Order Tracking by Square-Root Cubature Kalman Filter with Constraints

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Abstract. Condition monitoring of mechanical systems is an important topic for the industry because it helps to improve the machine maintenance and reduce the total operational cost associated. In that sense, the vibration analysis is an useful tool for failure prevention in rotating machines, and its main challenge is estimating on-line the dynamic behavior due to non-stationary operating conditions. Nevertheless, approaches for estimating time-varying parameters require the shaft speed reference signal, which is not always provided, or are oriented to off-line processing, being not useful on industrial applications. In this paper, a novel Order Tracking (OT) is employed to estimating both the instantaneous frequency (IF) and the spectral component amplitudes, which does not require the shaft speed reference signal and may be computed on-line. In particular, a nonlinear filter (Square-Root Cubature Kalman Filter) is used to estimate the spectral components from the vibration signal that provide the necessary information to detect damage on a machine under time-varying regimes. An optimization problem is proposed, which is based on the frequency constraints to improve the algorithm convergency. To validate the proposed constrained OT scheme, both synthetic and real-world application are considered. The results show that the proposed approach is robust and it successfully estimates the order components and the instantaneous frequency under different operating conditions, capturing the dynamic behavior of the machine.

Keywords: Order tracking · Non-stationary signals · Kalman filtering

1 Introduction

Vibration analysis of rotating machines is one of the most used techniques for fault diagnosis and condition monitoring due to its high performance and low implementation cost. Nowadays, the main challenge in vibration analysis is to

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track and reduce the influence of changes during time-varying operation conditions and loads. In this regard, the order tracking (OT) techniques had been proposed, oriented to obtain the fundamental component features of the shaft reference speed (called basic order) and capture the dynamics of the measured vibration signals. The OT have shown to be useful within the analysis of non-stationary vibration signals, condition monitoring and fault diagnosis [5]. This technique allows to identify the rotation speed and the spectral/order components, which are fundamental to describe the state of both, the machine and its conforming mechanisms, during changing loads and speed regimes [11].

Particularly, a suitable estimation strategy is carried out using Kalman filtering in [5]; its improved version with increased precision, termed Vold-Kalman filtering (VKF_OT), was proposed in [14]. However, it requires measuring the shaft speed, which makes the order analysis still complex. The measurement of the shaft speed implies installing additional equipment near to the machine, which in certain situations is inconvenient. In [4], another approach is discussed consisting of a nonlinear least minimum squares algorithm, which estimates the amplitude, frequency, and phase of a non-stationary sinusoid, but the principal shortcomings come from its lack of robustness in the estimation procedure. In [6,15], a frequency tracker based on an oscillatory model, is introduced, where its parameters are calculated by Extended Kalman Filtering (EKF), obtaining the amplitude, phase, and mainly the frequency of a harmonic signal for de-noising in non-stationary signals. However, the tuning of model parameters is complex and requires an expertise degree.

In [12] an extended version of EKF frequency tracker for non-stationary harmonic signals is presented, where the time-varying amplitude is another state variable attached to the oscillatory model, outperforming the conventional methods aforementioned. Nonetheless, the increment in the amount of state variables implies more computational cost affecting the on-line tracking task. In contrast, in [8] the time-varying amplitude is estimated assuming the state variables as the in-phase and quadrature components of the signal, and computing the quadratic mean between those components. Besides, the EKF is based on the linearization by using Taylor's series expansion, however, the computation of Jacobian matrix induces high running complexity, limiting the application capability. To overcome the drawbacks of EKF on estimation accuracy, stability, convergence, among others., a novel nonlinear filtering approach is proposed in [1], termed Cubature Kalman Filter (CKF). In order to improve the CKF performance, a extended version was proposed in [3], so-called square-root cubature Kalman filter (SRCKF). The SRCKF propagates the probability distribution function in a simple and efficient way and it is accurate up to second order in estimating mean and covariance [3]. Based on explained above approaches, this paper discusses an OT approach with improved IF estimation by means of SRCKF, which introduces a frequency tracker that allows to capture the signal intrinsic dynamics, and thus, the OT deals with non-stationarity associated to several parts of the machine when determining the fundamental frequency of a vibration signal. Additionally, a simple method to incorporate state constraints is presented to improve the precision and the tolerance to the parameters initialization.

2 Materials and Methods

From the input signal $y(n)$, Order Tracking provides the estimation of modes and amplitudes present in each oscillation. The machine shaft speed is the *basic order*, while superior orders are related as the shaft speed harmonics. Thus, the shaft speed, $\eta = 60f$, is equivalent to shaft fundamental frequency of the machine, where η is given in revolutions per minute (rpm) and f in *Hz*.

2.1 Oscillatory Model and Instantaneous Frequency Estimation

The vibration signal, $y(n) \in \mathbb{R}$, acquired from a rotating machine can be represented as a superposition of K sinusoidal functions (termed *order components*), as follows:

$$y(n) = \sum_{k=1}^{K} a_k(n) \cos(k\omega(n)n + \varphi_k(n)) \tag{1}$$

where $a_k(n)$ and $\varphi_k(n)$ denote the amplitude and the phase of the k-th order component, respectively; $\omega(n) = 2\pi f_0(n)$ is the angular frequency of a rotational frequency $f_0(n)$. The variables $a_k(n)$, $\varphi_k(n)$ and $\omega(n)$ are time-varying.

Accordingly to [9], it is possible to extract both the instantaneous frequency (IF) and the order component amplitudes by the following state-space model:

$$\begin{bmatrix} \boldsymbol{x}_1(n+1) \\ \vdots \\ \boldsymbol{x}_K(n+1) \\ \boldsymbol{x}_{K+1}(n+1) \end{bmatrix} = \begin{bmatrix} \boldsymbol{M}(x_{K+1}(n)) & & & \\ & \ddots & & \\ & & \boldsymbol{M}(Kx_{K+1}(n)) & \\ & & & 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{x}_1(n) \\ \vdots \\ \boldsymbol{x}_K(n) \\ \boldsymbol{x}_{K+1}(n) \end{bmatrix} + \begin{bmatrix} \boldsymbol{\xi}_1(n) \\ \vdots \\ \boldsymbol{\xi}_K(n) \\ \boldsymbol{\xi}_{K+1}(n) \end{bmatrix} \tag{2}$$

$$y(n) = \begin{bmatrix} \boldsymbol{h} \cdots \boldsymbol{h}\ 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{x}_1(n) \\ \vdots \\ \boldsymbol{x}_K(n) \\ \boldsymbol{x}_{K+1}(n) \end{bmatrix} + \begin{bmatrix} v_1(n) \\ \vdots \\ v_K(n) \\ v_{K+1}(n) \end{bmatrix} \tag{3}$$

where the remaining terms of the state transition matrix are zero filled, and $\boldsymbol{x}_k(n) = [x_c(n)\, x_d(n)]^T \in \mathbb{R}^{2\times 1}$ is the state variable vector, being $x_c(n) = a(n)\cos(\omega(n)n + \varphi(n))$ and $x_d(n) = a(n)\sin(\omega(n)n + \varphi(n))$, which are the in-phase and quadrature components, respectively. In consequence, the order component amplitude and IF estimation are computed by $a(n) = \sqrt{x_c(n)^2 + x_d(n)^2}$ and $\omega(n) = x_{K+1}(n)$. The matrix $\boldsymbol{M}(x_{K+1}(n)) \in \mathbb{R}^{2\times 2}$ is a rotation matrix that is defined as follows:

$$\boldsymbol{M}(\omega(kn)) = \begin{bmatrix} \cos\omega(kn) & -\sin\omega(kn) \\ \sin\omega(kn) & \cos\omega(kn) \end{bmatrix} \tag{4}$$

As regards the model parameters, $\boldsymbol{\xi}(n) \sim \mathcal{N}(\boldsymbol{0}, \boldsymbol{Q}(n)) \in \mathbb{R}^{2K\times 1}$ is the process noise, where $\boldsymbol{Q}(n) \in \mathbb{R}^{2K\times 2K}$ is the covariance matrix of process noise; $\boldsymbol{h} = [1\ 0]$

forms the measurement matrix $\boldsymbol{H} \in \mathbb{R}^{1\times 2K+1}$, $v(n) \sim \mathcal{N}(0, r(n)) \in \mathbb{R}$ is the measurement noise, and $r(n) \in \mathbb{R}$ is the measurement variance.

It is worth noting that the model described in Eqs. (2) and (3) could be applied under the assumption that the speed does not present strong changes neither discontinue behaviors [7], it means, the following approximations hold: $a(n+1) \cong a(n)$, $\varphi(n+1) \cong \varphi(n)$ and $\omega(n+1) \cong \omega(n)$.

For the sake of simplicity, the process equation (Eq. (2)) could be rewritten in short form as

$$\boldsymbol{X}(n+1) = \boldsymbol{\vartheta}(n, \boldsymbol{X}(n)) + \boldsymbol{w}(n) \quad (5)$$

where $\boldsymbol{\vartheta}(n, \boldsymbol{X}(n))$ is the state transition nonlinear function. In this case, the estimation of state variable vector implies a set of nonlinear equations. Therefore, a recursive solution can be computed by means of nonlinear Kalman filtering.

2.2 Estimation of Model Parameters

As seen in Eqs. (2) and (3), parameter computation implies a recursive nonlinear analysis allowing to get an approximated solution when Gaussian noise is assumed but avoiding calculation of corresponding Jacobians of state variables. To this end, the Square-Root Cubature Kalman Filter (SRCKF), which is based on the recursive propagation of variable state moments (mean and variance), is suggested in [2], under the assumption that implicated nonlinear function, $\boldsymbol{\vartheta}$, should be reasonably smooth. In this case, a quadratic function near the prior mean is used assuming that it could properly approximate the given nonlinear function. To this end, the error covariance matrix should be symmetric and positive definiteness to preserve the filter properties on each update cycle. So hence, SRCKF uses a forced symmetry on the solution of the Ricatti equation improving the numerical stability of the Kalman filter [10], whereas the underlying meaning of the covariance is embedded in the positive definiteness [2].

The SRCKF algorithm that is described in Table 1 carries out the QR decomposition (termed triangularization procedure, $\boldsymbol{S} = \text{tria}\{\cdot\}$), where the $\boldsymbol{S}$ is a lower triangular matrix and denotes a square-root factor [2]. Besides, aiming to parameterize the SRCKF is mandatory taking into account the process and measurement covariance parameters, where the main parameter is q because it comprises the information related with variances of the state estimates as follow:

$$diag(\mathrm{Q}) = \left[\, q_1^a \; q_1^a \cdots q_K^a \; q_K^a \; q_{K+1}^f \,\right] \quad (6)$$

where $q_i^a (i = 1, ..., K)$ denotes the amplitude variance of the order components and q_{K+1}^f denotes the frequency variance, which describes the dynamic behavior of the system.

2.3 State Estimation with Constraints

Constraints on states $\boldsymbol{x}(n)$ to be estimated are important model information that is often not used in state estimation. Typically, such constraints are due to

Table 1. SRCKF algorithm (Part 1), [9]

Initialization:

1. Define the input values
   $y_{1:N}$, $\widehat{\boldsymbol{x}}_0$, $\boldsymbol{P}_{0|0} = \boldsymbol{S}_{0|0}\boldsymbol{S}_{0|0}^\top$, $\boldsymbol{Q}_0$, $\boldsymbol{R}_0$
2. Define the cubature points
   $\boldsymbol{\phi}_i = \sqrt{m/2}\left\{\left[\boldsymbol{I}_{m\times m} \;\; -\boldsymbol{I}_{m\times m}\right]\right\}$

Tracking:

3. **for** $n = 1$ to N **do**

Time update

4. Evaluate the cubature points ($i = 1, 2, \ldots, m$), where $m = 2K + 1$,
   $\boldsymbol{\chi}_{i,n-1|n-1} = \boldsymbol{S}_{n-1|n-1}\boldsymbol{\phi}_i + \widehat{\boldsymbol{x}}_{n-1|n-1}$
5. Evaluate the propagated cubature points ($i = 1, 2, \ldots, m$)
   $\boldsymbol{\chi}^*_{i,n|n-1} = \boldsymbol{\vartheta}\left(\boldsymbol{\chi}_{i,n-1|n-1}\right)$
6. Estimate the predicted state
   $\widehat{\boldsymbol{x}}_{n|n-1} = \frac{1}{m}\sum_{i=1}^{m}\boldsymbol{\chi}^*_{i,n|n-1}$
7. Estimate the square-root factor of prediction error covariance
   $\boldsymbol{S}_{n|n-1} = \mathrm{tria}\left\{\left[\widehat{\boldsymbol{\mathcal{X}}}_{n|n-1} \;\; \boldsymbol{S}_{\boldsymbol{Q}|n}\right]\right\}$
   where $\widehat{\boldsymbol{\mathcal{X}}}_{n|n-1} = \frac{1}{\sqrt{m}}\left[\boldsymbol{\chi}^*_{1,n|n-1} - \widehat{\boldsymbol{x}}_{n|n-1} \;\; \boldsymbol{\chi}^*_{2,n|n-1} - \widehat{\boldsymbol{x}}_{n|n-1} \cdots \boldsymbol{\chi}^*_{m,n|n-1} - \widehat{\boldsymbol{x}}_{n|n-1}\right]$
   and $\boldsymbol{S}_{\boldsymbol{Q}|n}$ denotes a square-root factor of $\boldsymbol{Q}_{n-1}$

Measurement Update

8. Evaluate the cubature points ($i = 1, 2, \ldots, m$)
   $\boldsymbol{\chi}_{i,n|n-1} = \boldsymbol{S}_{n|n-1}\boldsymbol{\phi}_i + \widehat{\boldsymbol{x}}_{n|n-1}$
9. Evaluate the propagated cubature points ($i = 1, 2, \ldots, m$)
   $\boldsymbol{\psi}_{i,n|n-1} = \boldsymbol{h}\left(\boldsymbol{\chi}_{i,n|n-1}\right)$
10. Estimate the predicted state
   $\widehat{\boldsymbol{y}}_{n|n-1} = \frac{1}{m}\sum_{i=1}^{m}\boldsymbol{\psi}_{i,n|n-1}$
11. Estimate the square-root of the innovation covariance matrix
   $\boldsymbol{S}_{yy,n|n-1} = \mathrm{tria}\left\{\left[\mathcal{Y}_{n|n-1} \;\; \boldsymbol{S}_{\boldsymbol{R}|n}\right]\right\}$
   where $\mathcal{Y}_{n|n-1} = \frac{1}{\sqrt{m}}\left[\boldsymbol{\psi}_{1,n|n-1} - \widehat{\boldsymbol{y}}_{n|n-1} \;\; \boldsymbol{\psi}_{2,n|n-1} - \widehat{\boldsymbol{y}}_{n|n-1} \cdots \boldsymbol{\psi}_{m,n|n-1} - \widehat{\boldsymbol{y}}_{n|n-1}\right]$
   and $\boldsymbol{S}_{\boldsymbol{R}|n}$ denotes a square-root factor of $\boldsymbol{R}_n$
12. Estimate the cross-covariance matrix
   $\boldsymbol{P}_{xy,n|n-1} = \mathcal{X}_{n|n-1}\mathcal{Y}^\top{}_{n|n-1}$
   where $\mathcal{X}_{n|n-1} = \frac{1}{\sqrt{m}}\left[\boldsymbol{\chi}_{1,n|n-1} - \widehat{\boldsymbol{x}}_{n|n-1} \;\; \boldsymbol{\chi}_{2,n|n-1} - \widehat{\boldsymbol{x}}_{n|n-1} \cdots \boldsymbol{\chi}_{m,n|n-1} - \widehat{\boldsymbol{x}}_{n|n-1}\right]$
13. Estimate the Kalman gain
   $W_n = \left(\boldsymbol{P}_{xy,n|n-1}/\boldsymbol{S}^\top_{yy,n|n-1}\right)/\boldsymbol{S}_{yy,n|n-1}$
14. Estimate the updated state
   $\widehat{\boldsymbol{x}}_{n|n} = \widehat{\boldsymbol{x}}_{n|n-1} + W_n\left(y_n - \widehat{\boldsymbol{y}}_{n|n-1}\right)$
15. Estimate the square-root factor of the corresponding error covariance
   $\boldsymbol{S}_{n|n} = \mathrm{tria}\left\{\left[\mathcal{X}_{n|n-1} - W_n\mathcal{Y}_{n|n-1} \;\; W_n\boldsymbol{S}_{\boldsymbol{R}|n}\right]\right\}$
16. **end**

physical limitations on the states. In Kalman filter theory, there is no general way of incorporating these constraints into estimation problem. However, the constraints can be incorporated in the filter by projecting the unconstrained Kalman filter estimates onto the boundary of the feasible region at each time step [16,17]. The numerical optimization at each time step may be a challenge in time-critical applications. In this section, a simple method introduced in [13] is applied to handle state constraints in the SRCKF.

Assume that the constraints of state variables are represented by box constraints as follow:

$$\boldsymbol{x}_L(n) \leq \boldsymbol{x}(n) \leq \boldsymbol{x}_H(n) \tag{7}$$

where subindexes L and H denote the lower and upper boundaries, respectively. The method is illustrated for $\boldsymbol{x}(n) \in \mathbb{R}^2$. In the case of a second order system, the feasible region by the box constraints can be represented by a rectangle as in Fig. 1. It is showed the illustration of the steps of constraint handling of the SRCKF algorithm from one time step to the next. At $t = n - 1$, the actual state $\boldsymbol{x}_n$, its estimate $\widehat{\boldsymbol{x}}_{n-1}$ and state covariance are selected. The constraints information can be incorporated in the SRCKF algorithm in a simple way during the time-update step. After the propagation of the sigma points (step 5.), the (unconstrained) transformed sigma points which are outside the feasible region can be projected onto the boundary of the feasible region and continue the further steps. In Fig. 1, at $t = n$ two sigma points which are outside the feasible region are projected onto the boundary (right plot in the figure). The mean and covariance with the constrained sigma points now represent the a priori state variable (x_n^{-SRCKF}) and covariance, and they are further updated in the measurement update step. The advantage here is that the new a priori covariance includes information on the constraints, which should make the SRCKF estimate more efficient (accurate) compared to the SRCKF estimate without constraints. Extension of the proposed method to a higher dimension, d, is straightforward. Alternative linear constraints, e.g., $C\boldsymbol{x} \leq d$ are easily included by projecting the sigma point violating the inequality normally onto the boundary of a feasible region. It is observed that the new (constrained) covariance obtained at a time step is lower in size compared to the unconstrained covariance. If in case, the estimate after the measurement update is outside the feasible region, the same projection technique can be extended. In a practical point of view, the boundaries are fixed according to maximum and minimum values that could take the state variables. In consequence, in case of the order components $L^a = \min y(n)$ and $H^a = \max y(m)$, whereas the IF constraints depend on the approximated knowledge of the machine speed range, where L^f and H^f are usually fixed as zero or idle speed and maximum speed, respectively.

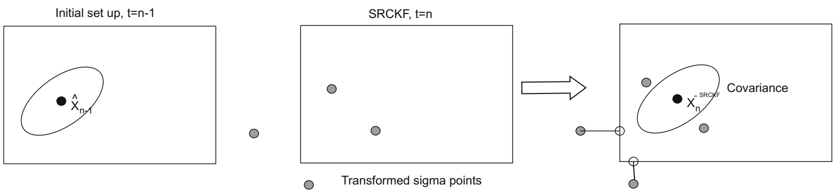


Fig. 1. Illustration of sigma constrained points.

3 Simulation Study

A synthetic signal is desing to validate the performance of considered OT schemes for the closed-order component identification, as recommended in [14]. The synthetic signal comprises three order components including 1, 4 and 4.2. The assumed reference shaft speed linearly increases from 0 to 1800 rpm for 5 s. A sampling frequency of 1 kHz is used through this simulation. The Table 2 illustrates the amplitudes assigned to these order components. Order amplitude level is set as time-varying, since it is assumed that, most of the machine mechanisms have different vibration levels.

Table 2. Spectral components composing the in synthetic signal

Order numbers	1	4	4.2
Amplitude	Linearly increasing from 0 to 10	Linearly increasing from 0 to 3	Linearly increasing from 0 to 2.5

In Fig. 2, the time-frequency representation of the synthetic signal is shown, as well as its generative time series. It can be observed the difficult to distinguish closed-order components using methods based on Fourier transforms. Also, it is worth noting that the amplitude differences between the first order and its harmonics make almost insignificant the low-frequency information. As a result, it generates a wrong representation of required components. Instead, when using OT techniques based on parametric models, it is possible to capture properly the information about the behavior of each order component. Computation parameters that influence tracking performance, such as the correlation matrix of process noise and the error covariance propagation, are investigated here. The algorithms are tested under different parameter values, i.e., two values for variance of process noise and two values for error covariance propagation, in order to shows the improvement achieved with constraints. The Fig. 3 shows the waveform reconstruction (WR), amplitude (A) and frequency estimation errors. It is possible to see a high performance (accuracy) in the WR, specially when SRCKF with constraints is used. It is important notice that the frequency estimation, by SRCKF without constraints (segmented line), presents a inverse behavior, which is a wrong interpretation of the algorithm and it may produce errors during the signal analysis.

3.1 Case Study: Wind Turbine if Estimation - CMMNO2014 Contest

This experiment consisted of estimating the instantaneous speed in *rpm*, or instantaneous frequency in Hz, from a wind turbine operating under non-stationary conditions. The information given hereafter, as well as the signal,

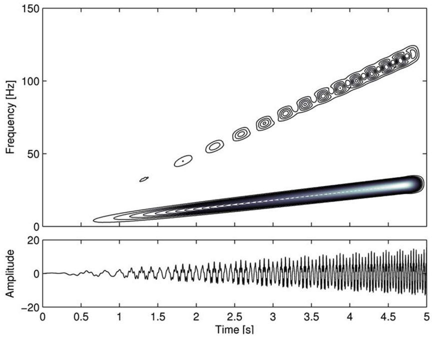
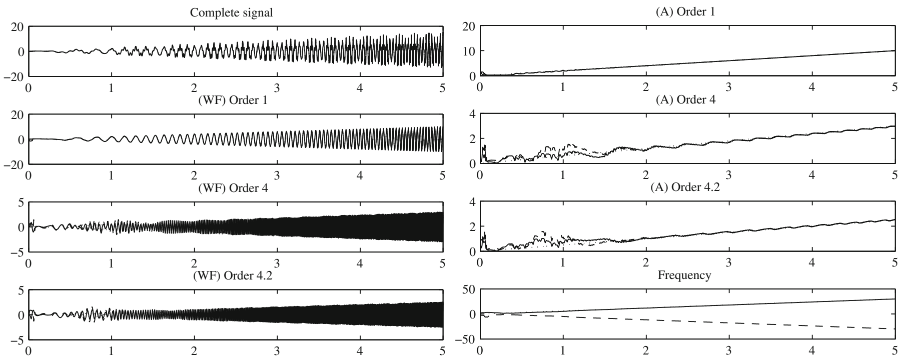


Fig. 2. Time-Frequency representation from synthetic signal obtained by STFT (hamming window, 512 frequency samples, and 50 % overlap)



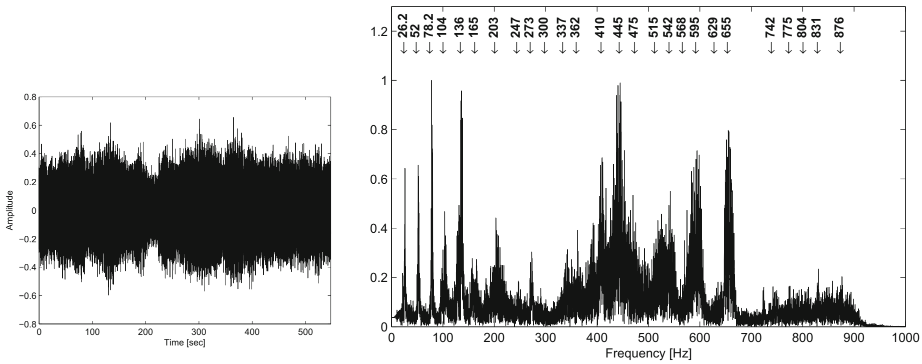
(a) Waveform reconstruction (b) Amplitude and Frequency

Fig. 3. Estimated order components of the synthetic signal using the parameters $p = 1e^{-1}$, $q^a = 1e^{-10}$, $q^f = 1e^{-12}$ and $r = 1e^{-9}$. (··) Original signal, (- -) unconstrained and (-) constrained estimation

have been kindly provided by **Maïa Eolis** to solve the contest in the framework of the International Conference on Condition Monitoring of Machinery in Non-stationary Operations (CMMNO), December 15–16, 2014 Lyon-France[1]. The provided signal comes from an accelerometer located on the rotor side of the gearbox (high speed shaft) casing in the radial direction, and the speed of the main shaft (also called low speed shaft) is between 13 and 15 *rpm* during the recording. The sampling frequency is 20 KHz and the acquisition time is 547 s approx. As regards to high-speed shaft estimation, from the kinematics of the machine the boundaries $[L^f, H^f]$ from the desired IF are defined between 25.99 Hz and 29.98 Hz. However, after to carried out the testing, it was found that the minimum boundary must be fixed at 15 Hz. In Fig. 4 is shown the

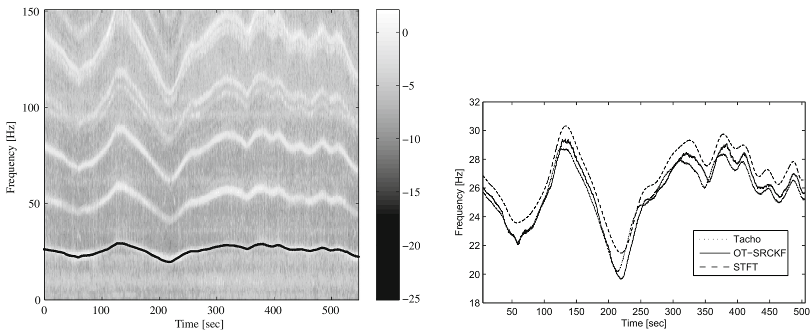
[1] Contest rules link: http://cmmno2014.sciencesconf.org/conference/cmmno2014/pages/cmmno2014_contest_V2.pdf.

provided signal, where it is possible to observe the signal in time and frequency ((a) and (b) parts, respectively). In frequency domain are marked the harmonics obtained using an harmonic algorithm discussed in [9], by the Fourier transform computation from 20 s signal segment. Here, the harmonic 26.2 Hz is used as the first order, obtaining in total a set of 26 orders. In addition, the SRCKF parameters associated to process and measurement covariances are fixed as $q_i^a = 10^{-3}$, $q^f = 10^{-10}$ and $r = 10^{-11}$.



(a) (b)

Fig. 4. Provided signal by CMMNO2014 contest in time and frequency domain.



(a) (b)

Fig. 5. IF estimated by IAS-OT model from CMMNO2014 contest wind turbine signal: (a) time-frequency representation highlighting the estimation with black line, and (b) a comparison with the tacho reference.

As a result, Fig. 5 displays the IF estimated using the proposed constrained OT model. Obtained IF is highlighted with a black line on the time-frequency representation Fig. 5(a), where it is possible to see that the estimation match with the high speed shaft, ranging from 20Hz to 30Hz, which confirms the boundaries fixed into the model. A comparison with the tacho reference is shown in Fig. 5(b), and besides, the IF estimation (red line) using a traditional method based on time-frequency representation (noted as STFT), which consists of tracking the maxima values in the STFT [18]. It is worth noting that using the aforementioned method was achieve the fifth place in the contest. In that sense, the proposed IAS-OT model allows to improve the result obtained using the based-STFT method, reaching a relative error under $\pm 3\,\%$ despite the fact that the intervals $[220-250]$ and $[320-350]$s there are a delay between the reference and the estimated IF.

4 Conclusions

The study proposes, derives and implements a novel constrained SRCKF approach based on a nonlinear state-space model, where it is possible simultaneously extract multiple order/spectral components together with IF estimation and decouple close orders. An improvement of the tracking algorithm is rendered incorporating constraints to the state variables, allowing to have a better performance of the SRCKF. The SRCKF captures the dynamic behavior of the system in terms of the IF estimation, which is an advantage when it is necessary to analyze machines where the reference shaft speed cannot be measured. Therefore, the proposed approach is an useful tool to compensate the non-stationary operating conditions of the machine and it contributes with the diagnostic analysis. The future work will be centered into optimization of the initialization of the model parameters and validation the proposed scheme in other kind of applications.

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