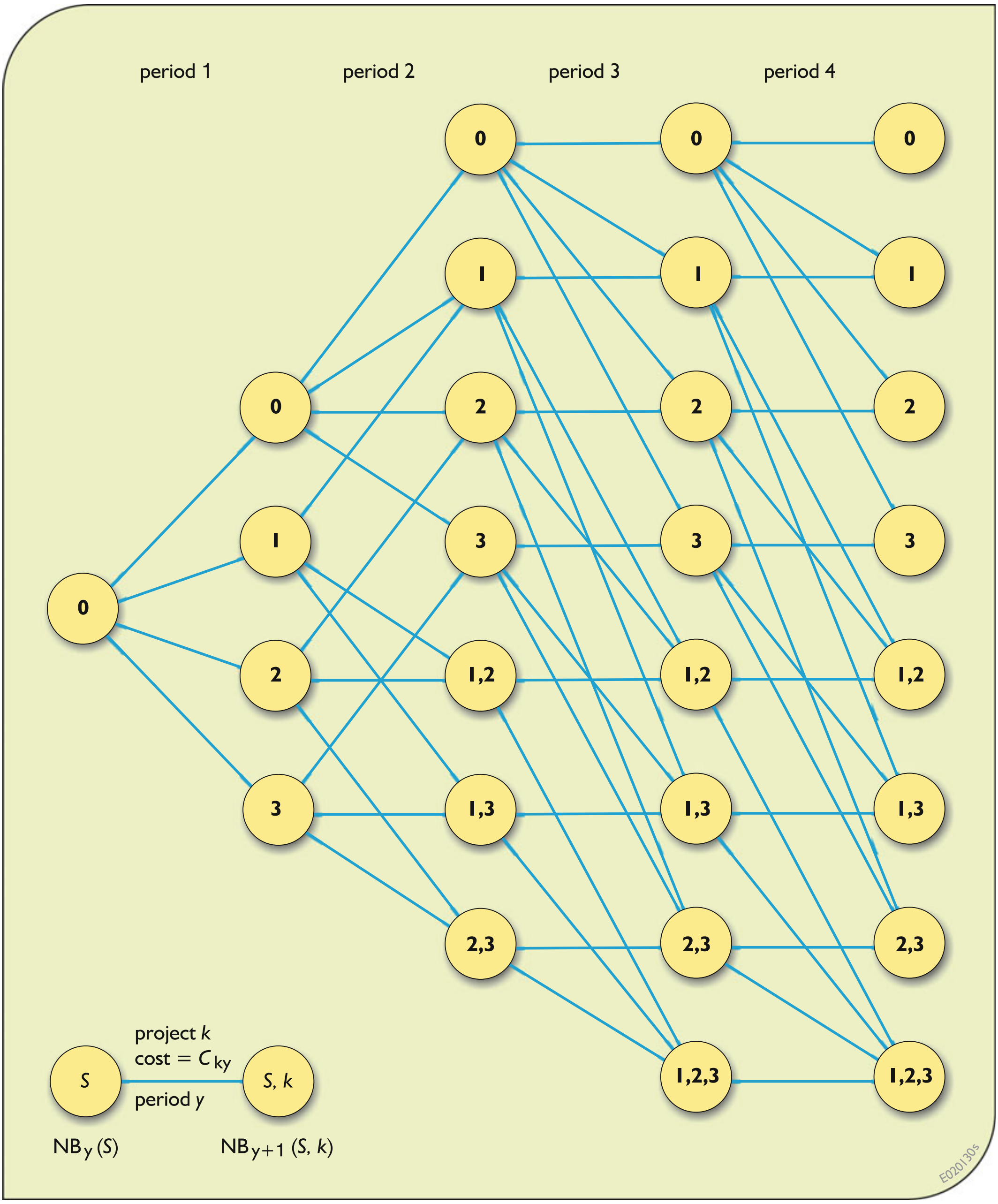


Fig. 11.29 Project scheduling options. Numbers in nodes represent existing projects. Links represent new projects, the difference between the existing projects at both connecting nodes

add more than one project in any single construction period. Let C_{ky} be the present value of the cost of implementing project k in construction period y.

The optimal is to find the best (maximum benefits less costs) paths through the network. Each link represents a net benefit, $NB_y(\boldsymbol{S})$ over the next 5-years obtained from the set of projects, $\boldsymbol{S}$, that exist less the cost of adding a new project k.

Using linear programming, one can define a continuous nonnegative unknown decision variable X_{ij} for each link between node i and node j. It will be an indicator of whether a link is on the optimum path or not. If after solving the model its value is 1, the link connecting nodes i and j represents the decision to make in that construction period. Otherwise its value is 0 indicating the link is not on the optimal path. The sequence of links having their X_{ij} values equal to 1 will indicate the most beneficial sequence of project implementations.

Let the net benefits associated with node i be designated NB_i (that equals the appropriate $NB_y(\boldsymbol{S})$ value), and the cost, C_{ky}, of the new project k associated with that link. The objective is to maximize the present value of net benefits less project implementation costs over all periods y.

$$\text{Maximize} \sum_i \sum_j (NB_i - C_{ij})X_{ij} \tag{11.84}$$

Subject to
Continuity at each node

$$\sum_h X_{hi} = \sum_j X_{ij} \tag{11.85}$$
for each node i in the network.

Sum of all decision variable values on the links in any one period y must be 1. For example in period 1

$$X_{00} + X_{01} + X_{02} + X_{03} = 1 \tag{11.86}$$

The sums in Eq. 11.86 are over nodes h having links to node i and over nodes j having links from node i.

The optimal path through this network can also be solved using dynamic programming. (Refer to the capacity expansion problem illustrated in Chap. 4). For a backward moving solution procedure, let

$\boldsymbol{s}$ = subset of projects k not contained in the set $\boldsymbol{S}$ ($\boldsymbol{s} \notin \boldsymbol{S}$).

$\$_y$ = the maximum project implementation funds available in period y.

$F_y(\boldsymbol{S})$ = the present value of the total benefits over the remaining periods, y, $y + 1$, …, 4.

$F_{Y+1}(\boldsymbol{S})$ = 0 for all sets of projects $\boldsymbol{S}$ following the end of the last period.

The recursive equations for each construction period, beginning with the last period, can be written

$$F_y(\boldsymbol{S}) = \text{maximum}\ \{NB_y(\boldsymbol{S}) - \sum_{k \in s} C_{ky} + F_{y+1}(\boldsymbol{S} + \boldsymbol{s})\}$$
$$\text{for all } \boldsymbol{S} \quad \mathbf{s} \notin \boldsymbol{S} \quad \sum_{k \in s} C_{ky} \le \$_y \tag{11.87}$$

Defining $F_y'(\boldsymbol{S})$ as the present value of the total benefits of all new projects in the set $\boldsymbol{S}$ implemented in all periods up to and including period y, and the subset $\boldsymbol{s}$ of projects k being considered in period y now belonging to the set $\boldsymbol{S}$ of projects existing at the end of the period, the recursive equations for a forward moving solution procedure beginning with the first period, can be written

$$F_y'(\boldsymbol{S}) = \text{maximum}\{\ NB_y(\boldsymbol{S} - \boldsymbol{s}) - \sum_{k \in s} C_{k,y} + F_{y-1}'(\boldsymbol{S} - \boldsymbol{s})\}$$
$$\text{for all } \boldsymbol{S} \quad \boldsymbol{s} \in \boldsymbol{S} \quad \sum_{k \in s} C_{ky} \le \$_y \tag{11.88}$$

where $F_0'(0) = 0$.

Like the linear programming model, the solutions of these dynamic programming models identify the sequencing of projects recognizing their interdependencies. Of interest, again, is what to do in this first constriction period. The only reason for looking into the future is to make sure, as best as one can that the first period's decisions are not myopic. Models like these can be developed and solved again with more updated estimates of future conditions when next needed.

Additional constraints and variables might be added to these scheduling models to enforce requirements that some projects precede others or that if one project is built another is infeasible.

These additional restrictions usually reduce the size of a network of feasible nodes and links, as shown in Fig. 11.29.

Another issue that these dynamic models can address is the sizing or capacity expansion problem. Frequently, the scale or capacity of a reservoir, pipeline, pumping station, or irrigation is variable and needs to be determined concurrently with the solution of the scheduling and timing problems. To solve the sizing problem, the costs and capacities in the scheduling model become variables.

This project scheduling problem by its very nature must deal with uncertainty. A relatively recent contribution to this literature is the work of Haasnoot et al. (2013), Walker et al. (2013).

11.12 Conclusions

This chapter on river basin planning models introduces some ways of modeling river basin components, separately and together within an integrated model. Ignored during the development of these different model types were the uncertainties associated with the results of these models. As discussed in Chaps. 7 and 8, these uncertainties may have a substantial effect on model solution and the decision taken.

Most of this chapter has been focused on the development of simplified screening models, using simulation as well as optimization methods, for identifying what and where and when infrastructure projects should be implemented, and of what capacity. The solution of these screening models, and any associated sensitivity and uncertainty analyses, can be of value prior to committing to more costly design modeling exercises.

Preliminary screening of river basin systems, especially given multiple objectives, is a challenge to accomplish in an efficient and effective manner. The modeling methods and approaches discussed in this chapter serve as an introduction to that art.

References

Haasnoot, M., Kwakkel, J. H., Walker, W. E., & ter Maat, J. (2013). Dynamic adaptive policy pathways: A method for crafting robust decisions for a deeply uncertain world. *Global environmental Change, 23*, 485–498.

Maidment, D. R. (1993). *Handbook of hydrology*. NY: McGraw-Hill Inc.

Rippl, W. (1883). The capacity of storage reservoirs for water supply. In *Proceedings of the Institute of Civil Engineers (Brit.)* (Vol. 71).

U.S. DOE. (2002, April). *Hydropower program*. http://hydropower.inel.gov or http://hydropower.id.doe.gov/

USACE. (1991). Hydrology and hydraulics workshop on Riverine Levee Freeboard, Monticello, Minnesota, Report SP-24, Hydrologic Engineering Center, US Army Corps of Engineers, 272 pp.

Walker, W. E., Haasnoot, M., & Kwakkel, J. H. (2013). Adapt or Perish: A review of planning approaches for adaptation under deep uncertainty. *Sustainability*, *5*, 955–979.

Additional References (Further Reading)

Becker, L., & Yeh, W. W. G. (1974). Timing and sizing of complex water resource systems. *Journal of the Hydraulics Division, ASCE , 100*(HYIO), 1457–1470.

Becker, L., & Yeh, W. W.-G. (1974a). Optimal timing, sequencing, and sizing of multiple reservoir surface water supply facilities. *Water Resources Research, 10* (1), 57–62.

Bedient, P. B., & Huber, W. C. (1992). *Hydrology and floodplain analysis* (2nd ed.). Reading: Addison Westley.

Basson, M. S., Allen, R. B., Pegram, G. G. S., & van Rooyen, J. A. (1994). *Probabilistic management of water resource and hydropower systems*. Highlands Ranch: Water Resources Publications.

Chin, D. A. (2000). *Water-resources engineering*. Upper Saddle River: Prentice Hall.

Erlenkotter, D. (1973a). Sequencing of interdependent hydroelectric projects. *Water Resources Research, 9* (1), 21–27.

Erlenkotter, D. (1973b). Sequencing expansion projects. *Operations Research, 21*(2), 542–553.

Erlenkotter, D. (1976). Coordinating scale and sequencing decisions for water resources projects. *Economic Modeling for Water Policy Evaluation, NorthHolland/TIMS Studies in the Management Sciences, 3*, 97–112.

Erlenkotter, D,. & Rogers, J. S. (1977). Sequencing competitive expansion projects. *Operations Research, 25*(6), 937–951.
Hall, W. A., Tauxe, G. W., & Yeh, W. W.-G. (1969). An alternative procedure for optimization of operations for planning with multiple river, multiple purpose systems. *Water Resources Research, 5*(6), 1367–1372.
Jacoby, H. D., & Loucks, D. P. (1972). Combined use of optimization and simulation models in river basin planning. *Water Resources Research, 8*(6), 1401–1414.
James, L. D., & Lee, R. R. (1971). *Economics of water resources planning*. New York: McGraw-Hill Book Company.
Linsley, R. K., Franzini, J. B., Freyberg, D. L., & Tchobanoglous, G. (1992). *Water-resources engineering*. NY: McGraw-Hill Inc.
Loucks, D. P. (1976). Surface water quantity management, Chap. 5. In A. K. Biswas (Ed.), *Systems approach to water management*. New York: McGraw-Hill Book Company.
Loucks, D. P., Stedinger, J. R., & Haith, D. A. (1981). *Water resource systems planning and analysis*. Englewood Cliffs: Prentice Hall Inc.
Major, D. C., & Lenton, R. L. (1979). *Applied water resources systems planning*. Englewood Cliffs: Prentice Hall Inc.
Maass, A., Hufschmidt, M. M., Dorfman, R., Thomas, H. A., Jr., Marglin, S. A., & Fair, G. M. (1962). *Design of water resource systems*. Cambridge: Harvard University Press.
Mays, L. W. (2001). *Water resources engineering*. NY: Wiley.
Mays, L. W., & Tung, Y.-K. (1992). *Hydrosystems engineering and management*. NY: McGraw-Hill Inc.
Morin, T. L. (1973). Optimal Sequencing of Capacity Expansion Projects. *Journal of the Hydraulics Division, ASCE, 99*(HY9), 1605–1622.
Morin, T. L., & Esogbue, A. M. O. (1974). A useful theorem in the dynamic programming solution of sequencing and scheduling problems occurring in capital expenditure planning. *Water Resources Research, 10*(1), 49–50.
National Research Council. (1999). *New strategies for America's Watersheds*. Washington, DC: National Academy Press.
National Research Council. (2000). *Risk analysis and uncertainty in flood damage reduction studies*. Washington, DC: National Academy Press.
O'Laoghaire, D. T., & Himmelblau, D. M. (1974). *Optimal expansion of a water resources system*. New York: Academic Press Inc.
ReVelle, C. (1999). *Optimizing reservoir resources*. NY: Wiley.
Thomas, H. A., Jr., & Burden, R. P. (1963). *Operations research in water quality management*. Cambridge: Harvard Water Resources Group.
USACE. (1991a). Benefit determination involving existing levees. Policy Guidance Letter No. 26, Headquarters, U.S. Army Corps of Engineers, Washington, DC.
USACE. (1999). Risk analysis in geotechnical engineering for support of planning studies, ETL 1110-2-556, Headquarters, U.S. Army Corps of Engineers, Washington, DC.
Viessman, W., Jr., & Welty, C. (1985). *Water management technology and institutions*. NY: Harper & Row, Pubs.
Wurbs, R. A., & James, W. P. (2002). *Water resources engineering*. Upper Saddle River: Prentice Hall.

Exercises

11.1 Using the following information pertaining to the drainage area and discharge in the Han River in South Korea, develop an equation for predicting the natural unregulated flow at any site in the river, by plotting average flow as a function of catchment area. What does the slope of the function equal?

Gage point	Catchment area (km^2)	Average flow (10^6 m^3/year)
First bridge of the Han River	25,047	17,860
Pal Dang dam	23,713	16,916
So Yang dam	2703	1856
Chung Ju dam	6648	4428
Yo Ju dam	10,319	7300
Hong Chun dam	1473	1094
Dal Chun dam	1348	1058
Kan Yun dam	1180	926
Im Jae dam	461	316

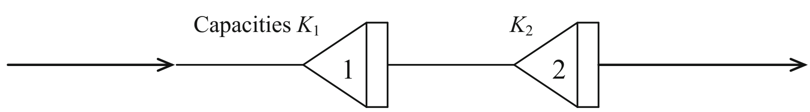
11.2 In watersheds characterized by significant elevation changes, one can often develop reasonable predictive equations for average annual runoff per hectare as a function of elevation. Describe how one would use such a function to estimate the natural average annual flow at any gage in a watershed which is marked by large elevation changes and little loss of water from stream channels due to evaporation or seepage.

11.3 Compute the storage-yield function for a single reservoir system by the mass diagram and modified sequent peak methods given the following sequences of annual flows: (7, 3, 5, 1, 2, 5, 6, 3, 4). Next assume that each year has two distinct hydrologic seasons, one wet and the other dry, and that 80% of the annual inflow occurs in season $t = 1$ and 80% of the yield is desired in season $t = 2$. Using the modified sequent peak method, show the increase in storage capacity required for the same annual yield resulting from within-year redistribution requirements.

11.4 Write two different linear programming models for estimating the maximum constant reservoir release or yield Y given a fixed reservoir capacity K, and for estimating the minimum reservoir capacity K required for a fixed yield Y. Assume that there are T time periods of historical flows available. How could these models be used to define a storage capacity-yield function indicating the yield Y available from a given capacity K?

11.5(a) Construct an optimization model for estimating the least-cost combination of active storage capacities, K_1 and K_2, of two reservoirs located on a single stream, used to produce a reliable constant annual flow or yield (or greater) downstream of the two reservoirs. Assume that the cost functions $C_s(K_s)$ at each reservoir site s are known and there is no dead storage and no evaporation. (Do not linearize the cost functions; leave them in their functional form.) Assume that 10 years of monthly unregulated flows are available at each site s.

(b) Describe the two-reservoir operating policy that you would incorporate into a model model to check the solution obtained from the optimization model.



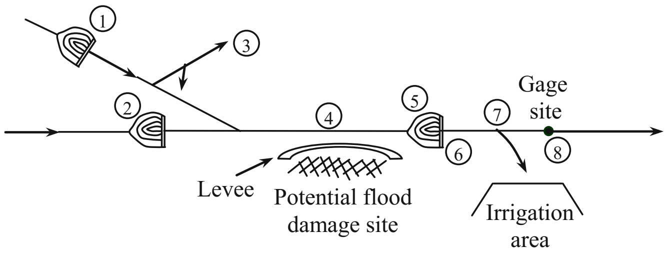
11.6 Given the information in the accompanying tables, compute the reservoir capacity that maximizes the net expected flood damage reduction benefits less the annual cost of reservoir capacity.

Reservoir capacity	Flood stage for flood of return period T					Annual capacity cost
	$T = 1$	$T = 2$	$T = 5$	$T = 10$	$T = 100$	
0	30	105	150	165	180	10[a]
5	30	80	110	120	130	25
10	30	55	70	75	80	30
15	30	40	45	48	50	40
20	30	35	38	39	40	70

[a]10 is fixed cost if capacity > 0; otherwise, it is 0

Flood stage	Cost of flood damage
30	0
50	10
70	20
90	30
110	40
130	50
150	90
180	150

11.7 Develop a deterministic, static, within-year model for evaluating the development alternatives in the river basin shown in the accompanying figures. Assume that there are $t = 1, 2, 3, \ldots, n$ within-year periods and that the objective is to maximize the total annual net benefits in the basin. The solution of the model should define the reservoir capacities (active + flood storage capacity), the annual allocation targets, the levee capacity required to protect site 4 from a T-year flood, and the within-year period allocations of water to the uses at sites 3 and 7. Clearly define all variables and functions used, and indicate how the model would be solved to obtain the maximum net benefit solution.



Site	Fraction of gage flow	
1	0	Potential reservoir for water supply
2	0.3	Potential reservoir for water supply, flood control
3	0.15	Diversion to a use, 60% of allocation returned to river
4	–	Existing development, possible flood protection from levee
5	0.6	Potential reservoir for water supply, recreation
6	–	Hydropower; plant factor = 0.30
7	0.9	Potential diversion to an irrigation district
8	1	Gage site

For simplicity, assume no evaporation losses or dead storage requirements. Omitting the appropriate subscripts t for time periods and s for site, let T, K, D, E, and P be the target, reservoir capacity, deficit, excess, and power plant capacity variables, respectively. Let Q_t and R_t be the natural streamflows and reservoir releases, and S_t be the initial reservoir storage volumes in period t. K_f will denote the flood storage capacity at site 2 that will contain a peak flow of QS and QR is the downstream channel flood flow capacity. The relationship between QS and K_f is defined by the function $\kappa(QS)$. The unregulated design flood peak flow for which protection is required is QN. KWH will be the kilowatt-hours of energy, H will be net storage head, h_t the hours in a period t. The variable q will be the water supply allocation. Benefit functions will be $B()$, $L()$ will denote loss functions and $C()$ will denote the cost functions.

11.8 List the potential difficulties involved when attempting to structure models for defining

(a) Water allocation policies for irrigation during the growing season.
(b) Energy production and capacity of hydroelectric plants.
(c) Dead storage volume requirements in reservoirs.
(d) Active storage volume requirements in reservoir.
(e) Flood storage capacities in reservoirs.
(f) Channel improvements for damage reduction.
(g) Evaporation and seepage losses from reservoirs.
(h) Water flow or storage targets using long-run benefits and short- run loss functions.

11.9 Assume that demand for water supply capacity is expected to grow as $t(60 - t)$, for t in years. Determine the minimum present value of construction cost of some subset of water supply options described below so as to always have sufficient capacity to meet demand over the next 30 years. Assume that the water supply network currently has no excess capacity so that some project must be built immediately. In this problem, assume that project capacities are independent and thus can be summed. Use a discount factor equal to exp(−0.07 t). Before you start, what is your best guess at the optimal solution?

Project number	Construction cost	Capacity
1	100	200
2	115	250
3	190	450
4	270	700

11.10 (a) Construct a flow diagram for a simulation model designed to define a storage-yield function for a single reservoir given known inflows in each month t for n years. Indicate how you would obtain a steady-state solution not influenced by an arbitrary initial storage volume in the reservoir at the beginning of the first period. Assume that evaporation rates (mm per month) and the storage volume/surface area functions are known.

(b) Write a flow diagram for a simulation model to be used to estimate the probability that any specific reservoir capacity, K, will satisfy a series of known release demands, r_t, downstream given unknown future inflows, i_t. You need not discuss how to generate possible future sequences of streamflows, only how to use them to solve this problem.

11.11 (a) Develop an optimization model for finding the cost-effective combination of flood storage capacity at an upstream reservoir and channel improvements at a downstream potential damage site that will protect the downstream site from a prespecified design flood of return period T. Define all variables and functions used in the model

(b) How could this model be modified to consider a number of design floods T and the benefits from protecting the potential damage site from those design floods? Let BF_T be the annual expected flood protection benefits at the damage site for a flood having return periods of T.

(c) How could this model be further modified to include water supply requirements of A_t to be withdrawn from the reservoir in each month t? Assume known natural flows Q_t^s at each site s in the basin in each month t.

(d) How could the model be enlarged to include recreation benefits or losses at the reservoir site? Let T^s be the unknown storage volume target and D_t^s be the difference between the storage volume S_t^s and the target T^s if $S_t^s - T^s > 0$, and E_t^s be the difference if $T^s - S_t^s > 0$. Assume that the annual recreation benefits $B(T^s)$ are a function of the target storage volume T^s and the losses $L^D(D_t^s)$ and $L^E(E_t^s)$ are associated with the deficit D_t^s and excess E_t^s storage volumes.

11.12 Given the hydrologic and economic data listed below, develop and solve a linear programming model for estimating the reservoir capacity K, the flood storage capacity K_f, and the recreation storage volume target T that maximize the annual expected flood control benefits, $B_f(K_f)$, plus the annual recreation benefits, $B(T)$, less all losses $L^D(D_t)$ and $L^E(E_t)$ associated with deficits D_t or excesses E_t in the periods of the recreation season, minus the annual cost $C(K)$ of storage reservoir capacity K. Assume that the reservoir must also provide a constant release or yield of $Y = 30$ in each period t. The flood season begins at the beginning of period 3 and lasts through period 6. The recreation season begins at the beginning of period 4 and lasts though period 7.

Period	1	2	3	4	5	6	7	8	9
Inflows to reservoir	50	30	20	80	60	20	40	10	70

$$B_f(K_f) = \begin{cases} 12K_f & \text{if} \quad K_f \le 5 \\ 60 + 8(K_f - 5) & \text{if} \quad 5 \le K_f \le 15 \\ 140 + 4(K_f - 15) & \text{if} \quad K_f \le 15 \end{cases}$$

$$C(K_f) = \begin{cases} 45 + 10K & \text{if} \quad 0 \le K_f \le 5 \\ 95 + 6(K_f - 5) & \text{if} \quad 5 \le K_f \le 20 \\ 185 + 10(K_f - 20) & \text{if} \quad 20 \le K_f \le 40 \\ 385 + 15(K - 40) & \text{if} \quad K_f \ge 40 \end{cases}$$

$B(T) = 9T$, where T is a particular unknown value of -reservoir storage

$L^D(D_t) = 4D_t$, where D_t is $T - S_t$ if $T \ge S_t$

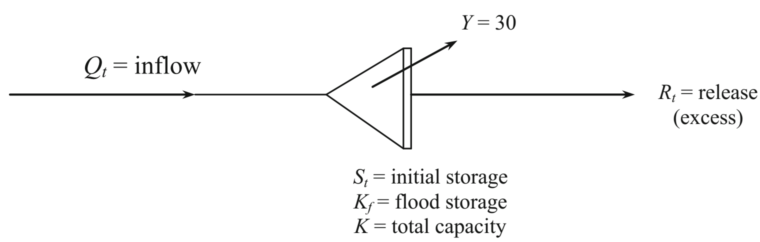
$L^E(E_t) = 2\,E_t$, where E_t is $S_t - T$ if $S_t \ge T$

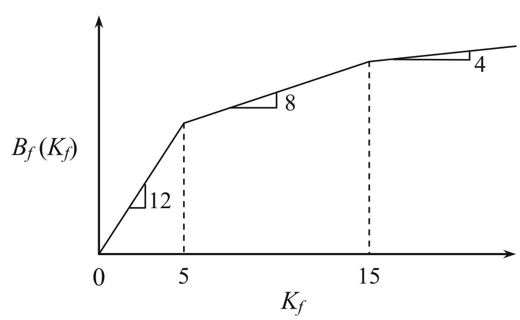
11.13 The optimal operation of multiple reservoir systems for hydropower production presents a very nonlinear and often difficult problem.

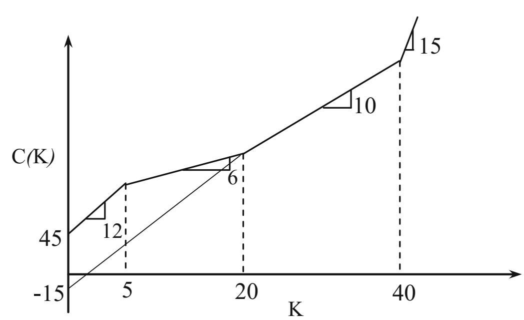
Use dynamic programming to determine the operating policy that maximizes the total annual hydropower production of a two-reservoir system, one downstream of the other. The releases R_{1t} from the upstream reservoir plus the unregulated incremental flow $(Q_{2t} - Q_{1t})$ constitute the inflow to the downstream dam. The flows Q_{1t} into the upstream dam in each of the four seasons along with the incremental flows $(Q_{2t} - Q_{1t})$ and constraints on reservoir releases are given in the accompanying two tables:

Upstream dam (flow in 10^6 m^3/period)			
Season t	Inflow Q_{1t}	Minimum release	Maximum release through turbines
1	60	20	90
2	40	30	90

(continued)







Upstream dam (flow in 10^6 m^3/period)			
Season t	Inflow Q_{1t}	Minimum release	Maximum release through turbines
3	80	20	90
4	120	20	90

Downstream dam (flow in 10^6 m^3/period)			
Season t	Incremental flow, $(Q_{2t} - Q_{1t})$	Minimum release	Maximum release through turbines
1	50	30	140
2	30	40	140
3	60	30	140
4	90	30	140

Note that there is a limit on the quantity of water that can be released through the turbines for energy generation in any season due to the limited capacity of the power plant and the desire to produce hydropower during periods of peak demand.

Additional information that affects the operation of the two reservoirs are the limitations on the fluctuations in the pool levels (head) and the storage-head relationships:

Data	Upstream dam	Downstream dam
Maximum head, H^{max} (m)	70	90
Minimum head, H^{min} (m)	30	60
Maximum storage volume, S^{max} (m^3)	150×10^6	400×10^6
Storage-net head relationship	$H = H^{max}(S/S^{max})^{0.64}$	$H = H^{max}(S/S^{max})^{0.62}$

In solving the problem, discretize the storage levels in units of 10×10^6 m^3. Do a preliminary analysis to determine how large a variation in storage might occur at each reservoir. Assume that the conversion of potential energy equals to the product R_iH_i to electric energy is 70% efficient independent of R_i and H_i. In calculating the energy produced in any season t at reservoir i, use the average head during the season

$$\bar{H}_i = \frac{1}{2}[H_i(t) + H_i(t+1)]$$

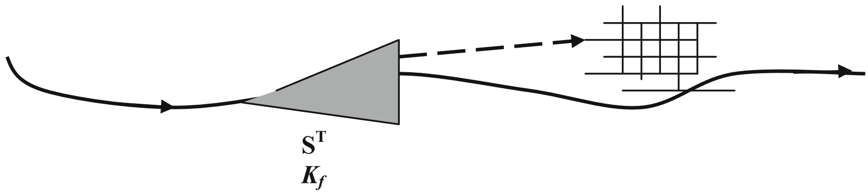
Report your operating policy and the amount of energy generated per year. Find another feasible policy and show that it generates less energy than the optimal policy.

Show how you could use linear programming to solve for the optimal operating policy by approximating the product term $R_i\bar{H}_i$ by a linear expression.

11.14 You are responsible for planning a project that may involve the building of a reservoir to provide water supply benefits to a municipality, recreation benefits associated with the water level in the lake behind the dam, and flood damage reduction benefits. First you need to determine some design variable values, and after doing that you need to determine the reservoir operating policy.

The design variables you need to determine include

- the total reservoir storage capacity (K),
- the flood storage capacity (K_f) in the first season that is the flood season,
- the particular storage level where recreation facilities will be built, called the storage target (S^T) that will apply in seasons 3, 4, and 5—the recreation seasons and finally,
- the dependable water supply or yield (Y) for the municipality.



Assume you can determine these design variable values based on average flows at the reservoir site in six seasons of a year. These average flows are 35, 42, 15, 3, 15, and 22 in the seasons 1–6, respectively.

The objective is to design the system to maximize the total annual net benefits derived from

- flood control in season 1,
- recreation in seasons 3–5, and
- water supply in all seasons,

less the annual cost of the

- reservoir and
- any losses resulting from not meeting the recreation storage targets in the recreation seasons.

The flood benefits are estimated to be 2 $K_f^{\ 0.7}$.

The recreation benefits for the entire recreation season are estimated to be 8 *ST*.

The water supply benefits for the entire year are estimated to be 20 *Y*.

The annual reservoir cost is estimated to be 3 $K^{1.2}$.

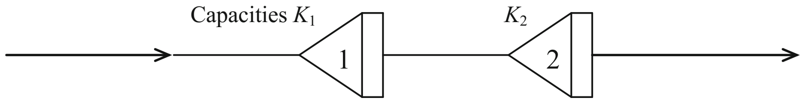
The recreation loss in each recreation season depends on whether the actual storage volume is lower or higher than the storage target. If it is lower the losses are 12 per unit average deficit in the season, and if they are higher the losses are 4 per unit average excess in the season. It is possible that a season could begin with a deficit and end with an excess, or vice versa.

Develop and solve a nonlinear optimization model for finding the values of each of the design variables: *K*, K_f, *ST*, and *Y* and the maximum annual net benefits. (There will be other variables as well. Just define what you need and put it all together in a model.)

Does the solution give you sufficient information that would allow you to simulate the system using a sequence of inflows to the reservoir that are different than the ones used to get the design variable values? If not how would you define a reservoir operating policy? After determining the system's design variable values using optimization, and then determining the reservoir operating policy, you would then simulate this system over many years to get a better idea of how it might perform.

11.15 Suppose you have 19 years of monthly flow data at a site where a reservoir could be located. How could you construct a model to estimate what the required over-year and within year storage needed to produce a specified annual yield *Y* that is allocated to each month t by some known fraction δ_t. What would be the maximum reliability of those yields? If you wanted to add to that an additional secondary yield having only 80% reliability, how would the model change? Make up 19 annual flows and assume that the average monthly flows are specified fractions of those annual flows. Just using these annual flows and the average monthly fractions, solve your model.

11.16



(a) Develop an optimization model for estimating the least-cost combination of active storage K^1 and K^2 capacities at two reservoir sites on a single stream that are used to produce a reliable flow or yield downstream of the downstream reservoir. Assume 10 years of monthly flow data at each reservoir site. Identify what other data are needed.

(b) Describe the two-reservoir operating policy that could be incorporated into a simulation model to check the solution obtained from the optimization model.

Define

$C^s(K^s)$	cost of active storage capacity at site s; where s = 1, 2
K_d^s	dead storage capacity of reservoir at site s; $K_d^s = 0$
S_t^s	storage volume at beginning of period t at site s
L^s	loss of water due to evaporation at site s; $L^s = 0$
R_t^{12}	release from reservoir at site 1 to site 2 in period t
Y_t	yield to downstream in period t
Q_t^s	10 years of monthly natural flows available at each site s
a_o^s	area associated with dead storage volume at site s
a^s	area per unit storage volume at site s
e_t^s	evaporation depth in period t at site s

11.17 Given inflows to an effluent storage lagoon that can be described by a simple first-order Markov chain in each of T periods t, and an operating policy that defines the lagoon discharge as a function of the initial volume and inflow, indicate how you would estimate the probability distribution of lagoon storage volumes.

11.18 (a) Using the inflow data in the table below, develop and solve a yield model for estimating the storage capacity of a single reservoir required to produce a yield of 1.5 that is 90% reliable in both of the two within-year periods t, and an additional yield of 1.0 that is 70% reliable in period t = 2.

(b) Construct a reservoir-operating rule that defines reservoir release zones for these yields.

(c) Using the operating rule, simulate the 18 periods of inflow data to evaluate the adequacy of the reservoir capacity and storage zones for delivering the required yields and their reliabilities. (Note that in this simulation of the historical record the 90% reliable yield should be satisfied in all the 18 periods, and the incremental 70% reliable yield should fail only two times in the 9 years.)

(d) Compare the estimated reservoir capacity with that which is needed using the sequent peak procedure.

Year	Period	Inflow
1	1	1.0
	2	3.0
2	1	0.5
	2	2.5
3	1	1.0
	2	2.0

(continued)

Year	Period	Inflow
4	1	0.5
	2	1.5
5	1	0.5
	2	0.5
6	1	0.5
	2	2.5
7	1	1.0
	2	5.0
8	1	2.5
	2	5.5
9	1	1.5
	2	4.5

11.19 One possible modification of the yield model of would permit the solution algorithm to determine the appropriate failure years associated with any desired reliability instead of having to choose these years prior to model solution. This modification can provide an estimate of the extent of yield failure in each failure year and include the economic consequences of failures in the objective function. It can also serve as a means of estimating the optimal reliability with respect to economic benefits and losses. Letting F_y be the unknown yield reduction in a possible failure year y, then in place of $\alpha_{py}Y_p$ in the over-year continuity constraint, the term $(Y_p - F_y)$ can be used. What additional constraints are needed to ensure (1) that the average shortage does not exceed $(1 - \alpha_{py})Y_p$ or (2) that at most there are f failure years and none of the shortages exceed $(1 - \alpha_{py})Y_p$.

11.20 In Indonesia there exists a wet season followed by a dry season each year. In one area of Indonesia all farmers within an irrigation district plant and grow rice during the wet season. This crop brings the farmer the largest income per hectare; thus they would all prefer to continue growing rice during the dry season. However, there is insufficient water during the dry season for irrigating all 5000 ha of available irrigable land for rice production. Assume an available irrigation water supply of 32×10^6 m^3 at the beginning of each dry season, and a minimum requirement of 7000 m^3/ha for rice and 1800 m^3/ha for the second crop.

(a) What proportion of the 5000 ha should the irrigation district manager allocate for rice during the dry season each year, provided that all available hectares must be given sufficient water for rice or the second crop?
(b) Suppose that crop production functions are available for the two crops, indicating the increase in yield per hectare per m^3 of additional water, up to 10,000 m^3/ha for the second crop. Develop a model in which the water allocation per hectare, as well as the hectares allocated to each crop, is to be determined, assuming a specified price or return per unit of yield of each crop. Under what conditions would the solution of this model be the same as in part (a)?

11.21 Along the Nile River in Egypt, irrigation farming is practiced for the production of cotton, maize, rice, sorghum, full and short berseem for animal production, wheat, barley, horsebeans, and winter and summer tomatoes. Cattle and buffalo are also produced, and together with the crops that require labor, water. Fertilizer, and land area (feddans). Farm types or management practices are fairly uniform, and hence in any analysis of irrigation policies in this region this distinction need not be made. Given the accompanying data develop a model for determining the tons of crops and numbers of animals to be grown that will maximize (a) net economic benefits based on Egyptian prices, and (b) net economic benefits based on international prices. Identify all variables used in the model.

Known parameters

C_i	miscellaneous cost of land preparation per feddan
P_i^{E}	Egyptian price per 1000 tons of crop i
P_i^{I}	international price per 1000 tons of crop i
v	value of meat and dairy production per animal
g	annual labor cost per worker
f^P	cost of P fertilizer per ton
f^N	cost of N fertilizer per ton
Y_i	yield of crop i, tons/feddan
α	feddans serviced per animal
β	tons straw equivalent per ton of berseem carryover from winter to summer
r^w	berseem requirements per animal in winter
s^{wh}	straw yield from wheat, tons per feddan
s^{ba}	straw yield from barley, tons per feddan
r^{s}	straw requirements per animal in summer
μ_i^N	N fertilizer required per feddan of crop i
μ_i^P	P fertilizer required per feddan of crop i
l_{im}	labor requirements per feddan in month m, man-days
w_{im}	water requirements per feddan in month m, 1000 m^3
h_{im}	land requirements per month, fraction (1 = full month)

Required Constraints. (Assume known resource limitations for labor, water, and land)

(a) Summer and winter fodder (berseem) requirements for the animals.
(b) Monthly labor limitations.
(c) Monthly water limitations.
(d) Land availability each month.
(e) Minimum number of animals required for cultivation.
(f) Upper bounds on summer and winter tomatoes (assume these are known).
(g) Lower bounds on cotton areas (assume this is known).

Other possible constraints

(a) Crop balances.
(b) Fertilizer balances.
(c) Labor balance.
(d) Land balance.

11.22 In Algeria there are two distinct cropping intensities, depending upon the availability of water. Consider a single crop that can be grown under intensive rotation or extensive rotation on a total of A hectares. Assume that the annual water requirements for the intensive rotation policy are 16,00 m^3 per ha, and for the extensive rotation policy they are 4000 m^3 per ha. The annual net production returns are 4000 and 2000 dinars, respectively. If the total water available is 320,000 m^3, show that as the available land area A increases, the rotation policy that maximizes total net income changes from one that is totally intensive to one that is increasingly extensive.

Would the same conclusion hold if instead of fixed net incomes of 4000 and 2000 dinars per hectares of intensive and extensive rotation, the net income depended on the quantity of crop produced? Assuming that intensive rotation produces twice as much produced by extensive rotation, and that the net income per unit of crop Y is defined by the simple linear function $5 - 0.05Y$, develop and solve a linear programming model to determine the optimal rotation policies if A equals 20, 50, and 80. Need this net income or price function be linear to be included in a linear programming model?