A New Approach for Orthogonal Representation of Lunar Contour Maps

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Abstract. Impact craters are typical lunar areas which can reflect the characteristics of lunar surface, so the studies of them are one of the key tasks of lunar exploration. A class of complete orthogonal piecewise polynomials in L^2 [0,1] called V-system is introduced in this paper and this new approach can be used to represent the contour lines of lunar DEM data. It is not only introduced for accurately representing the contour lines but also eliminating effectively the Gibbs phenomenon. Based on V-system, there is an algorithm for transferring a given contour to V-spectrum. The proposed algorithm is intuitive, easy and fast. Some examples of lunar contour maps' representation in V-system are given.

Keywords: Contour · Orthogonal · V-system · Frequency spectrum

1 Introduction

In recent decades, lunar exploration reached a brand new height, the morphological study of the moon has been one of the major lunar explorations. Impact craters are the most significant features of the planets like moon and mars. It is formed by the high-speed meteorites which impact lunar surface. Impact craters are the breakthrough window of the study of planetary inside material. It can provide the most direct evidence about the research for the status and evolution of celestial body by studying the impact craters, also this evidence can apply to the mechanism of pit and the impact effects. Throughout all the previous human space exploration missions, the identification of impact craters has been one of the focus researches.

Remote sensing image is the most common data source while doing research of the moon and its relevant methods become more and more mature. However, the research based on Lunar DEM (digital surface model) is relatively rare and its theories and methods are immature. At present, the related methods, theories and analysis on Earth's terrain extraction have been a very high level, but the applicability and effectiveness don't have been confirmed yet while using them to the lunar exploration. Thus, the methods of lunar terrain extraction and the technology based on lunar DEM need further research.

Contour is an extensive application. It can be used in geography or even in military. The contour can be seen as the intersection line of the horizontal planes of different altitudes and the actual surface. With contour lines, we can know how undulating the surface is and the characteristics of the surface such as which part belongs to peak,

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basin, ridge, valley, cliff and so on. In other word, contour can reflect the characteristics of the undulating momentum of surface and its structure. There are several successful workarounds [1–4] in geometric representation, storage, transmission and so on. It is worth noting that geometric problems have great connection with image processing. For example, in pattern recognition, after extracting the contours of one geometric shape, next step is to do the matches according to its characteristics. That is why it is necessary to discuss effective methods to deal with the data from geometry and make the appropriate frequency spectrum analysis. Supposed there is a proper orthogonal complete function system which can be used to accurately represent most of parametric geometric shapes, this orthogonal complete function system can be used to represent the contour maps. The orthogonal representation of lunar contour map is conducive to comprehensive analysis of lunar data.

A special complete orthogonal function in $L^2[0,1]$ called V-system, its application in representation for lunar contour maps through DEM data is introduced. It mainly establishes a new representation of lunar contour maps based on V-system. This representation is accurate theoretically; based on it we can do the spectrum analysis of geometric shapes.

This paper firstly introduces V-system, illustrates its structure and proves it is feasible. Then it explains the algorithm of orthogonal representation. At last it gives lunar contour maps for example to calculate the corresponding frequency spectrums.

2 Related Work

V-system [4–6] was proposed in 2005 based on U-system [7–10]. It maintains excellent properties of U-system and it is more flexible and convenient comparing to U-system. V-system is hierarchical, the complete description is "k times V-system, k = 0,1,2,3…". The k times V-system is one kind of $L^2[0,1]$ orthogonal complete function system created by k times piecewise polynomials. It includes infinitely differentiable function and different levels of intermittent which is exactly conductive to the representation of geometric information.

Here we discuss the k times V-system for preparation. On the interval of [0,1], if a set of limited functions $\{f_i(x), i = 1, 2, \ldots, m\}$ satisfies the conditions below:

1. $f_i(x)$ is piecewise k times polynomial which is using x = 1/2 to be the nodal point, and m = k+1;
2. $\langle f_i(x), f_i(x)\rangle = \delta_{ij}$, i, j $\in \{1, 2, \ldots, m\}$;
3. $\langle f_i(x), x^j\rangle = 0$, $i \in \{1, 2, \ldots, m\}, j \in \{0, 1, \ldots, k\}$,

then $\{f_i(x), i = 1, 2, \ldots, m\}$ is the generating element. Here $\langle \cdot, \cdot \rangle$ means inner product of $L^2[0,1]$.

Here are the steps of creating k times V-system:

1. Taking the first k + 1 Legendre polynomials on interval [0,1] to be the k times V-system's first k + 1 functions, marked as $V_{k,1}^1, V_{k,1}^2, \ldots, V_{k,1}^{k+1}$.

2. Creating the generating element $\left\{V_{k,2}^{i}, i = 1, 2, \ldots, k + 1\right\}$ which includes k + 1 piecewise k times polynomials, it satisfies the conditions (1)–(3) just mentioned, which means every function is not only orthogonal to each other in $\left\{V_{k,2}^{i}\right\}$, but also is orthogonal to $V_{k,1}^{1}, V_{k,1}^{2}, \ldots, V_{k,1}^{k+1}$. After putting $V_{k,2}^{i}$ behind $\left\{V_{k,1}^{1}, i = 1, 2, \ldots, k + 1\right\}$ in proper order, there is $V_{k,1}^{1}, V_{k,1}^{2}, \ldots, V_{k,1}^{k+1}, V_{k,2}^{1}, V_{k,2}^{2}, \ldots, V_{k,2}^{k+1}$.
3. Creating the rest by "Compression – Translation": Starting from $V_{k,2}^{1}$, each function can create two new functions. Then putting them behind the functions which have just mentioned. The rest can be deduced by analogy. At last we can get the k times V-system (Fig. 1).

Down below are the first 16 functions of 3 times V-system:

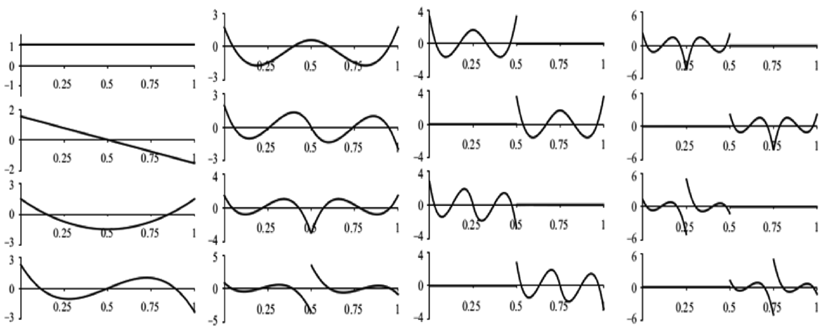

Fig. 1. 3 times V-system

Noticed that not every $L^2[0,1]$ orthogonal complete function is available for analysis of geometric information. The famous Fourier, Legendre and Chebyshev, all of them can't be used to represent geometric information because of Gibbs phenomenon (Fig. 2).

Here are some examples of reconstruction using Fourier and V-system. It is not hard to find out that Fourier results exist serious Gibbs phenomenon while doing the reconstruction of contour maps. In fact as long as the methods belong to successive orthogonal functions they all can't be used. Walsh and Haar are inappropriate to do finite term approximation because their strong discontinuity. These results show that V-system is a proper way to solve this problem. The properties of k times V-system are:

- It includes plenty of continuous and discontinuous information. In another word, it includes not only infinitely smooth functions, but also includes discontinuous functions of every level;
- Every piecewise polynomial can be accurately represented, this also called regeneration;
- Every function has local support and simple structure;
- The entire function system appears multi resolution.

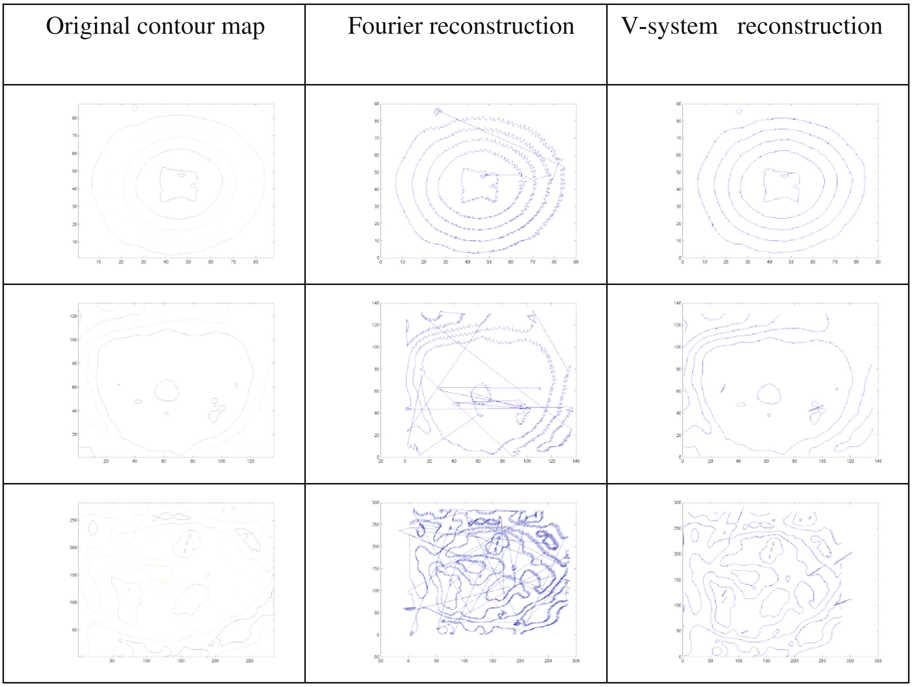

Fig. 2. Reconstruction

3 Algorithm of Orthogonal Representation for Contour Maps

To represent contour maps, the algorithm of orthogonal representation is introduced in this section. Let $t \in [0, 1]$, and supposed make [0,1] into 2n parts, the function which is approximated becomes to:

$$F_f(t) = f_i(t),\ t \in \left[\frac{i}{2^n}, \frac{i+1}{2^n}\right), i = 0, 1, \ldots, 2^n - 1, \tag{1}$$

with

$$\mathrm{P}(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = a_0 v_{0(t)} + a_1 v_{1(t)} + \ldots + a_{2^{n+1}-1} v_{2^{n+1}-1(t)}. \tag{2}$$

Change the given curve's representation into parametric way:

$$\begin{cases} x(t) = F_x(t) \\ y(t) = F_y(t) \end{cases}, \tag{3}$$

also have $a_j = \int_0^1 P(t)v_j(t)\,dt, \quad j = 0, 1, 2, \ldots, 2^{n+1} - 1$.

So far, as for a given $F_f(t)$, it can use P(t) to do precision representation with finite items, this also be called P(t) is the orthogonal representation of $F_f(t)$, $\{a_j | j = 1, 2, 3\ldots\}$ is $F_f(t)$'s V-spectrum. Because P(t) can accurately represent $F_f(t)$, there is eigenvalue defined as: $E = \left(\sum_{j=0}^{2^n - 1} a_j^2 \right)^{1/2}$.

Then let $b(j) = \frac{\|a_j\|}{\|a_1\|}$, $j = 1, 2, \ldots$, $b(j)$ is the j_{th} normalized descriptors. The normalized V-descriptors $b(j)$ are provided with translation, scaling and rotation invariance. Here is the proof:

Supposed there are a describe object P(t) and its V-descriptors a_j, if the translation amount is z_0, the level of scaling is β, the rotation angle is θ, after these transforms the object becomes to $\beta e^{i\theta}(P(t) + z_0)$, and its descriptors become

$$a_j' = \int_0^1 \beta e^{i\theta}(P(t) + z_0)v_j(t)\,dt = \beta e^{i\theta}\left[\int_0^1 P(t)v_j(t)dt + \int_0^1 z_0 v_j(t)\,dt\right] = \beta e^{i\theta}\left[a_j + z_0\delta(j)\right] \tag{4}$$

Here used

$$\int_0^1 v_j(t)\,dt = \delta(j) = \begin{cases} 0, j \neq 0 \\ 1, j = 0 \end{cases} \tag{5}$$

When $j \neq 0$, $a_j' = \beta e^{i\theta} a_j$, so $b'^{(j)} = b(j)$.

4 The Experimental Results and Analysis

This section provides some samples to explain the orthogonal representations of contour maps. Here we used DEM data from Chang-E 1 to revert the appearances of craters and marias, then we extracted their contour. After that V-system of degree 3 was used to represent these contour maps. One sample contains the original figure, the vertical view of figure, the contour map and frequency spectrum with or without orthogonalization. The sample arrangement will be shown at first, and then the followings are samples (Table 1 and Figs. 3, 4, 5, 6 and 7).

Table 1. Sample arrangement

Name of crater	Vertical view of figure	Contour map
Original figure	Frequency spectrum without orthogonalization	Frequency spectrum with orthogonalization

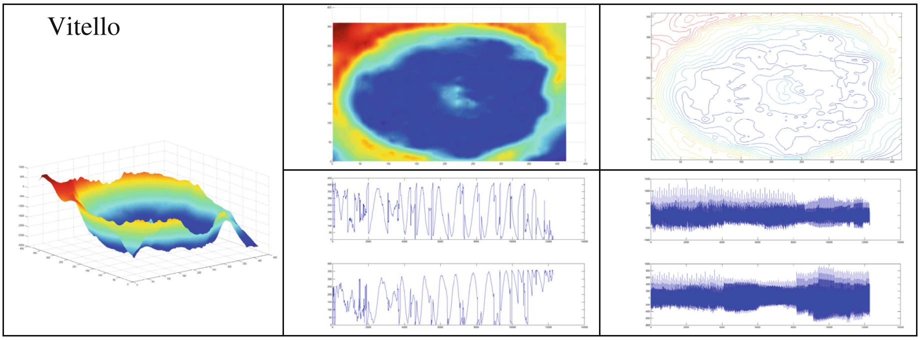

Fig. 3. Vitello

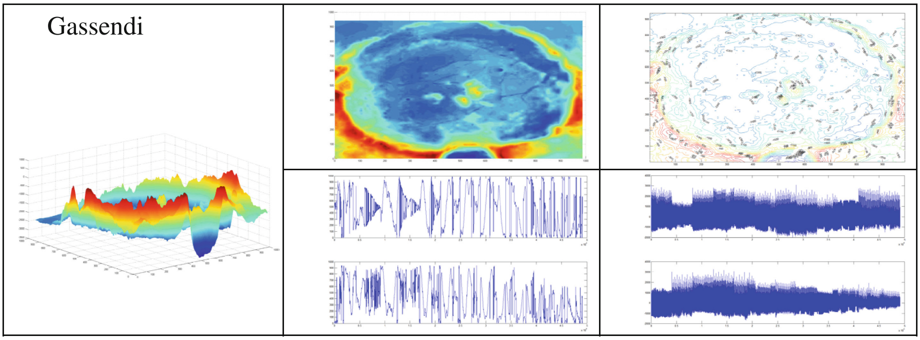

Fig. 4. Gassendi

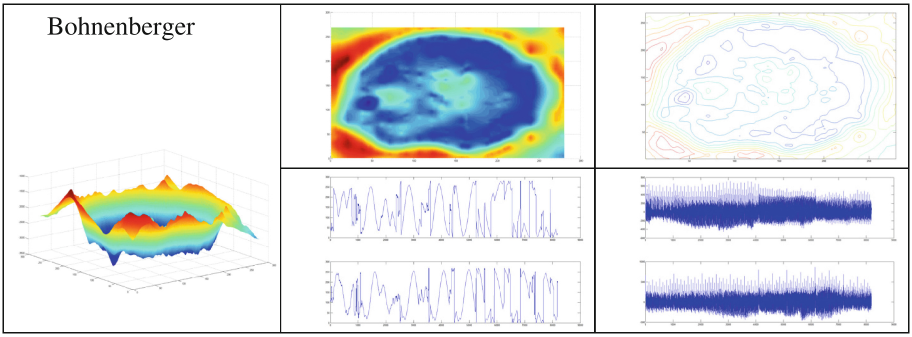

Fig. 5. Bohnenberger

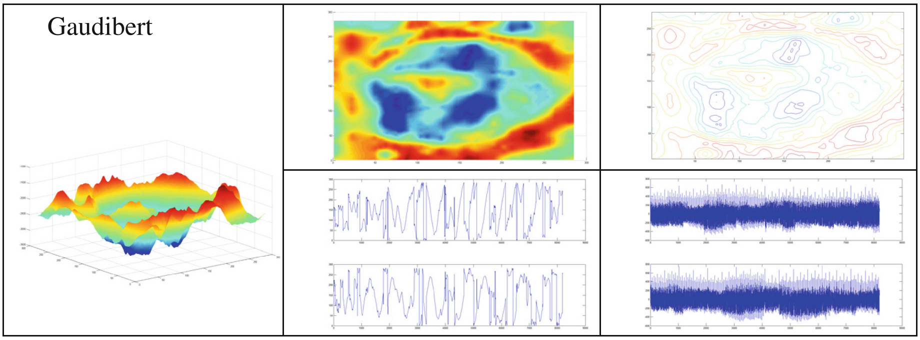

Fig. 6. Gaudibert

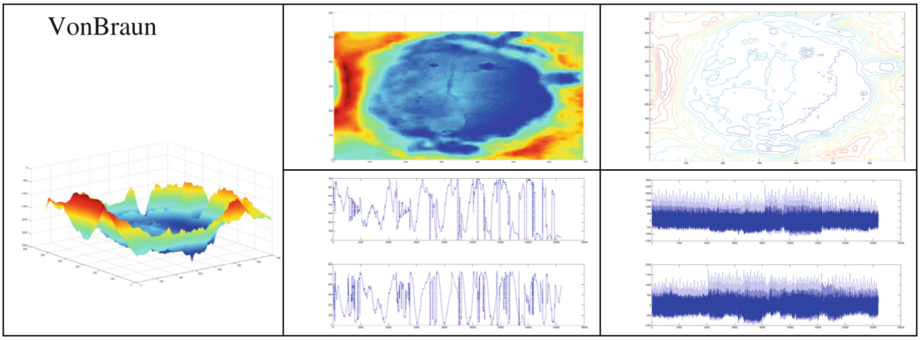

Fig. 7. VonBraun

5 Conclusion

In this paper, we have seen the effectiveness of information analysis and integrated application while using V-system to represent the lunar contour maps. The preliminary results indicate that V-system is one of the effective mathematical tools. Those orthogonal transform methods which are strong in signal processing, they can't accurately represent contour maps because they are highly smooth or strongly intermittent. The information conversion algorithm based on V-system shows that it is intuitive, convenient and fast. The works in this paper are some preparations for further research such as doing classifications and speculate the ages of craters through frequency spectrum, these are future work.

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References

1. Foley, J.D., et al.: Computer Graphics, Principles and Practice, 2nd edn. Engineering Industry Press, Beijing (2002)
2. Guo-Jin, W., et al.: Computer Aided Geometric Design. Higher Education Press, Beijing (2001)
3. Fa-Zhong, S.: Computer Aided Geometric Design and Non-Uniform Rational B-Spline. Higher Education Press, Beijing (2001)
4. Song, R., Ma, H., Wang, T., Qi, D.: Complete orthogonal V-system and its applications. Commun. Pure and Appl. Anal. **6**(3), 853 (2007)
5. Zhang, C.X., Miao, Z.H., Sun, F.M.: The algorithm for matching 2D contours based on V-system. In: Chinese Conference on Pattern Recognition, 2009. CCPR 2009 (2009)
6. Huang, C., Yang, L.H., Qi, D.X.: A new class of multi-wavelet bases: V-system. Acta Math. Sin. Engl. Ser. **28**(1), 105–120 (2011)
7. Feng, Y.-Y., Qi, D.-X.: A sequence of piecewise orthogonal polynomials. SIAM J. Math. Anal. **15**(4), 834–844 (1984)
8. Qi, D.-X., Feng, Y.-Y.: On the orthogonal complete system U. Acta Scientiarum Natur. Univ. Jilinensis **2**, 21–31 (1984)
9. Ma, H., Song, R.X., Qi, D.X.: Orthogonal complete U-system and its application in CAGD. In: 1st Korea-China Joint Conference on Geometric and Visual Computing (2005)
10. Qi, D., Tao, C., Song, R., Ma, H., Sun, W., Cai, Z.: Representation for a group of parametric curves based on the orthogonal complete U-system. Chinese J. Comput. (2006)