Nonlinear Dynamical Analysis of Eye Movement Characteristics Using Attractor Plot and First Lyapunov Exponent

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Abstract. The purpose of this study was to clarify eye movement characteristics during a visual search using nonlinear dynamics (chaos analysis). More concretely, the first Lyapunov exponent and the attractor plot were obtained for the time series data of *x*- and *y*-directional eye-gaze locations. An attempt was made to compare the first Lyapunov exponent and the attractor plot during a visual search task as a function of layout complexity of the display and to verify whether chaotic properties existed in the fluctuation of eye-gaze locations, and to examine how the scaling properties change as a function of the layout complexity. First Lyapunov exponent of the time series of eye-gaze locations took positive values, and tended to increase with the increase of search task difficulty (layout complexity). The attractor plot drew a trajectory like an ellipse, and the variation in attractor plots tended to be more complicated with the increase of task difficulty.

Keywords: Nonlinear dynamics · Eye movement · Attractor plot · First Lyapunov exponent · Layout complexity of display

1 Introduction

It has been widely known that nonlinear chaotic dynamics are ubiquitous in many biological systems such as Electroencephalography (EEG), body sway, and hear rate [1, 2]. Fairbanks and Taylor [3] proposed a method for measuring the scaling properties of temporal and spatial patterns of eye movements. Eye movements are classified into saccade, fixation, and micro-saccade. Micro-saccades generally occur over an angular range of typically 0.5° called dwell region. The characteristics of saccades are represented by ballistic jumps. It is regarded that saccades and micro-saccades are produced by different physiological mechanisms. Therefore, the nonlinear behaviors of these eye movements are expected to be different. It has not become clear whether the nonlinear characteristics are helpful for understanding eye movement further, and provide us with new information which the traditional analysis of eye movements cannot produce.

We look at some object to obtain information from it, and comprehend the situation around the object. Although eye gaze is relatively simple as compared with other communication means such as gesture or speech, it can provide us with abundant information on our perception and cognition so that we can conjecture our internal cognitive state.

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There are four types of eye movement. Eye gaze is typically directed to one location on a display for about 200–300 ms (this is called fixation), and then moves to another location extremely rapidly (in about 20–30 ms) (this is termed saccade). The angular rotation of saccade is about 600 deg/s. We are momentarily and effectively blind during the saccadic eye movement. Saccade jumps automatically to the location predetermined by the brain's visual system during the preceding fixations. If the movement is more than 15°, our head rotates automatically. Nystagmus is explained by reference to a common experience that of looking out of the window of a moving train and attempting to keep up with the view rather than find out some feature within the view. Nystagmus is a response to rapidly moving objects. The last eye movement is smooth pursuit to smoothly follow an object. However, it must be noted that there is a limit to the speed of such a movement.

The question of how people select fixation point as they move their eyes around a display in front of them might be addressed with reference to specific activities necessary for cognition. In daily life, nothing can be performed without the retrieval of information. The evaluation of eye movement characteristics and performance measures provides us with an important knowledge on the information strategy used in search tasks. Many studies on eye movement characteristics are conducted to clarify a variety of cognitive processes [5–14].

Murata et al. [4] investigated eye movement characteristics during a visual search using nonlinear dynamics (scaling properties) using the fractal dimensional analysis for the time series data of x- and y-directional gaze-locations. For both x- and y-directional eye movements, the scaling property represented by the fractal dimension tended to increase with the increase of difficulty Layout complexity) of a search task. The fractal dimension also tended to be smaller for the wide display than for the narrow display. On the basis of the result that the search time and the x- and y-directional fractal dimensions were not so strongly related, they conjectured that the search time and the fractal dimension stem from the different mechanism underlying a variety of search activities. However, they did not discuss whether chaotic behavior is observed in the fluctuation of time series of eye-gaze location.

The purpose of this study was to clarify eye movement characteristics during a visual search using nonlinear dynamics (chaos analysis). More concretely, the first Lyapunov exponent and attractor plot were obtained for the time series data of x- and y-directional gaze-locations. An attempt was made to compare the first Lyapunov exponent and attractor plot during a visual search task as a function of layout complexity of the display and to verify whether chaotic properties existed in the fluctuation of eye-gaze location, and to examine how the scaling properties change as a function of the layout complexity.

2 Method

2.1 Participants

Ten male undergraduate or graduate students from 22 to 24 years old took part in the experiment. All signed the document on informed consent after receiving a brief explanation of the aim and the contents of the experiment.

2.2 Apparatus

The eye movement during a search task was measured using an eye-tracker equipment (ViewTracker, DITECT). This eye-gaze measurement system makes use of infrared and visual camera technologies to determine the eye-gaze locations. The resolution of the computer display was 640 by 480 pixel. The sampling frequency of the eye-tracker equipment was 60 Hz.

2.3 Task, Design, and Procedure

The search task was to search for a target stimulus that consisted of three random letters. The layout complexity *LC* (Murata and Furukawa [15]) was calculated according to Eq. (1)

$$LC = -N\sum_{i=1}^{n} p_i \log_2 p_i \quad (1)$$

(a) *LC*=50bit

	1	2	3	4	5	6	7
1	IOB		ULK				
2	RTJ					DRX	
3		XOG					
4					JGV		
5							
6		PBE			EOA		BCX
7			MTH				

(b) *LC*=100bit

	1	2	3	4	5	6	7
1	IEU	EXC			YGH		RHG
2					HJI		
3	PBE				GAH		SZD
4		FAR	IZS	UAM			
5	LJW			BCX			
6		WZH		KTA			XOG
7		CQF	OTB	MYI			

(c) *LC*=150bit

	1	2	3	4	5	6	7
1		AJB	NYI		HNM	LJW	MRY
2		IOB		VZA	JWI		
3	RWV	YRJ				XJI	
4			DKV	FXT	ULK	QHJ	EFN
5		GIO			UXP	WAF	
6	TLA		OAC		BKL		KLU
7	CWJ		PBE	ZSP			SYT

(d) *LC*=200bit

	1	2	3	4	5	6	7
1	EOA	RTJ		OJY	IBG	GJK	HVT
2	ZNL		RHG	FTJ	UAM	BZN	
3	NUA				JGV		BCX
4	SRF	CDK	KTA	QZN		VWY	OTB
5	ALR	IEU	XOG	LKC		MHJ	
6	GAH		EXC	NDV	QRU	YTP	XSO
7	WUQ	PYR		LNA	DUK		TJM

Fig. 1. Display used in the search task. Each display (a)–(d) subtended vertically and horizontally 14.4° and 22.6°, respectively.

in which N is the number of objects on the display, n is the number of groups of similar objects, and p_i is the probability of selecting an object from group i. *LC*s for the horizontal and the vertical directions are calculated separately. In this study, four kinds of *LC*s (50 bits, 100 bits, 150 bits, and 200 bits) were used. The displays for *LC* of 50 bits, 100 bits, 150 bits, and 200 bits subtended vertically and horizontally 14.4° and 22.6°, respectively (For more detail, see Murata et al. [4]). In Fig. 1, the displays for *LC* = 50, 100, 150, and 200 bits are exemplified.

In this experiment, *LC* was a within-subject factor. The order of performance of four conditions of *LC* was randomized across the participants. Eye movements during the search task of each condition were recorded ten times for each participant.

The time series data were linearly interpolated so that data length is larger than 1000, and entered into R language package (tseries Chaos) in order to obtain first Lyapunov exponent and attractor plot.

3 Results

Figure 2 shows first Lyapunov exponent compared among *LC* conditions for embedding dimensions 3, 4, and 5. Figure 3 shows first Lyapunov exponent compared among embedding dimensions for each *LC* conditions. Here, the time series of x-directional eye gaze locations were used to calculate first Lyapunov exponent. The attractor plots for *LC* = 50, 100, 150, and 200 bits are shown in Figs. 4, 5, 6, and 7, respectively. As for the y-directional eye-gaze locations, similar results were obtained.

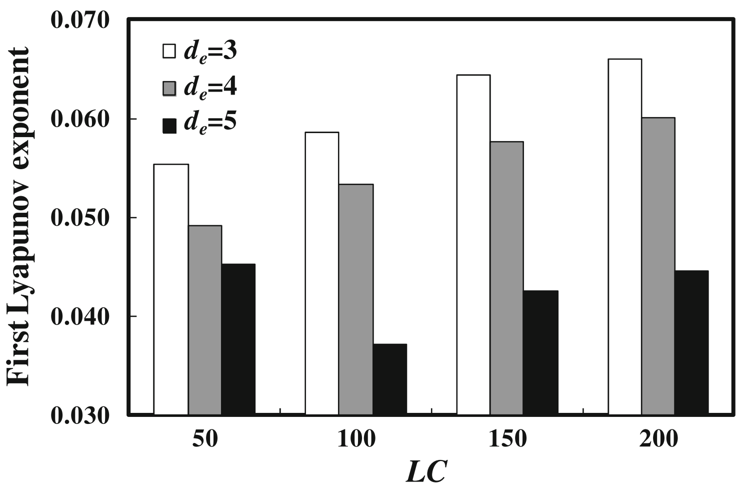

Fig. 2. First Lyapunov exponent compared among *LC* conditions for each embedding dimension.

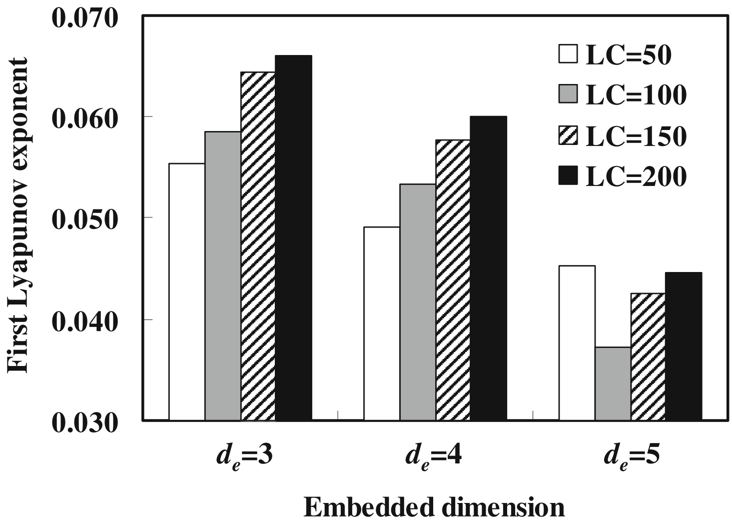

Fig. 3. First Lyapunov exponent compared among embedding dimensions for each *LC* conditions.

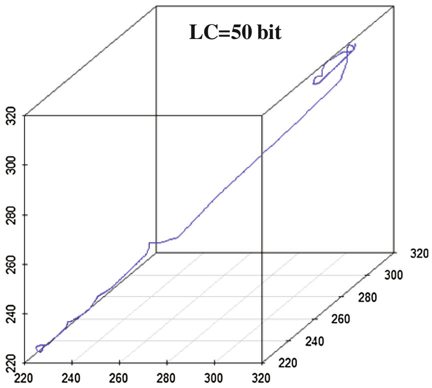

Fig. 4. Attractor plot (*LC* = 50 bits)

4 Discussion

The positive value of first Lyapunov exponent generally shows that the time series of eye-gaze locations include chaotic property. The chaotic phenomenon is characterized by two properties, that is, orbital instability and unpredictability. The larger first Lyapunov exponent is, the larger the width of variation of data gets larger. It was clarified that the search time, the x-directional and the y-directional first Lyapunov exponents tended to increase with the increase of search task difficulty (layout

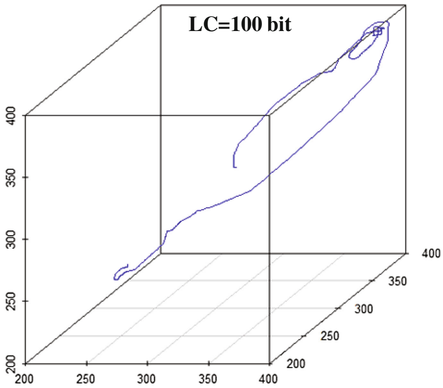

Fig. 5. Attractor plot (*LC* = 100 bits)

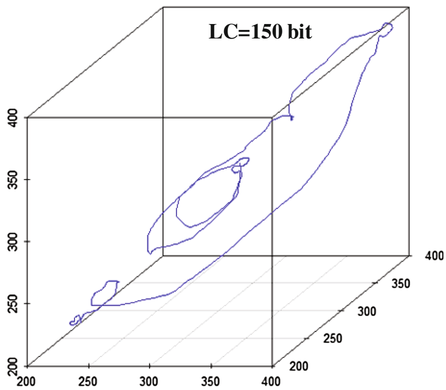

Fig. 6. Attractor plot (LC = 150 bits)

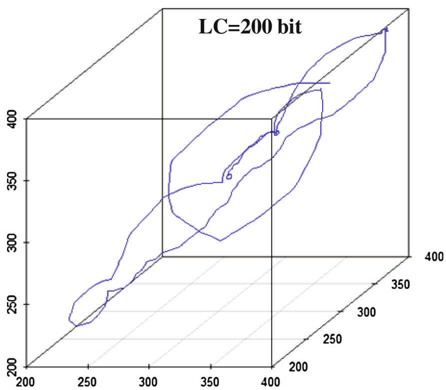

Fig. 7. Attractor plot (LC = 200 bits)

complexity). Therefore, it was investigated how the search time was related to the *x*- and *y*-directional first Lyanunov exponent.

The search time and the first Lyapunov exponent were not so strongly correlated. This must mean that the first Lyapunov exponent is variable based on different mechanism from the search time. The *x*- and *y*-directional first Lyapunov exponents were strongly correlated. Like the fractal dimension in Murata et al. [4], the variation behind the first Lyapunov exponent must be different from that of the search time, which means that the first Lyapunov exponent must be one of the important indices to get further insight into human's eye movement.

As for the attractor plot, in accordance with the tendencies of first Lyapinov exponent above, the width of variation in attractor plots tended to be wider and more complicated with the increase of task difficulty. As a whole, it seems that the attractor plot draws a trajectory like an ellipse as in Figs. 4, 5, 6, and 7. The ellipse-like trajectory got more and more complicated with the increase of *LC*.

Future research should examine the scaling properties of more practical visual activities such as Web search and conjunction search in order to generalize the scaling properties in eye movements.

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