What Is Mechanics?

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The International Union of Theoretical and Applied Mechanics is active both in Theoretical Mechanics and in Applied Mechanics. Beyond these, however, it is concerned with that fruitful interaction between them which, increasingly, has constituted one of the great scientific success stories of the past 300 years.

A. Theoretical Framework

1. Newtonian Foundations

Mechanics was the first science for which a systematic theoretical framework founded on mathematics was created, as pioneered in Newton's 'Principia' (first published in 1687). Although many already existing ideas were incorporated in 'Principia', it represented a fundamentally new overall conception of how a major science can be characterized in terms of laws which in form are of an explicitly mathematical character.

The underlying strategy developed by Newton for this purpose, and since carried much further by his successors, was a 'divide and rule' strategy. It proved equally appropriate to either of the two major parts of mechanics; that is, to statics, or dynamics; concerned with how motion is prevented, or governed, respectively.

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Thus, Newton's strategy requires first that we divide up or classify different types of *force*; representing different influences which in combination may act to prevent motion, or to govern it when it occurs. *Matter*, again, is to be thought of as divided up into particles of relatively small size, which are subject to forces of two main kinds: *external* forces (such as gravity, for example), along with those internal forces which act between a pair of particles and which necessarily assume equal and opposite values.

Motion may be prevented in a system where the sum (in a vectorial sense) of all the different forces acting on each and every particle of the system is zero. Wherever that sum is nonzero, however, its value specifies the rate of change of the *momentum* (mass times velocity) for the particle's motion; which, therefore, it governs.

Besides the twofold application of the 'Divide and Rule' strategy within statics (through classification of forces, and through the subdivision of matter into particles), dynamics necessitates a third subdivision of time into brief instants. During each such instant a particle's momentum changes by an amount equal to the instant's duration times the sum of the forces acting on the particle.

2. Mathematical Formulation of Newtonian Mechanics

For the dynamics of a body small enough (relative to an entire system) to be treated as a single particle, this principle can be expressed mathematically as an *ordinary differential equation* with time as the independent variable. Motions of planets and of their satellites, for example, lend themselves to interpretation in terms of solutions of such ordinary differential equations.

In other circumstances, when matter is viewed as subdivided into many particles, even the laws of statics may need to be expressed as *partial* differential equations with space coordinates as independent variables. Their form depends upon an assumed knowledge of *material properties*, including knowledge of how internal stresses (the internal forces acting between adjacent particles, divided by their area of contact) depend upon other factors including the body's degree of deformation. Such a partial differential equation may express the fact that the sum of all forces on a particle, including external forces as well as the internal forces of interaction with adjacent particles, is zero.

Dynamics, on the other hand, allows that sum to take a nonzero value which determines how the momentum of the particle concerned changes from one brief instant to the next. Analytical expression of results such as this takes the form of partial differential equations with space coordinates *and also the time* as independent variables. Modern treatments of such problems using high-speed computers may approximate the partial differential equations by means of finite differences; or may, alternatively, apply the basic principles of mechanics directly to carefully selected particles of matter known as *finite elements*.

Theoretical mechanics has continued over the years to make massive progress through (i) improved representation of material properties, (ii) incorporation of additional types of external force, and (iii) advances in analytical or numerical methods. Brief indications of these main lines of progress will now be given.

3. Improved Representation of Material Properties

For matter in the solid state a rather early initiation, and subsequent extended development, of a comprehensive theory of elasticity, allowing stress to depend linearly on the measure of deformation known as strain, led to impressive results in both statics and dynamics. It was then followed successively by a range of more refined theories (including, for example, the theory of plasticity) which especially need to be applied in cases of *large deformation*. Important newer developments have resulted from the treatment of 'materials with memory'.

For matter in either of the two fluid states (liquid or gas), no serious problems are posed by statics; mainly, because stresses in a fluid at rest take the form of a simple pressure acting equally in all directions, as explained in elementary hydrostatics texts. The dynamics of fluids, however, is affected by additional stresses associated for common fluids with their viscosity: and such stresses prove to be important even for fluids of very small viscosity. This is because the condition satisfied by a fluid motion at a solid boundary requires its velocity to vary steeply across a thin boundary layer, where viscous stresses are substantial and may significantly affect whether or not the *boundary layer*, as a result of separating prematurely from such a solid boundary, changes drastically the character of the whole flow.

Different relationships of stress to motion, more complicated than can be described by a simple viscosity coefficient, govern the flow properties of certain substances. Analysis of these relationships and their consequences forms the branch of mechanics known as rheology.

Other material properties which may be important include those familiar from thermodynamics. They are concerned with the convertibility of energy between its various forms, and with 'equations of state'. In a fluid, for example, these relate changes in the fluid density (mass per unit volume) to changes in pressure and also temperature. For gases, furthermore, kinetic theory (applying mechanical laws to the molecules themselves) may give valuable information about material properties which can critically affect gas dynamics, including not only viscous action but also *heat transfer* (which, besides influencing dynamical behaviour, is also of importance in its own right).

4. Consideration of Additional External Forces

External forces which in mechanics may need to be considered together with gravity include *electromagnetic* forces; whose effect can be substantial, especially for good conductors. These include both solid and liquid metals, and also ionized gases, some of whose electromagnetic properties are found to deserve study by means of kinetic theory.

Systems subjected to a general rotation may be analyzed by means of space coordinates related to a rotating frame of reference. Dynamics in such a rotating frame effectively incorporates additional forces (the centrifugal and Coriolis forces) which influence greatly the behaviour of technically important *gyroscopic* systems, and also the dynamics of the ocean or the atmosphere.

5. Advances in Mathematical Methods

Beginning with the analytical mechanics of Lagrange, mathematical theories founded upon energy considerations have proved to be of very general and wide-spread applicability. Some special mathematical methods of importance to theoretical mechanics include linear and nonlinear theories of *vibrations and waves*. These, for example, have proved to be of great value for acoustics by elucidating not only the types of vibrations which our ears detect but also their transmission to our ears as waves travelling through solid or fluid media.

Theories of *stability* are also most important. For statics, they can distinguish between an equilibrium which can persist and one which will collapse in response to a small disturbance of a particular shape (as when a solid shell buckles under load). For dynamics, they can distinguish between steady motions able to persist and those which will break up through instability into either regular or randomised disturbances. Description of instability, phenomena in terms of the generic process known as ‘bifurcation’ have proved increasingly fruitful.

More recently, general studies of *dynamical systems* have given the broad analytical background to two important tendencies: the tendency for certain continuously varying motions to develop discontinuities (‘catastrophes’, as when an acoustic wave develops into a shock wave); and the tendency for certain well ordered motions to develop into random motions (‘chaos’; a process long studied in the dynamics of fluids as the tendency for laminar flows to develop into turbulence, but now recognized as relevant also to many solid systems with only a few degrees of freedom). Much of the previously existing knowledge and understanding in these areas (for example, of shock waves or turbulence) has been valuably extended and systematized by these new theories.

In all periods, many important mathematical advances have been initiated by research workers in mechanics. Thus the science of mathematics itself was a major beneficiary from progress in theoretical mechanics.

B. Examples of the Applications of Mechanics

1. Applied Mechanics and its Growing Utilisation of Theoretical Mechanics

The applications of mechanics are found in many scientific fields, some of which (like astronomy, oceanography, and meteorology) have already been referred to; as well as in most of the principal subdivisions of engineering and technology. Into each particular scientific or technological field the process of penetration of any refined ideas from theoretical mechanics has been slow, for a good reason: the scientific phenomena needing to be elucidated, or the technological objectives needing to be met, were in most cases much too complicated for effective treatment by the methods of theoretical mechanics during their early stages of development.

In these circumstances, practitioners of the disciplines concerned needed to concentrate above all on devising ingenious *measurement techniques* appropriate to a particular area of application, and on amassing useful empirical rules for correlating data obtained with these techniques. Such rules might be expressed in phraseology using some of the simpler ideas from theoretical mechanics, but they were not based on detailed theoretical analysis.

Great achievements were to result from applied mechanics development using these empirical methods. In each area the approach in question has, furthermore, continued to make good progress, especially during periods immediately following the introduction of a new measurement technique.

At the same time the general progress in theoretical mechanics has successfully taken place, partly under its own momentum and partly under the stimulus of challenges posed by the complicated problems needing to be solved in particular areas of application. Gradually, within each field, theoretical analysis of ‘model problems’ has become recognized as making a truly valuable contribution to studies of the complicated ‘real problems’ which they were designed to model. Even when the agreement between experiment and theory was not very excellent such analysis might permit a most useful extrapolation of the available experimental data to other conditions for which additional experiments would be too difficult or costly or time-consuming. In time, the models became more and more comprehensive, and correspondingly more valuable.

These continuing processes, in which theoretical mechanics both makes valued contributions to, and is stimulated by challenges derived from, applied mechanics, have increasingly brought the two parts of mechanics much closer together in a fruitfully cooperative unity. Some examples of how this happened in particular areas of application are given in the rest of this note.

2. Structural Engineering

In structural engineering, early empirical developments had led to great achievements which included such refined designs as medieval cathedrals. Yet gradually methods became more analytical as it became possible to estimate loads and the *stress distributions* they would produce, first in single beams and then in frameworks; and to compare stresses so calculated with yield stresses characteristic of the materials used. *Stability* calculations were initiated too, by Euler, so as to yield estimates of critical loads for buckling.

Nevertheless, over long periods it was necessary to overdesign structures by 'factors of safety' as high as 10; that is, to design them so that calculated loads for yielding or buckling were ten times more than the estimated worst-case loads. This need arose from a great multiplicity of uncertainties; for example, in the field of stress analysis, or concerning the reliability of assumptions about material properties or load estimates.

Later, the study of frameworks and similar structures became much more thorough; it took increasingly into account the problems of stress concentrations at junctions between elements, and of detailed design to minimise these. Also, 'limit design' methods were developed (studying the behaviour of a framework after early applications of load have produced certain plastic deformations).

Furthermore, the inevitable presence in a structure of particular types of *imperfection* which adversely influence structural integrity was increasingly allowed for. These included geometrical imperfections powerfully affect buckling behaviour.

Again, small material imperfections including dislocations and cracks were intensely studied in relation to their capacity for growth leading to plastic deformation or to fracture. This, for example, allowed the phenomenon of metal *fatigue* under cyclic loading (first discovered empirically) to become clearly interpretable in terms of crack growth, and there were similar successes in the area of metal *creep*. In the meantime, the importance of composite materials (from concrete to carbon-fiber-reinforced plastics) became increasingly recognized, and this too stimulated many major new developments in theoretical mechanics.

The enormous economic advantage of structure-weight minimisation for the design of aircraft and spacecraft gave a special boost to refined structural analysis aimed at achieving accuracies such as would permit 'factors of safety' to be brought down from former figures like 10 to modern figures around 1.5 or even less. They also encouraged particular mathematical developments (for example, in the theory of shells) of very great subtlety.

At the same time, empirical methods advanced in parallel; particularly through the development of strain-gauge technology, of various optical techniques and of a wide range of convenient methods for loading simulation. The modern structural engineer uses an admirably well integrated blend of empirical activity (doing experiments and using compendious data from earlier experiments) with the analytic and computational methods of theoretical mechanics.

3. Hydraulics

The study of flow in man-made systems of pipes and channels goes back to the beginnings of history with the irrigation achievements in Egypt and Mesopotamia which made possible the first great civilisations. Then it developed further in response to successive needs, from (for example) the Roman Empire's large-scale aqueduct and hypocaust systems to the remarkable drainage and irrigation schemes carried out in the early years of the Dutch Republic.

Theoretical mechanics made its impact on the subject in the eighteenth century when Daniel Bernoulli showed how a steady stream, according to Newton's Laws, would in the absence of frictional forces conserve what we now call its *total head*. Much of the ensuing development of hydraulics (at first, mainly empirical) was expressed in terms of laws governing how much total head is frictionally dissipated by different types of steady stream under different conditions. Some puzzling features of these laws, which recognize (for example) how streams progressing along pipes lose total head far less steeply where the pipe cross-section contracts than where it expands, became intelligible in terms of theoretical mechanics only when the conditions governing *boundary layer separation* in a fluid flow had been elucidated both theoretically and empirically.

Other developments in the two-dimensional (and three-dimensional) modelling of fluid motions allowed improved design of pipe and channel junctions and detailed calculations of flows such as those in the neighbourhood of sluice-gates. More recently, turbulence modelling played a growing part in such work. In the meantime, the increasingly refined theory of *surface wave* phenomena had been most valuably applied so as to understand how stationary waves, standing waves and travelling waves are generated by flows in open channels. Also, *cavitation* theory elucidated effects due to the motion of a liquid generating locally negative pressures which cause the spontaneous appearance of bubbles.

In parallel with all the theoretical developments, laboratory studies utilising carefully instrumented flumes and water-tunnels and other hydraulic modelling devices have added greatly to knowledge in all parts of this discipline. Modern hydraulic design rests on calculations utilising a well integrated combination of results emanating both from theoretical mechanics and from experiments in the laboratory and in the field.

4. Mechanical Engineering

Building upon the long-established empirical technology of water-powered and wind-powered mill machinery, mechanical engineering began to develop rapidly after steam-power was introduced in the eighteenth century. Ideas from theoretical mechanics played an increasing role in these developments.

Thus, the part of dynamics known as kinematics (the *geometrical* description of relative motions) influenced the detailed design of trains of *gearing* mechanisms of all kinds; and, especially, the shaping of surfaces required to be in contact from gear-teeth to screws. The part known as kinetics (relation of motion to forces and to the *work* done by these) influenced the design not only of the actual means for transferring power but also of the devices for controlling the resulting motions. An increasingly important contribution was made by the theory of vibrations of mechanical systems, with its fundamental elucidation of the concept of normal modes of vibration each with its own natural frequency; and of the dominating influence of *resonance* (coincidence between forcing frequency and natural frequency) in generating these usually *unwanted* vibrations.

By the last quarter of the nineteenth century the mechanics of fluids had begun to exert a major influence on mechanical engineering design; as when, for example, a hydrodynamic theory of lubrication related the forces sustainable in a journal bearing to the geometry and viscous properties of the oil film. Above all, development of the *steam turbine* called for meticulous shaping of steam passages and of turbine blades, taking into account the interacting dynamics and thermodynamics of fast-moving steam. A new understanding of heat transfer, based on the fundamental mechanics of fluids, was at the same time being established.

Systems for efficient exchange of energy (in either direction) between streams of fluid and rotating blades were then progressively developed for the successive requirements of fans, propellers, fluid transmissions, compressors and gas turbines; with applications in aeronautics and many other fields. More and more, these developments rested upon refined representations of the dynamics of fluids and blades in relative motion. Other flow geometries needed to be analysed as the great liquid-fuelled rocket engines began to be designed. In many such developments the fluid dynamics of *combustion* played a critical part; while, once again, the associated cooling systems needed careful study by fluid-dynamic heat transfer theory. The same ingredients (chemical reactions, heat transfer and fluid dynamics, with the dynamics of mixing playing a dominant role) underlie many current developments in process engineering.

In the meantime the modern kinematic and kinetic analysis of machines and mechanisms has continued to make great progress, which has proved especially vital for the increasingly important technology of robotics. Gyro dynamics, again, has led to remarkable developments in control and navigation through the design of sensitive systems combining gyroscopes and accelerometers.

The modern mechanical engineer, to be sure, synthesizes knowledge from many different disciplines; including (for example) an even wider area of materials science than was touched on earlier in relation to structural engineering, and including also large parts of electronics that are essential for purposes both of instrumentation and control. Theoretical mechanics of solids and fluids continues, however, to form the fundamental basis of the mechanical engineer's art.

5. External Fluid Dynamics

The interaction of a solid body with an external fluid through which it moves is the subject of external fluid dynamics. Remarkable developments in external aerodynamics (the case where the fluid is air) were needed, in addition to several structural and power-plant developments that have been referred to earlier, to make possible the twentieth century's extraordinary achievements in aeronautical engineering.

Efficient horizontal flight requires the air pressures and viscous stresses around an aircraft to produce a resultant force with a large vertical component or *lift* to balance the weight, and yet with a low horizontal component or *drag* to be overcome by engine thrust. The viscosity of air is small but in external aerodynamics must not be neglected since classical theories on this assumption predict a zero resultant force on any solid body in steady motion through air. Accordingly, the discoveries by Prandtl and others, that well designed external shapes for aircraft could allow all the effects of viscosity to be confined to thin boundary layers and wakes, permitting drag to be low although (when the wakes contained intense trailing vortices) lift could be high, represented a vital first step forward in aerodynamic design; allowing major improvements in performance over that of the crude shapes of aviation's early years.

Later, after improvements in engines and in structural design as well as in aero-dynamics had raised aircraft speeds near to or above the speed of sound, aircraft began to experience an additional component of drag, associated with generation of an acoustic wave often called the 'sonic boom'. The corresponding feature in external hydrodynamics is observed when a ship's speed increases so that it experiences not only a wake-generating drag but also a *wave-making* drag. Wave theory for both acoustic waves in air and surface waves on water has been fruitfully applied to the study of shapes that will postponed, until substantially increased speeds, any significant rise in drag due to wave-making.

Other important parts of external fluid dynamics are concerned with the stability and control of the motion of aircraft through air or of ships or submarines through water. These parts achieved great success through applying the laws of dynamics to the solid vehicle in each case, taking into account fluid forces as affected by the vehicle's movement and by the action of control surfaces such as rudders, ailerons, etc.

At very high speeds, the integrity of an aircraft's structure may be threatened by frictional generation of heat in the boundary layer and this may once more demand careful heat transfer analyses. In the case of spacecraft re-entering the earth's atmosphere the problems are intensified and the analyses need to be at a very refined level.

Ingenious pressure-measuring and flow-visualisation techniques using copiously instrumented wind tunnels and other experimental facilities have been essential to the successes achieved in external fluid dynamics. Recently, the complete probing of a complicated velocity field in a fluid became possible through the development of Laser Doppler anemometry. Yet simultaneous improvements in computational fluid dynamics, based on major advances in fluid dynamic theory, have taken such a form that the balance of a designer's reliance on the twin supports of experimental data and theoretical analysis has continued to shift slowly but progressively in the direction of theory.

6. Planetary Sciences

After the astronomers of the sixteenth and early seventeenth centuries had amassed a remarkable body of accurate planetary observations, and Kepler had established a post-Copernican interpretation of these in terms of three empirical laws satisfied by the orbits of planets (and of their satellites), a first great success of the theoretical mechanics introduced by Newton was his demonstration that those laws must be interpreted in terms of an *inverse-square law* of gravitational attractive force. Much later, simultaneous attraction by more than one body was approximately allowed for, and used by Euler, Lagrange, Laplace and others to explain the principal departures from Kepler's laws which more accurate observations had brought to light.

The perturbation theory developed for this purpose proved invaluable after artificial satellites of earth were launched from 1959 onwards. Careful observation of the slow deviation of their orbits from those predicted by Kepler's laws allowed accurate determinations of the *perturbing forces* on a satellite (that is, forces other than simple attraction to the earth's centre). Important geophysical information was so obtained, including the best available data on departures of earth's shape from the spherical, and on winds in those highest parts of the atmosphere which include the orbits of many satellites.

Study of the lower parts of the earth's atmosphere is, of course, the province of meteorology. The invention of the barometer in the seventeenth century had allowed some empirical progress in the study of the weather, but subsequent advances rested firmly on knowledge of the theoretical mechanics of fluids in a *rotating frame of reference* and on improved understanding of the physics of moist air and of radiation. Admittedly, every big step forward in data acquisition, such as the radiosonde balloon, enlarged the potentialities of meteorology as a science; nevertheless, the data were only useful when interpreted in terms of ideas emanating in part from theoretical mechanics. Some of these ideas proved useful also in the study of planetary atmospheres.

The modern meteorologist is able, however, to go far beyond the simple introduction of ideas from fluid dynamics. The global collection of data from radiosondes and from meteorological satellites now permits *initial conditions* to be determined that allow a forward numerical integration of the partial differential equations for atmospheric motions (including the necessary radiation physics) to be carried out in finite-difference form and used successfully to calculate weather conditions a few days ahead. This rapidly developing field is one of today's particularly important areas of application of theoretical mechanics.

Similar stories can be told in two other branches of the study of *the earth's fluid envelope*: Oceanography, with its highly developed study of ocean tides and currents intimately related to hydrodynamic theory; and ionospheric dynamics, including the dynamics of the interaction of the earth's magnetosphere with the solar wind of charged particles emanating from the sun, Increasingly, the mechanics of the entire fluid envelope needs to be treated as a whole.

Solid-earth geophysics is also highly dependent on mechanics and on its well developed *seismological* theories, allowing information about the interior of the earth to be inferred from data gathered by sensitive seismographs on how waves generated by earthquakes propagate through it. Extensive knowledge of the material properties of the solid outer parts of the earth (the *crust* and *mantle*) were derived by these methods; which in addition, demonstrated the existence of the liquid core. Later, it became recognized that the magnetic field of the earth is generated by 'dynamo' motions within the electrically conducting core through mechanisms that in recent times have been in part clarified by magneto-fluid-dynamics. All of these studies of the earth's interior are found valuable also in investigations of the structure of the moon and of the other planets and their satellites.

7. Life Sciences

Beyond the widespread applications of mechanics to different branches of engineering and technology and also to planetary sciences that have been briefly mentioned so far, this note abstains from outlining the interactions of mechanics with other physical sciences; principally, because mechanical ideas have long been fully integrated within disciplines such as physics, physical chemistry and astrophysics (admittedly, with various important relativistic or quantum-theory modifications; and, to be sure, as rather modest parts of the whole in each case). Instead, this description of the applications of mechanics is concluded with an indication of how rapidly the life sciences, after, quite a slow start, have in the last decades been infiltrated by ideas from theoretical mechanics.

This was the period during which, first of all, the very diverse modes of animal locomotion were subjected to detailed analysis from the standpoint of the forces required and of how they are exerted by muscles and other motile organs. That work has comprehensively applied the principles of mechanics of solids to all of the different *gaits* (including creeping, walking, hopping, trotting, cantering, and galloping) used in terrestrial locomotion, Simultaneously, *aquatic locomotion* was being studied from the standpoint of hydrodynamics for a huge range of fishes and other vertebrate swimmers, for mobile crustaceans and molluscs, and for flagellar and ciliate microorganisms. Aerodynamics, again, was used to analyse the many and various modes of forward flight and its control, of takeoff and landing, and of climbing, soaring, diving and hovering used by birds, bats, and insects.

The biomechanics of bone and connective tissue was comprehensively investigated, with applications to the design of prosthetic and orthotic devices. Physiological fluid dynamics was developed, too, for the detailed analysis of *blood flow* in the cardiovascular system, of *air flow* in the respiratory tract, and of fluid motions in various specialised organs such as the ear. There are, furthermore, many other excellent possibilities for future applications of mechanics in the life sciences.

To summarize: The International Union of Theoretical and Applied Mechanics utilises its large International Congresses, along with its relatively smaller specialised Symposia and also its Joint Symposia held in cooperation with other international bodies active in engineering and the sciences, for three main purposes: to promote fundamental, mathematical and computational developments in theoretical mechanics; to advance those new experimental techniques that are needed for applied mechanics to be able to make progress in its diverse fields of application; and above all, to move towards the winning of yet more fruitful results from those powerful interactions between these two fields that have been briefly celebrated in this note.