Simultaneous Hedging of Regulatory and Accounting CVA

Christoph Berns

Abstract As a consequence of the recent financial crisis, Basel III introduced a new capital charge, the CVA risk charge to cover the risk of future CVA fluctuations (CVA volatility). Although Basel III allows for hedging the CVA risk charge, mismatches between the regulatory (Basel III) and accounting (IFRS) rules lead to the fact that hedging the CVA risk charge is challenging. The reason is that the hedge instruments reducing the CVA risk charge cause additional Profit and Loss (P&L) volatility. In the present article, we propose a solution which optimizes the CVA risk charge and the P&L volatility from hedging.

Keywords CVA risk charge · Accounting CVA · Hedging · Optimization

1 Introduction

Counterparty credit risk is the risk that a counterparty in a derivatives transaction will default prior to expiration of the trade and will therefore not be able to fulfill its contractual obligations. Before the recent financial crisis many market participants believed that some counterparties will never fail ("too big to fail") and therefore counterparty risk was generally considered as not significant. This view changed due to the bankruptcy of Lehman Brothers during the financial crisis and market participants realized that even major banks can fail. For that reason, counterparty risk is nowadays considered to be significant for investment banks. The International Financial Reporting Standards (IFRS) demand that the fair value of a derivative incorporates the credit quality of the counterparty. This is achieved by a valuation adjustment which is commonly referred to as credit valuation adjustment (CVA), see e.g. [3–5]. The CVA is part of the IFRS P&L, i.e. losses (gains) caused by changes of the counterparties credit quality reduce (increase) the balance sheet equity.

Basel III requires a capital charge for future changes of the credit quality of derivatives, i.e. CVA volatility. Banks can either use a standardized approach to

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compute this capital charge or an internal model [2]. The latter charge is commonly referred to as CVA risk charge. Many banks have implemented a CVA desk in order to manage actively their CVA risk. CVA desks buy CDS protection on the capital markets to hedge the counterparty credit risk of uncollateralized derivatives which have been bought by the ordinary trading desks. Recognizing that banks actively manage CVA positions, Basel III allows for hedging the CVA risk charge using credit hedges such as single name CDSs and CDS indexes. However, the recognition of hedges is different depending on whether the standardized approach or an internal model is used [2].

Summarizing, we can look at counterparty credit risk from two different perspectives: the regulatory (Basel III) and the accounting (IFRS) one. Depending whether we consider counterparty risk from a regulatory or accounting perspective, different valuation methods are applied for this risk. In general, the regulatory treatment of counterparty risk is more conservative than the accounting one, cf. [6]. The difference between the regulatory and the accounting treatment of counterparty risk causes the following problem in hedging the CVA risk charge: eligible hedge instruments such as CDSs would lead to a reduction of the CVA risk charge. On the other hand, under IFRS, a CDS is recognized as a derivative and thus accounted at fair value through profit and loss and therefore introducing further P&L volatility.

The current accounting and regulatory rules expose banks to the situation that they cannot achieve regulatory capital relief and low P&L volatility simultaneously. Deutsche Bank, for instance, has largely hedged the CVA risk charge in the first half of 2013. The hedging strategy that reduced the CVA risk charge has caused large losses due to additional P&L volatility, cf. [7]. This example illustrates the mismatch between the regulatory and accounting treatment of CVA.[1] The mismatch demands for a trade-off between these two regimes, cf. [8]. For this reason, we propose in this article an approach which leads to an optimal allocation between CVA risk charge reduction and P&L volatility. Our considerations are restricted to the standardized CVA risk charge.

We start with an explanation of the standardized CVA risk charge, i.e. the regulatory treatment of CVA. Afterwards, we show that the standardized CVA charge can be interpreted as a (scaled) volatility/variance of a portfolio of normally distributed positions. This interpretation reveals the modeling assumptions of the regulator and will be crucial for the later considerations. In a next step, we explain the counterparty risk modeling from an accounting perspective and we compute the impact of the hedge instruments (used to reduce the CVA risk charge) to the overall P&L volatility, assuming that the risk factor returns are normally distributed. Without the mismatch between the regulatory and the accounting regime, the hedge instruments would move anti-correlated to the corresponding accounting CVAs and the resulting common volatility would be small. Due to the mismatch, the CVA and the hedge instrument changes will not offset completely. For this reason we introduce a

[1]Due to the exclusion of DVA from the Basel III regulatory calculation, the mismatch potentially intensifies.

synthetic[2] total volatility σ_{syn} consisting basically of the sum of the additional accounting P&L volatility σ_{hed} caused by fair value changes of the hedge instruments (hedge P&L volatility) and the regulatory CVA volatility $\sigma_{CVA,reg}$ (i.e. basically the CVA risk charge)[3]:

$$\sigma_{syn}^2 = \sigma_{hed}^2 + \sigma_{CVA,reg}^2. \tag{1}$$

Hence, (1) defines a steering variable describing the common effects of CVA risk charge hedging and resulting P&L volatility. One should mention that formula (1) may suggest statistical independence of the two quantities. However, there exists a dependence in the following sense: both the regulatory CVA volatility and the hedge P&L volatility depend on the hedge amount. The more we hedge, the smaller the $\sigma_{CVA,reg}$. On the other hand, the more we hedge, the larger the σ_{hed}. The definition of the synthetic volatility as a sum of σ_{hed}^2 and $\sigma_{CVA,reg}^2$ can be motivated by the following consideration: the term $\sigma_{CVA,reg}^2$ is related to the regulatory capital demand for CVA risk. The other term, σ_{hed}^2, can be interpreted as capital demand for market risk of the hedge instruments. Although the hedge instruments are excluded from the regulatory capital demand computation for market risk, they potentially reduce the balance sheet equity and therefore may reduce the available regulatory capital. The sum in (1) is now motivated by the additivity of the total capital demand.

In the following we will consider σ_{syn} as function of the hedge amount and search for its minimum. The hedge amount minimizing σ_{syn} leads to the optimal allocation between CVA risk charge relief and P&L volatility. We will derive analytical solutions. The discussion of several special cases will provide an intuitive understanding of the optimal allocation. For technical reasons we exclude index hedges in the derivation of the optimal hedge strategy. However, it is easy to generalize the results to the case where index hedges are allowed.

2 Counterparty Risk from a Regulatory Perspective: The Standardized CVA Risk Charge

In this section we introduce the standardized CVA risk charge. A detailed explanation of all involved parameters is given in the Basel III document [2]. The formula for the standardized CVA risk charge is prescribed by the regulator and is used to determine the amount of regulatory capital which banks must hold in order to absorb possible losses caused by future deteriorations of the counterparties credit quality. We will see that the standardized CVA risk charge can be interpreted as volatility (i.e. standard deviation) of a normally distributed random variable. More precisely, we will show that the CVA risk charge can be interpreted as the 99 % quantile of a portfolio of

[2]We use the word synthetic since σ_{syn} mixes a volatility measured in regulatory terms and a volatility measured in accounting terms.

[3]This connection will be explained later.

positions subject to normally distributed CVA changes (i.e. CVA P&L) only. This gives some insights into the regulators modeling assumptions for future CVA. It is worth to mention that the regulators modeling assumptions may hold or not hold. A detailed look at the regulators modeling assumptions can be found in [6].

In order to be prepared for later computations, we introduce in this section some notations and recall some facts about normally distributed random variables.

The standardized CVA risk charge K is given by [2]:

$$K = \beta\sqrt{h}\Phi^{-1}(q) \tag{2}$$

with

- $h = 1$, the 1-year time horizon,
- Φ the cumulative distribution function of the standard normal distribution
- $q = 99\,\%$ the confidence level and
- β defined by[4]

$$\beta^2 = \left(\sum_{i=1}^{n} 0.5 \cdot \omega_i \left(M_i EAD_i - M_i^{hed} B_i\right) - \omega_{ind} M_{ind} B_{ind}\right)^2 + \sum_{i=1}^{n} 0.75 \cdot \omega_i^2 \left(M_i EAD_i - M_i^{hed} B_i\right)^2 \tag{3}$$

with

- ω_i a weight depending on the rating of the counterparty i, n is the number of counterparties
- M_i, M_i^{hed}, and M_{ind} the effective maturities for the ith netting set (corresponding to counterparty i), the hedged instrument for counterparty i and the index hedge
- EAD_i the discounted regulatory exposure w.r.t. counterparty i
- B_i, B_{ind} the discounted hedge notional amounts invested in the hedge instrument (CDS) for counterparty i and the index hedge.

Formula (2) is determined by the regulator. In order to get a better understanding of this formula, we will derive a stochastic interpretation of it. Before that, we need to recall a fact about normal distributions: if the random vector $\vec{X}$ has a multivariate normal distribution, i.e. $\vec{X} \sim \mathcal{N}(0, \Sigma)$ with mean 0 and covariance matrix Σ, then, for a deterministic vector $\vec{a}$, the scalar product

$$\langle \vec{a}, \vec{X} \rangle := \sum_i a_i X_i \tag{4}$$

[4]For simplicity we consider only one index hedge. The results in this article can easily be generalized to more than one index hedge.

has a univariate normal distribution with mean 0 and variance

$$\sigma^2 = \langle \vec{a}, \Sigma \vec{a} \rangle. \tag{5}$$

Now we are able to derive the stochastic interpretation of the CVA risk charge, more precise the interpretation as volatility.

2.1 Standardized CVA Risk Charge as Volatility

In this section we will show that the regulators' modeling assumptions behind the standardized CVA risk charge are given by normally distributed CVA returns which are aggregated by using a one-factor Gaussian copula model.[5] We consider n counterparties. By R_i, we denote the (one year) CVA P&L (i.e. those P&L effects caused by CVA changes) w.r.t. counterparty i.

Lemma 1 *If one assumes $R_i \sim \mathcal{N}(0, \sigma_i^2)$ and further, if one assumes that the random vector*[6]

$$\vec{R} = (R_1, \ldots, R_n)^t$$

is distributed according to a one-factor Gaussian copula model, i.e. $\vec{R} \sim \mathcal{N}(0, \Gamma)$ with $\Gamma_{ii} = \sigma_i^2$ and $\Gamma_{ij} = \rho\sigma_i\sigma_j$ with ρ independent of i and j for $i \neq j$, then the 99 % quantile of the distribution of $\vec{R}$ is equal to the CVA risk charge (2).

Proof Using (4) and (5), we find that the aggregated CVA return (common CVA P&L) $R_{CVA,reg} := \sum_{i=1}^n R_i = \langle \vec{1}, \vec{R} \rangle$[7] has the distribution $\mathcal{N}(0, \sigma_{CVA,reg}^2)$ with

$$\sigma_{CVA,reg}^2 = \langle \vec{1}, \Gamma \vec{1} \rangle = \sum_{i,j=1}^n \Gamma_{i,j} = \left(\sqrt{\rho} \sum_{i=1}^n \sigma_i \right)^2 + (1-\rho) \sum_{i=1}^n \sigma_i^2 \tag{6}$$

If we compare the above expression with (3), we see that this expression is equal to β^2 (with $B_{ind} = 0$, i.e. no index hedges) if we set $\rho = 0.25$ and $\sigma_i = \omega_i(M_i EAD_i - M_i^{hed} B_i)$. The quantile interpretation of the CVA risk charge (i.e. Formula (2)) follows from standard properties of the normal distribution.

The above lemma shows that the standardized CVA risk charge is basically the volatility of the sum $\sum_i R_i$ of n normally distributed random variables. The normally distributed random variables are equicorrelated: $\rho(R_i, R_j) = 0.25$. Each CVA return R_i has the volatility

$$\sigma_i = \omega_i(M_i EAD_i - M_i^{hed} B_i). \tag{7}$$

[5] This is a very strong assumption that might not be true in reality.

[6] By $\cdot^t$ we denote the transpose of a vector/matrix.

[7] By $\vec{1}$ we denote the vector $(1, \ldots, 1)^t$.

Hence, buying credit protection on counterparty i reduces the corresponding CVA volatility. If we assume $M_i = M_i^{hed}$, the optimal hedge w.r.t. counterparty i is given by a CDS with notional amount B_i equals

$$B_i = EAD_i. \tag{8}$$

3 Counterparty Risk from an Accounting Perspective

As explained in the introduction, counterparty risk from an accounting perspective is quantified by a fair value adjustment called credit valuation adjustment (CVA). The CVA reduces the present value (PV) of a derivatives portfolio in order to incorporate counterparty risk:

$$PV = PV_{riskfree} - CVA,$$

whereby $PV_{riskfree}$ denotes the market value of the portfolio without counterparty risk and CVA is the adjustment to reflect counterparty risk. For the modeling of CVA, banks have some degrees of freedom. Typically, the accounting CVA is computed by means of the following formula (see e.g. [4]):

$$CVA = \int_0^T D(t)EE(t)dP(t) \tag{9}$$

with T the effective maturity of the derivatives portfolio, $D(t)$ the risk-free discount curve, $EE(t) = E[\max\{0, PV(t)\}]$ the (risk-neutral) expected positive exposure at (future time point) t, and $dP(t)$ is the (risk-neutral) default probability of the counterparty in the infinitesimal interval $[t, t + dt]$. For the implementation of (9), a discretization of the integral is necessary. Many banks assume a constant EE profile (i.e. $EE(t) = EE^*$ for all t). In that case, (9) simplifies to

$$CVA = EE^* \int_0^T D(t)dP(t). \tag{10}$$

Further, the (risk-neutral) default probabilities are typically modeled by a hazard rate model, i.e. one assumes that the default time is exponentially distributed with parameter λ. Using this assumption, we can write:

$$CVA = \lambda EE^* \int_0^T D(t)e^{-\lambda t}dt. \tag{11}$$

The approximation (11) will be helpful in the next section, where we describe the hedging of CVA from an accounting perspective

3.1 CVA Hedging from an Accounting Perspective

In previous sections we have seen that the regulatory CVA hedging (i.e. CVA risk charge hedging) can be achieved by buying credit protection. Effectively, (7) says that the regulatory exposure is reduced by the notional amount of the bought credit protection. At this place, we describe CVA hedging from an accounting perspective.

Let us consider a derivatives portfolio with a single counterparty. In order to hedge the corresponding counterparty risk, one can buy, for example, a single name CDS such that the CVA w.r.t. the counterparty together with the CDS is Delta neutral (i.e. up to first order, CVA movements are neutralized by the CDS movements). The condition for Delta neutrality is

$$\Delta CVA = \Delta CDS \tag{12}$$

whereby Δ describes the derivative of the CVA and CDS respectively (w.r.t. the credit spread of the counterparty). To be more precise, the default leg of the CDS should compensate the CVA movements. Using a standard valuation model for a CDS (see e.g. [4]) and computing the derivatives in (12), it is easy to see that (12) is equivalent to

$$B = EE^*, \tag{13}$$

i.e. the optimal hedge amount is given by EE^*. Typically, EE^* is given by the average of the expected positive exposures $EE(t)$ at future time points t:

$$EE^* = \frac{1}{T}\int_0^T EE(t)dt. \tag{14}$$

If we compare (13) with (8) we see that the optimal hedge notional amount for hedging CVA risk from a regulatory perspective is the regulatory exposure EAD, while the optimal hedge notional amount for hedging accounting CVA risk is given by EE^*. In general it holds $EAD > EE^*$, due to conservative assumptions made by the regulator[8] (we refer to [6] for a detailed comparison of these two quantities). Thus, hedging CVA risk differs whether it is considered from an accounting or a regulatory perspective. This mismatch causes additional P&L volatility in the accounting framework, if the CVA risk is hedged from a regulatory perspective (i.e. if the CVA risk charge is hedged).

Finally we remark that we can write the CVA sensitivities $\Delta_{CVA} = \frac{d}{ds}CVA$ as

$$\Delta_{CVA} = EE^* \Delta_{CDS}, \tag{15}$$

whereby Δ_{CDS} is the sensitivity of (the default leg of) a CDS with notional amount $B = 1$.

[8]For example, the alpha multiplier in the IMM context overstates the EAD by a factor of 1.4. Further, the non-decreasing constraint to the exposure profile leads to an overstatement, see [6] for details.

4 Portfolio P&L

As explained above, the hedge instruments reduce the (regulatory) counterparty credit risk. But they may cause new market risk due to additional P&L volatility. However, although in accordance with Basel III eligible hedge instruments are excluded from market risk RWA calculations, the additional P&L volatility of the hedge instruments leads to fluctuations in reported equity. In order to describe the effects of hedging to the overall P&L, we introduce in the present section the corresponding framework. We divide the overall P&L in different parts: the P&L of the hedge instruments, the P&L of the remaining positions, and the CVA P&L. The framework will be helpful later on, when we want to quantify the impact of the CVA risk charge hedges to the accounting P&L.

4.1 Portfolio P&L Without CVA

Let us assume that a bank holds derivatives with n different counterparties for which single name CDS exists. The bank has to decide to which extent it hedges the counterparty risk w.r.t. these counterparties by either single name CDS or index hedges. By Σ we denote the correlation matrix (of dimension $N \times N$, $N > n$) of all risk factors r_i, $i = 1, \ldots, N$ the banks (trading) portfolio is exposed to. Without loss of generality, we assume that the correlations between the CDS of the considered n counterparties are given by the first $n \times n$ components of Σ, i.e. $\Sigma_{i,j} = \rho(CDS_i, CDS_j)$, $i, j = 1, \ldots, n$. Further, $\Sigma_{n+1,i}$ denotes the correlation between the index hedge and the CDS on counterparty $i \in \{1, \ldots, n\}$. The whole portfolio Π of the bank contains the hedge instruments (CDS and index hedge) as well as other instruments (e.g. bonds): $\Pi = \Pi_{hed} \cup \Pi_{rest}$. The sub-portfolio Π_{hed} is driven by the credit spreads of the counterparties. Note that Π_{rest} may depend on some of these credit spreads as well. In the following, we will assume the P&L of the portfolio Π is given by:

$$P\&L = \sum_{i=1}^{n} (B_i \Delta_i + \Delta_{i,rest}) dr_i + B_{ind} \Delta_{ind} dr_{ind} + \sum_{j=n+2}^{N} \Delta_j dr_j, \tag{16}$$

whereby Δ_i denotes the sensitivity of CDS_i w.r.t. the corresponding credit spread, $\Delta_{i,rest}$ denotes the sensitivity of the remaining positions which are sensitive w.r.t. the credit spread of counterparty i as well,[9] B_i (resp. B_{ind}) denotes the notional of CDS_i (resp. the notional of the index hedge), and dr_i describes the change of the risk factor r_i (the first n risk factors are the credit spreads) in the considered time period.

[9]For example, if Π_{rest} contains a bond emitted by the counterparty i, then (ignoring the Bond-CDS Basis) $\Delta_{i,rest} = -\Delta_i$).

4.2 Impact with CVA

This section extends the above considerations to the case where we allow for a CVA component. We define the total P&L as the difference between the P&L given by (16) and the CVA P&L:

$$P\&L_{tot} = P\&L - P\&L_{CVA}, \tag{17}$$

whereby $P\&L_{CVA}$ is defined in a similar manner as in (16)[10]:

$$P\&L_{CVA} = \sum_{i=1}^{n+1} \Delta_{i,CVA} dr_i. \tag{18}$$

In (18), the risk factors r_i are the same risk factors which appear in the first $n+1$ summands of (16). This is because the CVAs are driven by the same risk factors as the corresponding hedge instruments. Recall that in a setup where counterparty risk is completely hedged, the P&L of the hedge instruments is canceled out by the P&L of the CVAs. This is the case, if the corresponding sensitivity is equal. In Sect. 3.1 we have shown how one can achieve this (using the condition of Delta neutrality) by choosing the right hedge notional amounts.

4.3 Impact of CVA Risk Charge Hedging on the Accounting P&L Volatility

The additional P&L volatility caused by the hedge instruments is basically given by the residual volatility of the hedge instruments which is not canceled by the CVAs. In order to derive an expression for this volatility, we start with the derivation of the volatility of the total portfolio P&L. The residual volatility will consist of those parts of the total volatility which are sensitive w.r.t. the hedge instruments.

In order to proceed, we have to introduce the following notations: the vector $\vec{\Delta}_{CVA} \in \mathbb{R}^{n+1}$ contains the CVA sensitivities and the return vector $\vec{dr} \in \mathbb{R}^N$ describes the changes of the N risk factors the trading book is exposed to. We further introduce the sensitivity vectors[11] $\vec{\Delta} = (\Delta_1, \ldots, \Delta_{ind}, \ldots, \Delta_N)^t \in \mathbb{R}^N$

[10] We consider only the credit spreads as risk factors. Exposure movements due to changes in market risk factors are not considered. This is unproblematic for the considerations in this article since we will end up with dynamic CVA hedging strategy (cf. Sect. 5) which incorporates the exposure changes.

[11] The first n components of $\vec{\Delta}$ are the CDS sensitivities w.r.t. credit spread changes and the $n+1$th component is the sensitivity of the index hedge.

and[12] $\vec{\Delta}_{rest} = (\Delta_{1,rest}, \ldots, \Delta_{n,rest})^t \in \mathbb{R}^n$, the notional vector $\vec{B} = (B_1, \ldots, B_n, B_{ind})^t \in \mathbb{R}^{n+1}$ and the diagonal matrix $Q_\Delta = diag(\Delta_1, \ldots, \Delta_n, \Delta_{ind}) \in \mathbb{R}^{(n+1)\times(n+1)}$.

Lemma 2 *If we assume that the portfolio P&L is given by (17) and if we further assume $\vec{dr} \sim \mathcal{N}(0, \Sigma)$ (for some correlation matrix Σ), then the squared volatility (i.e. the variance) of (17) is given by*[13]

$$\begin{aligned}\sigma^2_{P\&L_{tot}} &= \left\langle \begin{pmatrix} Q_\Delta \vec{B} \\ \vec{0} \end{pmatrix} \Sigma \begin{pmatrix} Q_\Delta \vec{B} \\ \vec{0} \end{pmatrix} \right\rangle + \left\langle \begin{pmatrix} \vec{\Delta}_{CVA} \\ \vec{0} \end{pmatrix} \Sigma \begin{pmatrix} \vec{\Delta}_{CVA} \\ \vec{0} \end{pmatrix} \right\rangle \\ &+ \left\langle \begin{pmatrix} \vec{\Delta}_{rest} \\ \vec{\Delta}_{N-n-1} \end{pmatrix} \Sigma \begin{pmatrix} \vec{\Delta}_{rest} \\ \vec{\Delta}_{N-n-1} \end{pmatrix} \right\rangle - 2\left\langle \begin{pmatrix} Q_\Delta \vec{B} \\ \vec{0} \end{pmatrix} \Sigma \begin{pmatrix} \vec{\Delta}_{CVA} \\ \vec{0} \end{pmatrix} \right\rangle \\ &+ 2\left\langle \begin{pmatrix} Q_\Delta \vec{B} \\ \vec{0} \end{pmatrix} \Sigma \begin{pmatrix} \vec{\Delta}_{rest} \\ \vec{\Delta}_{N-n-1} \end{pmatrix} \right\rangle - 2\left\langle \begin{pmatrix} \vec{\Delta}_{rest} \\ \vec{\Delta}_{N-n-1} \end{pmatrix} \Sigma \begin{pmatrix} \vec{\Delta}_{CVA} \\ \vec{0} \end{pmatrix} \right\rangle. \end{aligned} \tag{19}$$

Proof With the above defined vectors, we can write:

$$\begin{aligned} P\&L_{tot} &= \langle Q_\Delta \vec{B} - \vec{\Delta}_{CVA}, \vec{dr}_{n+1} \rangle + \langle\, \vec{\Delta}_{rest}, \vec{dr}_{n+1} \rangle + \langle \vec{\Delta}_{N-n-1}, \vec{dr}_{N-n-1} \rangle \\ &= \left\langle \begin{pmatrix} Q_\Delta \vec{B} \\ \vec{0}_{N-n-1} \end{pmatrix} - \begin{pmatrix} \vec{\Delta}_{CVA} \\ \vec{0}_{N-n-1} \end{pmatrix} + \begin{pmatrix} \vec{\Delta}_{rest} \\ \vec{\Delta}_{N-n-1} \end{pmatrix}, \vec{dr} \right\rangle \\ &= \langle \vec{a} - \vec{b} + \vec{c}, \vec{dr} \rangle \end{aligned} \tag{20}$$

whereby $\vec{dr}_{n+1}$ denotes the $n+1$-dimensional vector consisting of the first $n+1$ components of $\vec{dr}$, $\vec{dr}_{N-n-1}$ consists of the remaining $N-n-1$ components of $\vec{dr}$, $\vec{\Delta}_{N-n-1}$ denotes the vector of the remaining $N-n-1$ sensitivities, and $\vec{0}_{N-n-1}$ is the $N-n-1$-dimensional vector whose components are all equal to 0.[14] Clearly, the vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ coincide with the respective summands of the left hand side of the scalar product in (20). If we use $\vec{dr} \sim \mathcal{N}(0, \Sigma)$, it follows from (4) to (5):

$$\begin{aligned} \sigma^2_{P\&L_{tot}} &= \langle \vec{a} - \vec{b} + \vec{c}, \Sigma(\vec{a} - \vec{b} + \vec{c}) \rangle \\ &= \langle \vec{a}, \Sigma\vec{a} \rangle + \langle \vec{b}, \Sigma\vec{b} \rangle + \langle \vec{c}, \Sigma\vec{c} \rangle - 2\langle \vec{a}, \Sigma\vec{b} \rangle + 2\langle \vec{a}, \Sigma\vec{c} \rangle - 2\langle \vec{c}, \Sigma\vec{b} \rangle. \end{aligned} \tag{21}$$

If we plug in the expressions for $\vec{a}$, $\vec{b}$ and $\vec{c}$, we obtain (19). □

In order to be prepared for later computations, we will further simplify Expression (19). To this end, we introduce the following notations: by Σ_{n+1} we denote the $(n+1) \times (n+1)$ matrix consisting of the first $n+1$ column and row entires of Σ only, i.e. $\Sigma_{i,j}$, $i, j = 1, \ldots n+1$. The matrix $\Sigma_{N,n+1}$ is the $N \times (n+1)$ matrix

[12] The vector $\vec{\Delta}_{rest}$ contains the n sensitivities w.r.t. credit spread changes of those trading book positions which are different from the CDSs used for hedging but are sensitive w.r.t. to the credit spreads of the hedge instruments as well.

[13] The vector $\vec{\Delta}_{N-n-1}$ is defined in the proof.

[14] In the following, we will omit the index $N-n-1$ and simply write $\vec{0}$.

obtained from Σ by deleting the last $N-n-1$ columns and $\Sigma^t_{N,n+1}$ denotes its transpose matrix. With this notation and using that $\vec{0}$ cancels many components in (19), we can write:

$$\begin{aligned}\sigma^2_{P\&L_{tot}} &= \langle Q_\Delta \vec{B}, \Sigma_{n+1} Q_\Delta \vec{B}\rangle + \langle \vec{\Delta}_{CVA}\Sigma_{n+1}, \vec{\Delta}_{CVA}\rangle - 2\langle Q_\Delta \vec{B}, \Sigma_{n+1}\vec{\Delta}_{CVA}\rangle \\ &\quad + 2\left\langle \vec{B}, Q_\Delta \Sigma^t_{N,n+1}\begin{pmatrix}\vec{\Delta}_{rest}\\ \vec{\Delta}_{N-n-1}\end{pmatrix}\right\rangle - 2\left\langle \vec{\Delta}_{CVA}, \Sigma^t_{N,n+1}\begin{pmatrix}\vec{\Delta}_{rest}\\ \vec{\Delta}_{N-n-1}\end{pmatrix}\right\rangle \\ &\quad + \left\langle \begin{pmatrix}\vec{\Delta}_{rest}\\ \vec{\Delta}_{N-n-1}\end{pmatrix} \Sigma \begin{pmatrix}\vec{\Delta}_{rest}\\ \vec{\Delta}_{N-n-1}\end{pmatrix}\right\rangle.\end{aligned} \tag{22}$$

In (22), the first summand describes the volatility of the hedge instruments if they are considered as isolated from the remaining positions (i.e. those positions which are different from the hedge instruments). Analogously, the other quadratic terms (i.e. the second and the last summand in (22)) represent the volatility of the CVA and the remaining positions respectively. The cross terms (third, fourth, and fifth summand) describe the interactions between the volatility of the hedge instruments, the CVA and the remaining positions. For example, the third term describes the interaction between the CVA and the hedge instruments.

The P&L volatility σ^2_{hed} caused by the hedge instruments is given by those terms of (22) which depend on the hedge instruments, i.e. those terms which depend on $\vec{B}$. These are the first, the third, and the fourth term of (22), i.e.

$$\sigma^2_{hed} = \langle Q_\Delta \vec{B}, \Sigma_{n+1} Q_\Delta \vec{B}\rangle - 2\langle Q_\Delta \vec{B}, \Sigma_{n+1}\vec{\Delta}_{CVA}\rangle + 2\left\langle \vec{B}, Q_\Delta \Sigma^t_{N,n+1}\begin{pmatrix}\vec{\Delta}_{rest}\\ \vec{\Delta}_{N-n-1}\end{pmatrix}\right\rangle. \tag{23}$$

The other terms of (22) describe the volatility caused by the remaining positions.

In order to simplify the notation, we write σ^2_{hed} in the following way:

$$\sigma^2_{hed} = \langle A\vec{B}, \vec{B}\rangle + \langle \vec{B}, \vec{b}\rangle \tag{24}$$

with

$$A = Q_\Delta \Sigma_{n+1} Q_\Delta \tag{25}$$

and

$$\vec{b} = Q_\Delta \Sigma^t_{N,n+1}\begin{pmatrix}\vec{\Delta}_{rest}\\ \vec{\Delta}_{N-n-1}\end{pmatrix} - Q_\Delta \Sigma_{n+1}\vec{\Delta}_{CVA}. \tag{26}$$

Note that σ^2_{hed} is not simply given by a quadratic form but also incorporates a linear part. The quadratic form describes the volatility of a portfolio consisting of the hedge instruments, while the linear part describes the correlations of the hedge instruments with the remaining positions and with the CVAs.

4.3.1 Definition of the Steering Variable

We now define a steering variable aiming to define a unified framework for CVA risk charge hedging and P&L volatility. The steering variable is given by a synthetic volatility consisting of the sum of the regulatory CVA volatility and the volatility of the accounting P&L caused by the hedge instruments:

$$\sigma_{syn}^2 = \sigma_{CVA,reg}^2 + \sigma_{hed}^2. \tag{27}$$

The synthetic volatility unifies both the regulatory and the accounting framework. It can be considered as a function of the hedge notional amounts. The minimum of $\sigma_{tot,syn}^2$ describes the optimal allocation between CVA risk charge reduction and P&L volatility. Note that $\sigma_{tot,syn}^2$ contains now the matrices Γ and Σ, who describe the correlations between the same risk factors. This mismatch can be resolved, if the advanced CVA risk charge is used [2]. However, the use of different CVA sensitivities cannot be resolved. The most significant differences arise due to different exposure definitions: while the exposures EAD_i contained in the regulatory CVA sensitivities are based on the effective EPE and multiplied by the alpha multiplier (for IMM banks), this is not the case for the exposures used to compute the accounting CVA sensitivities. In general, these mismatches will lead to smaller accounting CVA sensitivities. Thus, a complete hedging of the CVA risk charge leads to an overhedged accounting CVA. See [6] for a complete description of the sources of the mismatch. Another source of potential overhedging is the following: if accounting CVA is already hedged by instruments which are not eligible hedge instruments in the sense of Basel III, additional hedge instruments are necessary for the hedging of the CVA risk charge. These hedge instruments will cause additional P&L volatility, since their offsetting counterparts (i.e. the CVAs) are not present (since they are already hedged).

5 Determination of the Optimal Hedge Strategy

This section describes concretely how the mismatch between the regulatory regime and the accounting regime can be mitigated. The result will be a dynamic CVA hedging strategy based on an optimization principle of the steering variable introduced in the previous section. We will ignore index hedges but all results can easily be generalized to the case where index hedges are included.

As opposed to the previous sections, the vector $\vec{B}$ will not contain the component B_{ind} in this section. As explained before, we want to minimize the synthetic volatility[15]

$$\sigma_{syn}^2(\vec{B}) = \sigma_{hed}^2(\vec{B}) + \sigma_{CVA}^2(\vec{B}) \tag{28}$$

[15] We ignore the index tot.

as a function of $\vec{B}$. The component B_i^* of the minimum $\vec{B}^*$ describes the optimal notional amounts of CDS_i, used to hedge the counterparty risk w.r.t. counterparty i. We now determine $\vec{B}^*$ by computing the zeros of the first derivative of σ_{syn}^2.

Theorem 1 *Under the same assumptions as in Lemma 2, the minimum $\vec{B}^*$ of (28) is given by*[16]

$$\vec{B}^* = H^{-1}\vec{f} \tag{29}$$

with

$$H := 2(A + Q_{M^{hed}}\Gamma Q_{M^{hed}}) \tag{30}$$

and

$$\vec{f} := 2Q_{M^{hed}}\Gamma Q_M\overrightarrow{EAD} - \vec{b}. \tag{31}$$

Proof In order to keep the display of the computations clear, we introduce the diagonal matrices $Q_M := diag(\omega_1 M_1, \ldots, \omega_n M_n)$ and $Q_{M^{hed}} := diag(\omega_1 M_1^{hed}, \ldots, \omega_n M_n^{hed})$ and the n-dimensional vector $\overrightarrow{EAD}$ whose components are given by the counterparty exposures. Using these definitions, we can write:

$$\sigma_{CVA}^2 = \langle Q_M\overrightarrow{EAD} - Q_{M^{hed}}\vec{B}, \Gamma(Q_M\overrightarrow{EAD} - Q_{M^{hed}}\vec{B})\rangle. \tag{32}$$

whereby Γ describes the constant correlation between the CVAs (all diagonal elements given by 1). Using (32) and (24), we can write:

$$\begin{aligned}\frac{\partial\sigma_{syn}^2}{\partial\vec{B}} &= \frac{\partial}{\partial\vec{B}}(\langle A\vec{B}, \vec{B}\rangle + \langle\vec{b}, \vec{B}\rangle)\\ &\quad + \frac{\partial}{\partial\vec{B}}\langle Q_{M^{hed}}\vec{B}, \Gamma Q_{M^{hed}}\vec{B}\rangle\\ &\quad - 2\frac{\partial}{\partial\vec{B}}\langle Q_{M^{hed}}\vec{B}, \Gamma Q_M\overrightarrow{EAD}\rangle\\ &= 2A\vec{B} + \vec{b} + 2Q_{M^{hed}}\Gamma Q_{M^{hed}}\vec{B} - 2Q_{M^{hed}}\Gamma Q_M\overrightarrow{EAD}\\ &= H\vec{B} - \vec{f},\end{aligned} \tag{33}$$

where we have used the notations (30) and (31). This shows (29). Further, we note that the matrix H is derived from correlation matrices and therefore positive semi-definite. As a result, H is indeed invertible. Moreover, it holds

$$\frac{\partial^2\sigma_{syn}^2}{\partial^2\vec{B}} = H.$$

Hence, the second derivative of σ_{syn}^2 is positive semi-definite and B^* is indeed a minimum.

[16] All terms are introduced in the proof.

Remark The implementation of the optimal hedge strategy works as follows: one has to compute on a regular basis (e.g. daily, weekly, etc.) the optimal solution (29). To do this one needs the CVA sensitivities,[17] the trading book sensitivities, and the correlation matrix of the risk factors.[18] Afterwards, the CVA desk needs to buy credit protection described by the optimal solution. This reduces the capital demand for counterparty risk and (by construction) minimizes the accounting P&L of the bought credit protection.

The approach presented in this article is based on many simplifying assumptions and restricted to the standardized CVA risk charge. Obviously, one could relax these assumptions and apply a comparable optimization principle. In such a case, it would possibly be hard to derive an analytical solution. Instead, one would obtain a numerical solution.

5.1 Special Cases

For illustration purposes, we consider the case $n = 1$, i.e. the special case of a single netting set. In that case both H and $\vec{f}$ are scalars:

$$H = 2\Delta^2 \Sigma_{1,1} + 2\omega^2 (M^{hed})^2$$

and

$$f = 2\omega^2 M M^{hed} EAD + 2\Delta\Delta_{CVA}\Sigma_{1,1} - \left(\Delta\Sigma_{1,1}\Delta_{rest} + \Delta \sum_{j=2}^{N} \Sigma_{1,j}\Delta_j \right), \quad (34)$$

whereby Δ describes the sensitivity of the hedge instrument of the considered counterparty, Δ_{rest} the sensitivity of the remaining positions (i.e. all positions without the CDS used for hedging purposes), Δ_{CVA} the sensitivity of accounting CVA and Δ_j are the sensitivities to the risk factors of the remaining positions. Thus, the optimal solution is

$$B^* = \frac{2\omega^2 M M^{hed} EAD + 2\sigma^2 \Delta\Delta_{CVA} - \left(\Delta\sigma^2 \Delta_{rest} + \Delta \sum_{j=2}^{N} \Sigma_{1,j}\Delta_j \right)}{2\Delta^2\sigma^2 + 2\omega^2 (M^{hed})^2} \quad (35)$$

where we have used that $\Sigma_{1,1}$ is equal to the volatility σ^2 of the hedge instrument. First, in order to get a better understanding of B^*, let us assume that the risk factor (credit spread) of the hedge instrument is independent of the remaining positions, i.e.

[17] Banks which actively manage their CVA risk usually compute these sensitivities.

[18] Larger banks usually have these data available, e.g. for market risk management purposes.

$\Delta_{rest} = 0$ and $\Sigma_{1,j} = 0$, for $j = 2, \ldots, N$. In that case (35) (we assume additionally $M = M^{hed}$) becomes

$$B^* = \frac{2\omega^2 M^2 EAD + 2\Delta\Delta_{CVA}\sigma^2}{2\omega^2 M^2 + 2\Delta^2\sigma^2}. \tag{36}$$

We see already that B^* is (at least from a certain volatility level) a decreasing function in σ^2, as we would expect it. Obviously, if we ignore the fact that the hedge instrument introduces further volatility (i.e. we assume $\sigma^2 = 0$), it holds

$$B^* = EAD.$$

It is easy to see that this is the optimal hedge amount if we minimize the CVA risk charge alone. As explained above, the most significant differences between the IFRS CVA and the regulatory CVA are the different exposure computation methodologies. In (36), these differences are reflected in EAD and Δ_{CVA}: while EAD is based on the regulatory methodology, Δ_{CVA} is based on accounting CVA methodology.[19] For illustration purposes, let us assume that Δ_{CVA} is based on the same exposure methodology as the regulatory CVA sensitivities (and that the modeling assumptions Sect. 3 holds). This means, that cf. (15)

$$\Delta_{CVA} = EAD\Delta, \tag{37}$$

i.e. we use the regulatory exposure EAD in (15) instead of the economical exposure EE^*. If we plug in (37) in (36), we obtain:

$$B^* = \frac{(2\omega^2 M^2 + 2\Delta^2\sigma^2)EAD}{2\omega^2 M^2 + 2\Delta^2\sigma^2} = EAD. \tag{38}$$

Thus, if we ignore the mismatch between the accounting and the regulatory CVA, the optimal hedge solution is given by the optimal hedge solution of the CVA risk charge only. If we include the mismatch, we can approximate the accounting CVA sensitivity by (cf. (15))

$$\Delta_{CVA} = EE^*\Delta. \tag{39}$$

As explained in Sect. 4.3.1, EE^* is smaller than EAD. Using (36) and (39) yields:

$$B^* = \frac{2\omega^2 M^2 EAD + 2\Delta\sigma^2 EE^*}{2\omega^2 M^2 + 2\Delta^2\sigma^2} < \frac{2\omega^2 M^2 EAD + 2\Delta\sigma^2 EAD}{2\omega^2 M^2 + 2\Delta^2\sigma^2} = EAD. \tag{40}$$

Hence, the mismatch leads to a smaller optimal hedge amount than the current regulatory exposure.

[19]Note that Δ_{CVA} depends on the exposure as well (while Δ is based on a unit exposure, cf. (16)). But this exposure is computed based on accounting methodology. This is the main source of differences between the accounting and regulatory regimes.

We remark that it cannot be excluded that B^* becomes negative. This is the case if the risk factors of the remaining positions are strongly correlated to the risk factor of the hedge instrument. In such a situation it seems to be reasonable to set $B^* = 0$.

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