IPMCs as EAPs: Models

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Abstract
The primary objectives of modeling IPMCs are to facilitate researchers with IPMC fabrication and performance prediction, to better understand the underlying principles of IPMC actuation and sensing, and to develop functional real-time control systems for IPMC devices. This chapter provides an overview of IPMC modeling with specific emphasis on physics-based models and control models. The underlying governing equations are presented and methods of solving are discussed.

Keywords
IPMC • Modeling • Controls

1 Physics-Based Model

Effective application of IPMCs in robotic devices relies on the ability to predict performance and achieve real-time control of IPMC actuators. Several studies have focused on this by developing models for IPMCs. Purely empirical models, or black-box models, have been developed to describe the bending response for a given voltage input of an IPMC actuator. Although these models can predict performance for a calibrated IPMC sample, they provide no insight into the underlying physics of IPMC electromechanical transduction. Semi-empirical models, or gray-box models, have been presented which incorporate some IPMC physics while using experimental data to accommodate model shortcomings. Physics-based models, or white-box models, provide a thorough understanding of IPMC electromechanical phenomena; however, these models are more complex and often limited to numerical solving methods. This section focuses on physics-based models of IPMCs and presents the governing partial differential equations which are most commonly employed. Contributions from various research groups are discussed.

The fundamental theory of IPMC electromechanical transduction (actuation) and mechanoelectrical transduction (sensing) will be presented using the same governing equations. Differences between actuation and sensing phenomena will be highlighted throughout.

1.1 Ionic Current in the Polymer Membrane

The polymer membrane of an IPMC consists of anions fixed to the polymer backbone and freely positioned positive ions, or cations. In a hydrated state, micro-channels in the polymer expand to allow free transport of cations and attracted water molecules through the membrane. When a voltage is applied to IPMC electrodes, the cations migrate away from the anode, dragging attracted water molecules. This causes osmotic pressure change with localized swelling near the cathode interface and contraction near the anode, which results in an overall...
deformation of the IPMC. A basic schematic of an IPMC polymer structure and transduction phenomenon is shown in Fig. 1.

In the case of sensing, a given deformation causes ionic rearrangement, which in turn results in a measurable voltage at the electrodes. This phenomenon is understood to be roughly the reverse transduction of an IPMC actuator. At the same time, there are various ways in which the physics behind mechanoelectrical transduction of IPMCs is understood. Physics-based models have been proposed based on ionic motion (Tadokoro et al. 2000). Other authors have explained the underlying causes of the transduction with solvent fluxes and pressure gradients based on the standard Onsager relations (Gennes et al. 2000). In such systems, the induced electric field is proportional to the applied bending torque, which causes a pressure gradient in the polymer. A different approach to describe the mechanoelectrical transduction is to consider the electrostatic interaction within the polymer (Nemat-Nasser and Li 2000). While the charge sensing in this model is connected to ionic dipoles that are induced in the polymer clusters, it has been proposed that the charge density on the surface of the polymer is proportional to the induced stress (Farinholt and Leo 2004). Using the same governing equations, the model was later expanded to add the effects of electrode resistance (Chen et al. 2007). This resulted in a compact, transfer-function representation of the physics-based model which will be further discussed in Sect. 8.2.
The governing equations presented herein comprise a fundamental physics-based model of IPMCs. The underlying cause of the phenomenon of IPMC electromechanical and mechanoelectrical transduction is induced ionic current and resulting nonzero spatial charge in the vicinity of the electrodes. Ionic current in the polymer for both cases can be described by the Nernst–Planck equation (Pugal et al. 2013):

$$\frac{\partial C}{\partial t} + \nabla \cdot (-D\nabla C - z\mu FC\nabla \phi - \mu C\Delta V\nabla P) = 0 \quad (1)$$

where $C$ is cation concentration, $\mu$ is the mobility of the cations, $D$ is the diffusion coefficient, $F$ is the Faraday constant, $z$ the is charge number, $\Delta V$ is the molar volume which quantifies the cation hydrophilicity, $P$ is the solvent pressure, and $\phi$ is the electric potential in the polymer.

An important difference between IPMC electromechanical and mechanoelectrical transduction is the magnitude, direction, and significance of individual terms in Eq. 1. Besides the concentration time derivative, the terms of Eq. 1 consist of three flux terms governed by the field gradients of electric potential, concentration, and solvent pressure. In case of actuation, the electric potential gradient term is significantly more prevalent than the solvent pressure flux, that is, $zF\nabla \phi \gg \Delta V\nabla P$; therefore, the pressure flux term is often neglected in actuation model implementation. However, in case of sensing, these terms are of similar significance and neither should be neglected. It is also interesting to note the direction of the electric potential gradient is opposite for sensing as compared to actuation because the ionic current is governed by an induced pressure gradient rather than an applied voltage. Conceptual models of IPMC actuation and sensing are shown in Fig. 2. Field gradient directions are indicated.

Fig. 2 Conceptual models of IPMC electromechanical (left) and mechanoelectrical (right) transduction with the formed field gradients (Pugal et al. 2013. © IOP Publishing. Reproduced by permission of IOP Publishing. All rights reserved)
The electric potential gradient term can be described by Poisson’s equation (Pugal et al. 2013):

$$-\nabla^2 \phi = \frac{F\rho}{\varepsilon}$$ (2)

where $\varepsilon$ is the absolute dielectric permittivity and $\rho$ is charge density defined as:

$$\rho = C - C_a$$ (3)

where $C_a$ is local anion concentration. While the cation concentration $C$ is governed by the Nernst–Planck Eq. 1, the anion concentration is related to local volumetric strain. The volume changes in the polymer matrix affect the local anion concentration as anions are fixed to the polymer backbone. Hence, the anion concentration $C_a$ is expressed:

$$C_a = C_0 \left[ 1 - \left( \frac{\partial u_1}{\partial x} + \frac{\partial u_2}{\partial y} + \frac{\partial u_3}{\partial z} \right) \right]$$ (4)

where $u_1$, $u_2$, and $u_3$ are local displacements in the $x$, $y$, and $z$ directions respectfully, and $C_0$ is the initial ion concentration.

Equations 1 and 2 make up what is commonly called the Poisson–Nernst–Planck (PNP) model for IPMCs and describes the fundamental physics within the polymer membrane.

### 1.2 Solid Mechanics Physics

The linear elastic material model can be used to describe the deformation of IPMCs. The constitutive relation of Hooke’s Law can be used to relate stress and strain in the polymer as:

$$
\begin{bmatrix}
\sigma_{11} \\
\sigma_{22} \\
\sigma_{33} \\
\sigma_{12} \\
\sigma_{23} \\
\sigma_{13}
\end{bmatrix} =
\begin{bmatrix}
2\mu + \lambda & \lambda & \lambda & 0 & 0 & 0 \\
\lambda & 2\mu + \lambda & \lambda & 0 & 0 & 0 \\
\lambda & \lambda & 2\mu + \lambda & 0 & 0 & 0 \\
0 & 0 & 0 & \mu & 0 & 0 \\
0 & 0 & 0 & 0 & \mu & 0 \\
0 & 0 & 0 & 0 & 0 & \mu
\end{bmatrix}
\begin{bmatrix}
\varepsilon_{11} \\
\varepsilon_{22} \\
\varepsilon_{33} \\
2\varepsilon_{12} \\
2\varepsilon_{23} \\
2\varepsilon_{13}
\end{bmatrix}
$$ (5)

where $\varepsilon_{ij}$ is the normal strain in the $i$-direction for $i = j$ and a shear strain for $i \neq j$. Stress terms $\sigma_{ij}$ are defined similarly. The constants $\mu$ and $\lambda$ are Lame’s constants, defined as

$$\mu = \frac{E}{2(1 + \nu)}, \quad \lambda = \frac{\nu E}{(1 + \nu)(1 - 2\nu)}$$ (6)
where $E$ is Young’s modulus and $\nu$ is Poisson’s ratio.

The system is in equilibrium if Navier’s displacement equations are satisfied, given by the relation:

$$-\nabla \cdot \sigma = F$$  \hspace{1cm} (7)

where $F$ is the body force vector per unit volume.

Newton’s second law is used to describe time-dependent deformation:

$$\rho_p \frac{\partial^2 u}{\partial t^2} - \nabla \cdot \sigma = F$$  \hspace{1cm} (8)

where $u$ is the local displacement vector and $\rho_p$ is the polymer density.

In 2D Cartesian coordinates, the Navier’s displacement equations take the form:

$$\left(\lambda + \mu\right) \left(\frac{\partial^2 u_1}{\partial x^2} + \frac{\partial^2 u_2}{\partial y^2}\right) + \mu \left(\frac{\partial^2 u_1}{\partial x \partial y} + \frac{\partial^2 u_2}{\partial y \partial x}\right) + F_1 = 0$$  \hspace{1cm} (9)

$$\left(\lambda + \mu\right) \left(\frac{\partial^2 u_1}{\partial x \partial y} + \frac{\partial^2 u_2}{\partial y \partial x}\right) + \mu \left(\frac{\partial^2 u_2}{\partial x^2} + \frac{\partial^2 u_2}{\partial y^2}\right) + F_2 = 0$$  \hspace{1cm} (10)

In the case of IPMC electromechanical transduction, $F_2 = 0$ and $F_1$ can be expressed as a function of cation concentration, $F_1 = A(C - C_a)$, where $A$ is a constant (Pugal et al. 2011). For mechanoelectrical transduction, the body force can be neglected and the bending is governed by appropriate boundary conditions.

### 1.3 Simplified Model

Although the provided equations described both transduction types of IPMCs, some terms are more prevalent than others for different transductions. For instance, it is reasonable to neglect the third flux term in Eq. 1 in the case of electromechanical transduction as it is very small compared to the second term. Also, while the local anion concentration can be expressed as a constant in the case of electromechanical transduction, it plays an important role in mechanoelectrical transduction where field gradients are significantly smaller.

In the following example, coupling of the Navier equations and the Nernst–Planck equation is presented based on a simplified model where $C_a = C_0$ is a constant and $\Delta V\Delta P \approx 0$. This simplification is generally assumed acceptable in the case of electromechanical transduction.

IPMC material is modeled via a multi-physics coupled problem, consisting of the PNP system of equations coupled to the Navier equation. Finite element methods of solving are typically utilized in simulating the solution.

For modeling an IPMC strip, a rectangular 2D domain $\Omega \subset \mathbb{R}^2$ with boundaries $\partial \Omega_1 \ldots A \subset \partial \Omega$ as shown in Fig. 3 can be considered.
The Nernst–Planck equation with simplifications as detailed above has the form

\[
\frac{\partial C}{\partial t} + \nabla \cdot (-D \nabla C - z\mu FC \nabla \phi) = 0 \tag{11}
\]

As there is no flow through the domain’s boundary, Eq. 11 is equipped with a Neumann boundary condition:

\[
-D \frac{\partial C}{\partial n} - \mu FC \frac{\partial \phi}{\partial n} = 0 \tag{12}
\]

For normal actuation of the IPMC, a positive voltage \( V_{\text{pos}} \) boundary condition is prescribed on \( \partial \Omega_1 \) and a zero voltage boundary condition on \( \partial \Omega_3 \):

\[
\phi_{\partial \Omega_1} = V_{\text{pos}}, \quad \phi_{\partial \Omega_3} = 0 \tag{13}
\]

On the rest of the boundary, \( \phi \) has zero normal derivatives, and it is thus equipped with Neumann boundary conditions:

\[
\frac{\partial \phi_{\Omega_2}}{\partial n} = \frac{\partial \phi_{\Omega_4}}{\partial n} = 0 \tag{14}
\]

For the Navier equations 9 and 10, the following Dirichlet boundary conditions can be applied:

\[
u_{1,\partial \Omega_2} = u_{2,\partial \Omega_2} = 0 \tag{15}\]

If no external forces are considered, zero Neumann boundary conditions can be applied on \( \partial \Omega \) for the Navier equations.

### 2 Control Models

#### 2.1 Physics-Based and Control-Oriented Model for IPMC Actuators

In this section, a physics-based and control-oriented actuation model for IPMC actuators is presented. The model combines the seemingly incompatible advantages of both the white-box models (capturing key physics) and the black-box models
(amenable to control design). The proposed modeling approach provides an interpretation of the sophisticated physical processes involved in IPMC actuation from a systems perspective. The model development starts from the governing PDE (Nemat-Nasser and Li 2000; Farinholt 2005) that describes the charge redistribution dynamics under external electrical field, electrostatic interactions, ionic diffusion, and ionic migration along the thickness direction. The model incorporates the effect of distributed surface resistance, which is known to influence the actuation behavior of IPMCs (Shahinpoor and Kim 2000). Moreover, by converting the original PDE into the Laplace domain, an exact solution is obtained, leading to a compact, analytical model in the form of infinite-dimensional transfer function. The model can be further reduced to low-order models, which again carry physical interpretations and are geometrically scalable. Most of the modeling work was published in Chen and Tan (2008).

2.1.1 Governing Equation
Let $D$, $E$, $\phi$, and $\rho$ denote the electric displacement, the electric field, the electric potential, and the charge density, respectively. The following equations hold:

\[
\nabla \cdot D = \rho = F(C^+ - C^-) \quad (16)
\]

\[
E = -\nabla \phi = \frac{D}{\kappa_e} \quad (17)
\]

where $\kappa_e$ is the effective dielectric constant of the polymer, $F$ is Faraday’s constant, and $C^+$ and $C^-$ are the cation and anion concentrations, respectively. The ion transportation can be captured by a second-order linear PDE in terms of charge density (Nemat-Nasser and Li 2000; Farinholt 2005):

\[
\frac{\partial \rho}{\partial t} - d \Delta^2 \rho + \frac{F^2 d C^-}{k_e RT} (1 - C^- \Delta V) \rho = 0. \quad (18)
\]

Nemat–Nasser and Li assumed that the induced stress is proportional to the charge density (Nemat-Nasser and Li 2000):

\[
\sigma = \alpha_0 \rho \quad (19)
\]

where $\alpha_0$ is the coupling constant. To ease the equation,

\[
K \triangleq \frac{F^2 d C^-}{k_e RT} (1 - C^- \Delta V). \quad (20)
\]

Farinholt investigated the current response of a cantilevered IPMC beam when the base is subject to step and harmonic actuation voltages (Farinholt 2005). A key assumption is that the ion flux at the polymer/metal interface is zero, which serves as a boundary condition for Eq. 18, leading to
2.1.2 Electrical Impedance Model

From Eq. 19, the stress induced by the actuation input is directly related to the charge density distribution \( \rho \). Therefore, as a first step in developing the actuation model, we will derive the electrical impedance model in this section. While the latter is of interest in its own right, one also obtains the explicit expression for \( \rho \) as a by-product of the derivation. Consider Fig. 4, where the beam is clamped at one end \( z = 0 \) and is subject to an actuation voltage producing the tip displacement \( w(t) \) at the other end \( z = L \). The neutral axis of the beam is denoted by \( x = 0 \), and the upper and lower surfaces are denoted by \( x = h \) and \( x = -h \), respectively.

Performing Laplace transform for the time variable of \( \rho(x, z, t) \) (noting the independence of \( \rho \) from the \( y \) coordinate), one converts Eq. 18 into the Laplace domain:

\[
\frac{\partial^3 \rho(x, z, s)}{\partial x^3} - \frac{K}{d} \left. \frac{\partial \rho}{\partial x} \right|_{x=\pm h} = 0.
\]  

(21)

where \( s \) is the Laplace variable. Define \( \beta(s) \) such that \( \beta(s) = (s + K)/d \). The generic solution for Eq. 22 can be obtained as

\[
\rho(x, z, s) = 2c_2(z, s)\sinh(\beta(s)x).
\]  

(23)

The \( z \)-dependent transfer function \( c_2(z, s) \) depends on the boundary condition of the PDE Eq. 21. Using Eq. 23 and the field Eqs. 16 and 17, one can derive the expressions for the electric field \( E \) and then for the electric potential \( \varphi \) in the Laplace domain:

\[
E(x, z, s) = 2c_2(z, s) \frac{\cosh(\beta(s)x)}{\kappa \beta(s)} + a_1(z, s).
\]  

(24)
\[ \varphi(x, z, s) = 2c_2(z, s) \frac{\sinh(\beta(s) x)}{\kappa e \beta^2(s)} - a_1(z, s)x + a_2(z, s) \quad (25) \]

where \( a_1(z, s) \) and \( a_2(z, s) \) are appropriate functions to be determined based on the boundary conditions on \( \varphi \), which are affected by the distributed surface resistance on the electrodes.

The surface electrode of an IPMC typically consists of aggregated nanoparticles formed during chemical reduction of noble metal salt (such as platinum salt) (Kim and Shahinpoor 2003). The surface resistance is thus non-negligible and has an influence on the sensing and actuation behavior of an IPMC (Shahinpoor and Kim 2000). In this modeling work, the effect of distributed surface resistance is incorporated into the impedance model, as illustrated in Fig. 5.

Let the electrode resistance per unit length be \( r_1 \) in \( z \) direction and \( r_2 \) in \( x \) direction. One can further define these quantities in terms of fundamental physical parameters: \( r_1' = r_1/W, r_2 = r_2/W \), with \( r_1 \) and \( r_2 \) representing the surface resistance per \{unit length \cdot unit width\} in \( z \) and \( x \) directions, respectively. In Fig. 5, \( i_p(z, s) \) is the distributed current per unit length going through the polymer due to the ion movement, \( i_h(z, s) \) represents the leaking current per unit length, and \( i_s(z, s) \) is the surface current on the electrodes. \( R_p \) denotes the through-polymer resistance per unit length, which can be written as \( R_p' = R_p/W \), with \( R_p' \) being the polymer resistance per \{unit length \cdot unit width\}. Note that by the continuity of current, the current \( i_s(z, s) \) on the top surface equals that on the bottom surface but with an opposite direction. The surface current \( i_s(0, s) \) at \( z = 0 \) is the total actuation current \( I(s) \).

The following equations capture the relationships between \( i_s(z, s), i_p(z, s), i_h(z, s), \) \( \varphi_{\pm}(z, s) \):
\[
\frac{\partial \phi_\pm(z, s)}{\partial z} = \mp \frac{r'_1}{W} i_s(z, s) \tag{26}
\]
\[
\frac{\partial i_s(z, s)}{\partial z} = -(i_p(z, s) + i_k(z, s)) \tag{27}
\]

From the potential condition at \( z = 0 \), i.e., \( \phi_\pm(0, s) = \pm V(s)/2 \), the boundary conditions for Eq. 25 are derived as
\[
\phi(\pm h, z, s) = \phi_\pm(z, s) \mp i_p(z, s) r'_2/W. \tag{28}
\]

With Eqs. 26 and 28, one gets
\[
\phi(\pm h, z, s) = \frac{\pm V(s)}{2} \mp \int_0^z \frac{r'_1}{W} i_s(\tau, s) d\tau - \frac{r'_2}{W} i_p(z, s). \tag{29}
\]

Combining Eq. 29 with Eq. 25, one can solve for the functions \( a_1(z, s) \) and \( a_2(z, s) \) in the generic expression for \( \phi(x, z, s) \). With consideration of the boundary condition Eq. 21, one can solve for \( c_2(z, s) \). With \( a_1(z, s) \), \( a_2(z, s) \), and \( c_2(z, s) \), one obtains \( E(h, z, s) \) from Eq. 24
\[
E(h, z, s) = -\frac{\phi(h, z, s)}{h} \frac{\gamma(s)(s + K)}{\gamma(s)s + K\tanh(\gamma(s))} \tag{30}
\]

where \( \gamma(s) \triangleq h\beta(s) \). Define the actuation current along the negative \( x \)-axis direction to be positive. The current \( i_p \) due to the ion movement can be obtained as
\[
i_p(z, s) = -sWD(h, z, s) = -sWkE(h, z, s). \tag{31}
\]

The leaking current \( i_k \) can be obtained as
\[
i_k(z, s) = \frac{\phi_+(z, s) - \phi_-(z, s)}{R'_p/W}. \tag{32}
\]

With Eqs. 30, 31, and 32, one can solve the PDE Eq. 27 for the surface current \( i_s(z, s) \) with the boundary condition \( i_s(L, s) = 0 \). The total actuation current \( I(s) = i_s(0, s) \) can be obtained, from which the transfer function for the impedance can be shown to be
\[
Z(s) = \frac{V(s)}{I(s)} = \frac{2\sqrt{B(s)}}{A(s)\tanh(\sqrt{B(s)L})} \tag{33}
\]

where
\[ A(s) \triangleq \frac{\theta(s)}{1 + r'_{2} \theta(s)/W} + \frac{2W}{R_{p}'}, \quad B(s) \triangleq \frac{r'_{1}}{W} A(s), \]  
\[ \theta(s) \triangleq \frac{sWk_{e}\gamma(s)(s + K)}{h(s\gamma(s) + Ktanh(\gamma(s)))}. \]  

(See Chen and Tan (2008) for the detailed derivation.)

### 2.1.3 Actuation Model

First, we derive the transfer function \( H(s) \) relating the free-tip displacement of an IPMC beam, \( w(L, s) \), to the actuation voltage \( V(s) \), when the beam dynamics (inertia, damping, etc.) is ignored. From Eqs. 19 and 23, one obtains the generic expression for the stress \( \sigma(x, z, s) \) generated due to actuation

\[ \sigma(x, z, s) = 2a_{0}c_{2}(z, s)\sinh(\beta(s)x). \]  

Note that \( c_{2}(z, s) \) is available from the derivation of the impedance model. When considering the surface resistance, the bending moment \( M(z, s) \) is obtained as

\[ M(z, s) = \int_{-h}^{h} x\sigma(x, z, s)Wdx = \frac{2a_{0}KWk_{e}(\gamma(s) - tanh(\gamma(s)))\phi(h, z, s)}{(s\gamma(s) + Ktanh(\gamma(s)))}. \]

From the linear beam theory (Gere and Timoshenko 1997)

\[ \frac{\partial^2 w(z, s)}{\partial z^2} = \frac{M(z, s)}{YI} \]  

where \( Y \) is the effective Young modulus of the IPMC and \( I = 2/3Wh^3 \) is the moment of inertia of the IPMC. Solving Eq. 38 with boundary conditions \( w(0, s) = 0 \) and \( w'(0, s) = 0 \), one can get

\[ w(L, s) = \frac{f(s)}{L^2} \cdot \frac{V(s)L^2 - 4\int_{0}^{L} \int_{0}^{z} r'_{1}/Wd\tau dz'dz}{1 + r'_{2}\theta(s)/W} \]  

where

\[ f(s) \triangleq -\frac{\alpha_{0}WL^2}{YI} \frac{Kk_{e}(\gamma(s) - tanh(\gamma(s)))}{(\gamma(s)s + Ktanh(\gamma(s)))}. \]
After doing three times integration of $i_s(\tau, s)$ in Eq. 39, one thus obtains the transfer function:

$$H(s) = \frac{w(L, s)}{V(s)} = f(s) \cdot g(s) \cdot X(s)$$  \hspace{1cm} (41)

where

$$g(s) = \frac{2}{1 + r_2' \theta(s)/W}$$  \hspace{1cm} (42)

$$X(z, s) = \frac{1 - \sec h(\sqrt{B(s)L}) - \tanh(\sqrt{B(s)L}) \sqrt{B(s)L}}{B(s)}.$$  \hspace{1cm} (43)

Note that the blocking force output $F(s)$ at the tip can be derived via $F(s) = w(L, s) K_0$, where $K_0 = 3YL^3$ denotes the spring constant of the beam.

Back to the free bending case, in order to accommodate the vibration dynamics of the beam, we cascade $G(s)$ to $H(s)$, as illustrated in Fig. 6. As the output of $G(s)$ represents the bending displacement (as that of $H(s)$ does), $G(s)$ will have a dc gain of 1. Since the actuation bandwidth of an IPMC actuator is relatively low (under 10 Hz), it often suffices to capture the mechanical dynamics $G(s)$ with a second-order system (first vibration mode):

$$G(s) = \frac{\omega_n^2}{s^2 + 2\xi \omega_n s + \omega_n^2}$$  \hspace{1cm} (44)

where $\omega_n$ is the natural frequency of the IPMC beam and $\xi$ is the damping ratio. The natural frequency $\omega_n$ can be further expressed in terms of the beam dimensions and mechanical properties (Voltera and Zachmanoglou 1965).

### 2.1.4 Model Reduction

An important motivation for deriving a transfer function-type actuation model is its potential use for real-time feedback control. For practical implementation of feedback control design, the model needs to be finite-dimensional, i.e., being a finite-order, rational function of $s$. However, in the actuation model derived earlier, $H(s)$ is infinite dimensional since it involves nonrational functions including sinh($\cdot$), cosh($\cdot$), $\sqrt{\cdot}$, etc. A systematic approach to model reduction is Padé approximation (Baker and
Graves-Morris 1996), where one can approximate $H(s)$ with a rational function of specified order. However, the computation involved is lengthy and the resulting coefficients for the reduced model can be complex. Therefore, in this chapter, a much simpler, alternative approach is proposed for model reduction by exploiting the knowledge of physical parameters and specific properties of hyperbolic functions.

Based on the physical parameters (see Chen and Tan 2008), $|\gamma(s)| \gg 10$, and $K \gg 10^6$, which allows one to make the approximation in the low-frequency range ($<100$ Hz): $\tanh(\gamma(s)) \approx 1$ and $\gamma(s) \approx h\sqrt{K/d} =: \gamma$. With the above approximations, one can simplify $f(s)$, $\theta(s)$, and $g(s)$ as

$$f(s) \approx -\frac{L^2a_0W \, k_e(\gamma - 1)}{2YL}$$

$$\theta(s) \approx \frac{sWk_e(\gamma + K)}{h(\gamma s + K)}$$

$$g(s) \approx \frac{2h(\gamma s + K)}{r_2^2k_e\gamma(s + K) + h(\gamma s + K)}$$.

The Taylor series expansions of $\sinh(a)$ and $\cosh(a)$ will be used for approximation $X(s)$:

$$X(s) \approx 1 + \sum_{n=0}^{m} \left( a^{2n+2}/(2n + 1)! - a^{2n}/(2n)! \right)$$

with $a = \sqrt{B(s)L}$, for some finite integer $m$. When $|s|$ is small (low-frequency range) and $2r_1/R_p$ is small (which is indeed the case, see Chen and Tan 2008), $|\sqrt{B(s)L}|$ is small and Eq. 34 approximates $X(s)$ well with a small integer $m$. Note that only even-degree terms appear in Eq. 48, and hence Eq. 48 is a function of $B(s)L^2$ instead of $\sqrt{B(s)L}$. Finally, since $B(s)$ is a rational function of $\theta(s)$ and $\theta(s)$ is approximated by a rational function Eq. 46, one can obtain an approximation to $X(s)$ by a rational function of $s$.

Combining Eqs. 45 and 47 and the approximation to $X(s)$, one gets a rational approximation to $H(s)$. Since the mechanical dynamics $G(s)$ is already rational, one obtains a finite-dimensional of actuation model. Note that a reduced model is still a physical model. In particular, it is described in terms of fundamental physical parameters and is, thus, geometrically scalable. This represents a key difference from other low-order, black-box models, in which case the parameters have no physical meanings and one would have to reidentify the parameters empirically for every actuator.
2.2 Empirical Model for Mechanical Simulations

If we consider an element of linear type actuator and an integrated artificial muscle actuator as shown in Fig. 7, physical model using partial differential equations is too complicated for numerical simulation of mechanical systems with such actuator. Also, properties of the constructed model can be different from an actual one (Yamakita et al. 2004, 2008; Kamamichi et al. 2007; Nishida et al. 2012). In those cases, empirical model of the system is useful for numerical simulations. However, it should be noted that the empirical model should give relationships both between control voltage and displacement and between applied force and displacement.


In order to identify both properties, one can construct an experimental system as illustrated in Fig. 8.

We assume the structure of a transfer function shown in Fig. 9, consisting of two subsystems $P_1(s)$ and $P_2(s)$ that are connected in series.

The transfer functions are identified as in the following steps:

1. Identification of $P_1(s)$:
   Measure a response from input voltage $v$ to displacement $y$, then compute the system $P(s) = P_2(s)P_1(s)$ from input–output data using a subspace identification algorithm.

2. Identification of $P_2(s)$:
   Measure a response from load $f_i$ to displacement $y$, then compute the system $P_2(s)$ from input–output data using a subspace identification algorithm.

3. Computation of $P_1(s)$:
   Compute the system $P_1(s)$ as $P_2(s)^{-1}P(s)$.

The identification of the system curve fit() or n4sid() in Matlab system can be used. If a force sensor is available, $P_1(s)$ can be identified independently.

One example of the identified transfer functions are given as

$$P_1(s) = -\frac{1.50 \times 10^{-3}s^2 + 1.09 \times 10^{-2}s + 3.98 \times 10^{-2}}{s^3 + 6.13s^2 + 3.23 \times 10s + 7.12 \times 10}, \quad (49)$$

$$P_2(s) = -\frac{3.49 \times 10^3s^2 + 1.23 \times 10^6s + 3.81 \times 10^6}{s^4 + 7.19s^3 + 6.49 \times 10^4s^2 + 4.14 \times 10^5s + 1.33 \times 10^6}, \quad (50)$$

The fitting result of the identified model is shown in Fig. 10, indicating rather large deviation from the experimental result.

It is reasonable to assume that the actuator has many nonlinear characteristics and one of the most influential factors is the dynamics of load since the bending characteristics of the film highly depend on the load. In order to include the effect, static nonlinear block can be inserted in front of the linear dynamics of $P_2(s)$, i.e., Hammerstein model for $P_2(s)$ (see Fig. 11).

For the static nonlinearity, any nonlinear function can be used. One of the examples is RBF (radial basis function) as


\[
N_1 = \sum_{i=0}^{n_1} \alpha_i \phi_i^1, \quad N_2 = \sum_{i=0}^{n_2} \beta_i \phi_i^2 \tag{51}
\]

\[
\phi_i^1 = \exp \left( -\frac{(f_i - \bar{f}_i)^2}{2\sigma^2} \right) f_1, \quad i = 0 \rightarrow 2, \tag{52}
\]

\[
\phi_i^2 = f_i', \quad i = 0 \rightarrow 2, \tag{53}
\]
\[ \bar{f}_0 = 0.0, \bar{f}_1 = 1.16 \times 10^{-3}, \bar{f}_2 = 2.37 \times 10^{-3}, \sigma = \frac{(\bar{f}_1 + \bar{f}_2)}{2} \]  

(54)

If we use the nonlinear structure, the response of the model is improved (see Fig. 12).

When the LTI model of the actuator applies a force vector \( f \) to a mechanical system where state space representations of the actuator and the mechanical system are represented as follows:

### 2.3 Actuator Model

\[
\begin{align*}
\dot{x}_a &= A_a + b_a v - b_f f \\
y_a &= C_a x_a
\end{align*}
\]  

(55)

### 2.4 Mechanical Model

\[
\begin{align*}
\dot{x}_m &= f_m(x_m) + g_m(x_m)f \\
y_m &= h_m(x_m)
\end{align*}
\]  

(56)

where \( f \) is an interaction force vector, then \( f \) can be solved by a velocity constraint:

\[ \dot{y}_a = \dot{y}_m \]  

(57)
as

\[ C_a \dot{x}_a = \frac{\partial h_m}{\partial x_m} \dot{x}_m \rightarrow C_a (A_a x_a + b_v v - b_f f) = \frac{\partial h_m}{\partial x_m} (f_m(x_m) + g_m(x_m)f) \]

\[ f = \left( \frac{\partial h_m}{\partial x_m} g_m(x_m) + C_a b_f \right)^{-1} \left( C_a (A_a x_a + b_v v) - \frac{\partial h_m}{\partial x_m} f_m(x_m) \right). \]  

(58)

If the actuator model contains input nonlinearity, \( f \) should be solved numerically to satisfy the velocity constraint in general.

### 3 Conclusions

In this chapter, latest physics-based models and control models of IPMC were reviewed. Physics-based models provide thorough understanding of the underlying physics of IPMC transduction phenomena. However, these models are more complex and often limited to numerical solving methods. The fundamental theory of IPMC electromechanical and mechanoelectrical transduction were presented using the same governing equations. Differences between actuation and sensing phenomena were explicitly explained.

Physics-based and control-oriented actuation model was presented that combines the seemingly incompatible advantages of both the white-box models (capturing key physics) and the black-box models (amenable to control design). This modeling approach provides an interpretation of the sophisticated physical processes involved in IPMC actuation from a systems perspective. Also, an empirical model for mechanical simulations was described that can be implemented for complex mechanical systems when physical model using partial differential equations is too complicated for numerical simulation.

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**References**


