A Novel Penta-Valued Descriptor for Color Clustering

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Abstract. This paper proposes a new color representation. This representation belongs to the penta-valued category and it has three chromatic components (*red, blue and green*) and two achromatic components (*black* and *white*). The proposed penta-valued representation is obtained by constructing a fuzzy partition in the RGB color space. In the structure of the penta-valued representation, it is defined the well known negation operator and supplementary, two new unary operators: the dual and the complement. Also, using the Bhattacharyya formula, it is defined a new inter-color similarity. Next, the obtained inter-color similarity is used in the framework of k-means clustering algorithm. On this way, it results a new color image clustering method. Some examples are presented in order to prove the effectiveness of the proposed multi-valued color descriptor.

Keywords: Key words: penta-valued representation, color similarity, intuitionistic fuzzy sets, color clustering, fuzzy color space.

1 Introduction

A color image generally contains tens of thousands of colors. Therefore, most color image processing applications first need to apply a color reduction method before performing further sophisticated analysis operations such as segmentation. The use of color clustering algorithm could be a good alternative for color reduction method construction. In the framework of color clustering procedure, we are faced with two color comparison subject. We want to know how similar or how different two colors are. In order to do this comparison, we need to have a good coordinate system for color representation and also, we need to define an efficient inter-color similarity measure in the considered system. The color space is a three-dimensional one and because of that for a unique description there are necessary only three parameters. Among of the most important color systems there are the following: *RGB*, *HSV*, *HSI*, *HSL*, *Luv*, *Lab*, *I1I2I3*. This paper presents a system for color representation called *rgbwk* and it belongs to the multi-valued color representation [14]. The presented system is obtained by constructing a penta-valued fuzzy partition in *RGB* color space. The sum of the parameters r,g,b,w,k verifies the condition of partition of unity and we can apply the Bhattacharyya similarity. Thus, one obtains a new calculus formula for inter-color similarity/dissimilarity. The paper has the following structure: Section 2 presents the construction modality for obtaining of the penta-valued color

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representation *rgbwk*, the inverse transform from the *rgbwk* color system to *RGB* one, and the definition in the framework of the proposed color representation for the unary operators like negation, dual and complement. The definition is accompanied by some color and image examples; Section 3 presents the new similarity/dissimilarity formulae based on the *rgbwk* components and an extension of the k-means algorithm for color clustering. The presentation is accompanied with some experimental results. Finally, Section 4 outlines some conclusions.

2 The Construction of a Penta-Valued Color Representation

For representing colors, several color spaces can be defined. A color space is a definition of a coordinate system where each color is represented by a single vector. The most commonly used color space is *RGB* [6]. It is based on a Cartesian coordinate system, where each color consists of three components corresponding to the primary colors *red*, *green* and *blue*. Other color spaces are also used in the image processing area: linear combination of *RGB* (similar to *I1I2I3* [9]), color spaces based on human color terms like *hue*, *saturation* and *luminosity* (similar to *HIS* [4], *HSV* [15], *HSL* [8]), or perceptually uniform color spaces (similar to *Lab* [5], *Luv* [3].

2.1 The Fuzzy Color Space rgbwk

We will construct this new representation starting from the *RGB* (*red*, *green*, *blue*) color system. We will suppose that the three parameters take value in the interval [0,1]. We will define the maximum *M*, the minimum *m*, the luminosity *L* [12] and the saturation *S* [10]:

$$M = \max(R, G, B) \tag{1}$$

$$m = \min(R, G, B) \tag{2}$$

$$L = \frac{M}{1 + M - m} \tag{3}$$

$$S = \frac{2(M - m)}{1 + |m - 0.5| + |M - 0.5|} \tag{4}$$

Firstly, we will define a fuzzy partition with two sets: the fuzzy set of chromatic colors and the fuzzy set of achromatic colors. These two fuzzy sets will be defined by the following two membership functions:

$$c = S \tag{5}$$

$$a = 1 - S \tag{6}$$

We obtained the first fuzzy partition for the color space:

$$c + a = 1 \tag{7}$$

We can say that c is the index of chromaticity and a is the index of achromaticity. Next in the framework of the chromatic colors, we will define the reddish, bluish and greenish color sets by the following formulae:

$$r = \frac{(R-G)_+ + (R-B)_+}{1 + |m - 0.5| + |M - 0.5|} \tag{8}$$

$$g = \frac{(G-R)_+ + (G-B)_+}{1 + |m - 0.5| + |M - 0.5|} \tag{9}$$

$$b = \frac{(B-R)_+ + (B-G)_+}{1 + |m - 0.5| + |M - 0.5|} \tag{10}$$

where $(x)_+ = \max(x, 0)$. There exists the following equality:

$$r + g + b = c \tag{11}$$

After that, in the framework of the achromatic colors, we define two subsets: one related to the white color and the other related to the black color:

$$w = a \cdot L \tag{12}$$

$$k = a \cdot (1 - L) \tag{13}$$

There exists the following equality:

$$w + k = a \tag{14}$$

From (7), (11) and (14) it results the subsequent formula:

$$r + g + b + w + k = 1 \tag{15}$$

We obtained a penta-valued fuzzy partition of the unit and in the same time we obtained a penta-valued color descriptor having the following five components: r (*red*), g (*green*), b (*blue*), w (*white*) and k (*black*). We must observe that among the three chromatic components r, g, and b at least one of them is zero, explicitly $\min(r, g, b) = 0$.

2.2 The Inverse Transform from rgbwk to RGB

In this section, we will present the calculus formulae for the *RGB* components having as primary information the *rgbwk* components. Firstly we will compute the *HSL* components and then the *RGB* ones. Thus for the calculus of luminosity *L*, we will use the achromatic components *w,k*.

$$L = \frac{w}{w + k} \tag{16}$$

For the calculus of saturation S and hue H, we will use the chromatic components *r,g,b*.

$$S = r + g + b \tag{17}$$

$$H = \arctan 2\left(\frac{(\omega_G - \omega_B)}{\sqrt{2}}, \frac{2\omega_R - \omega_G - \omega_B}{\sqrt{6}}\right) \tag{18}$$

where

$$\omega_R = r + \min(r, b + g) \tag{19}$$

$$\omega_G = g + \min(g, b + r) \tag{20}$$

$$\omega_B = b + \min(b, r + g) \tag{21}$$

For the *RGB* components, we have the following formulae:

$$R = (M - m)\frac{\omega_R}{S} + m \tag{22}$$

$$G = (M - m)\frac{\omega_G}{S} + m \tag{23}$$

$$B = (M - m)\frac{\omega_B}{S} + m \tag{24}$$

The parameters M and m can be determined solving the system of equations (3) and (4) and taking into account (16) and (17).

2.3 The Negation, The Dual, The Complement in the rgbwk and RGB Spaces

In the following we consider the negation of color $Q = (R, G, B)$, namely $\overline{Q} = (1 - R, 1 - G, 1 - B)$. Using (8), (9), (10), (12) and (13) it results:

$$\overline{r} = \frac{(G - R)_+ + (B - R)_+}{1 + |m - 0.5| + |M - 0.5|} \tag{25}$$

$$\overline{g} = \frac{(R - G)_+ + (B - G)_+}{1 + |m - 0.5| + |M - 0.5|} \tag{26}$$

$$\overline{b} = \frac{(R - B)_+ + (G - B)_+}{1 + |m - 0.5| + |M - 0.5|} \tag{27}$$

$$\overline{w} = a \cdot (1 - L) \tag{28}$$

$$\overline{k} = a \cdot L \tag{29}$$

We must highlight that the pair $(R, \overline{R})$ defines a fuzzy set [16] for the reddish color and it verifies the condition of fuzzy sets, namely $R + \overline{R} = 1$. The pair $(r, \overline{r})$ defines an Atanassov's intuitionistic fuzzy set [1] for the reddish colors and it verifies the condition $r + \overline{r} \le 1$. In addition, it results the hesitation index $i_r = 1 - r - \overline{r}$. Similarly, the pair $(b, \overline{b})$ defines an Atanassov's intuitionistic fuzzy set for bluish colors, the pair $(g, \overline{g})$ defines an Atanassov's intuitionistic fuzzy set for greenish colors while the pair (w, k) defines an Atanassov's intuitionistic fuzzy set for the white color. Thus for the color $Q = (0.3, 0.5, 0.8)$, one obtains the fuzzy set $R = 0.3$ and $\overline{R} = 0.7$, while for intuitionistic fuzzy description one obtains $r = 0$, $\overline{r} = 0.53$. The intuitionistic description is better than fuzzy description because the color Q is a bluish one and then reddish membership degree must be zero. More than that for the white color $Q = (1,1,1)$ one obtains for the fuzzy set description $R = 1$ and $\overline{R} = 0$ while for intuitionistic description one obtains $r = 0$, $\overline{r} = 0$ and $i_r = 1$. Again, the intuitionistic description is better than the fuzzy one. Thus the fuzzy description is identically with that of the red color while in the framework of intuitionist fuzzy description, the intuitionistic index is 1. This value is a correct value for an achromatic color like the white color. In addition to the *negation* $\overline{q} = (\overline{r}, \overline{g}, \overline{b}, \overline{w}, \overline{k})$ we can define the *dual* [13], where only the achromatic components are negated, namely $\ddot{q} = (r, g, b, \overline{w}, \overline{k})$ and the *complement* [13], $\tilde{q} = (\overline{r}, \overline{g}, \overline{b}, w, k)$ where only the chromatic components are negated. In the *RGB* space the *dual* $\ddot{Q}$ and the *complement* $\tilde{Q}$ are defined by the following formulae:

$$\ddot{R} = 1 + R - M - m, \quad \ddot{G} = 1 + G - M - m, \quad \ddot{B} = 1 + B - M - m \tag{30}$$

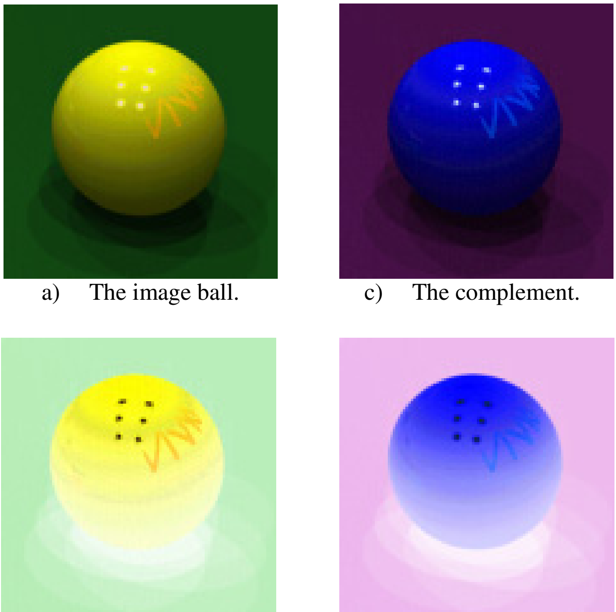
$$\tilde{R} = M + m - R, \quad \tilde{G} = M + m - G, \quad \tilde{B} = M + m - B \tag{31}$$

Figure 1 shows the image "ball" and its complement, dual and negation. In figure 2 we can see the colors Q1, Q2, Q3 in the first row, their negation in the second row, the dual in the third row, the complement in the forth row and the penta-valued representation in the fifth row.

3 Color Clustering in the rgbwk Space Using the Bhattacharyya Similarity

For any two colors $q_1 = (r_1, g_1, b_1, w_1, k_1)$, $q_2 = (r_2, g_2, b_2, w_2, k_2)$ we compute the Bhattacharyya similarity [2]:

$$F(q_1, q_2) = \sqrt{r_1 r_2} + \sqrt{g_1 g_2} + \sqrt{b_1 b_2} + \sqrt{w_1 w_2} + \sqrt{k_1 k_2} \tag{32}$$



a) The image ball. c) The complement.

b) The dual. d) The negation.

Fig. 1. The image ball

and its dissimilarity:

$$D(q_1, q_2) = \sqrt{1 - F(q_1, q_2)} \tag{33}$$

Using the dissimilarity defined by (33) in the framework of k-means algorithm, one obtains a color clustering algorithm. The algorithm k-means [7], [11] is one of the simplest algorithms that solve the clustering problem. The procedure classifies a given data set through a certain number of clusters fixed a priori. The main idea is to define k centroids, one for each cluster. The next step is to take each point belonging to the data set and associate it to the nearest centroid. After that, the cluster centroids are recalculated and k new centroids are obtained. Then, a new binding has to done between the same data set points and the nearest new centroid. A loop has been generated. This algorithm aims at minimizing an objective function, in this case a squared error function. The objective function is defined by:

$$J = \sum_{j=1}^{k} \sum_{i=1}^{n} D^2(x_i^{(j)}, c_j) \tag{34}$$

where $D^2(x_i^{(j)}, c_j)$ is a chosen dissimilarity measure between a data point x_i^j and the cluster center c_j. The function J represents an indicator of the dissimilarity of the n data points from their respective cluster centers. Using in (34) the dissimilarity (33),

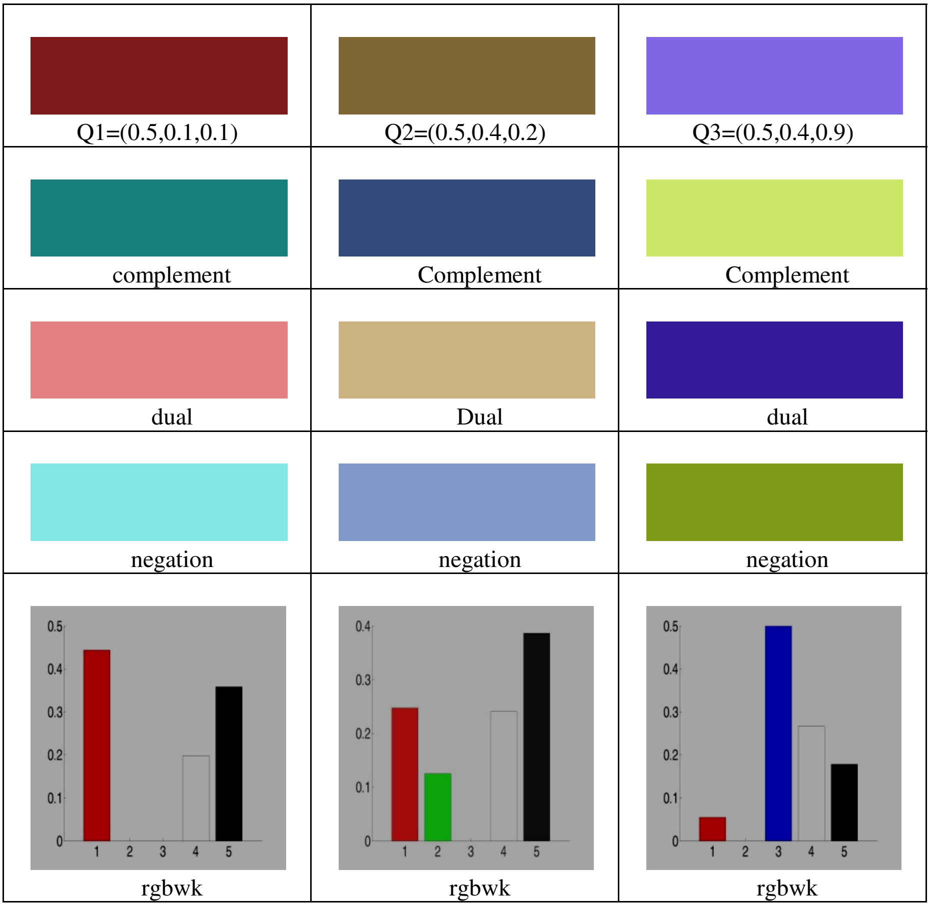


Fig. 2. The colors Q1, Q2 and Q3

we obtained the experimental results shown in figures 3, 4 and 5. Figure 3 shows the synthetic image “cyan-magenta” and its clustered variants. In the figure 3f and 3g we can see the strong asymmetry for the *Lab* and *Luv* systems. Also in the case of *HSL* system (figure 3c), the grey color does not appear in the right-bottom corner. We remark the strong symmetry for the variant obtained using the *RGB* system. The systems *HSI*, *HSV*, *I1I2I3*, *RGB* and *rgbwk* supplied the same four colors but having different distribution on the image support. For the image “bird” shown in figure 4, only in the case (i) for the *rgbwk* system the orange color was separated. For the image “bird”, the uniform green background was well sepaarted for the *HIS*, *HSV*, *Lab*, *Luv* and *rgbwk* color systems. For the image “flower” shown in figure 5, the orange color was separated in the case (h) for the *Lab* system and in the case (i) for the *rgbwk* system. For the image “flower”, the uniform gray background was well separated using the *Lab*, *Luv* and *rgbwk* color systems.

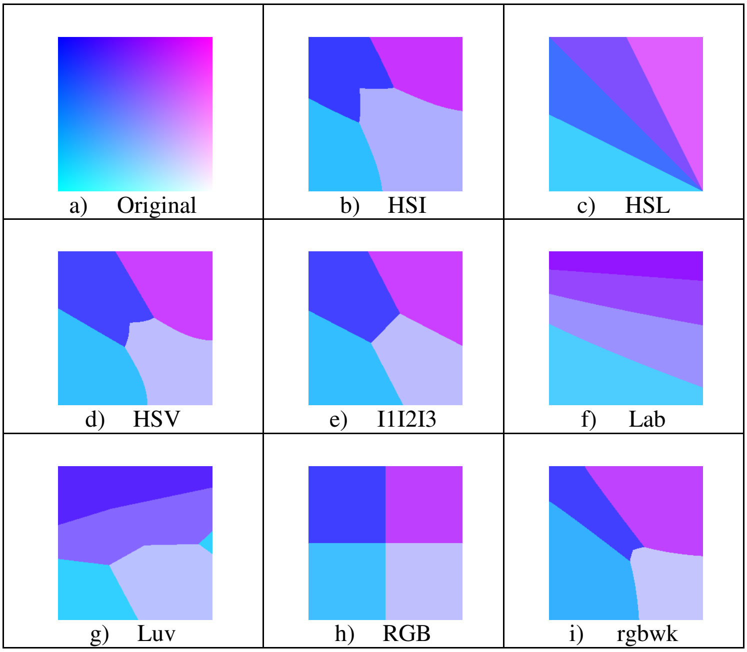


Fig. 3. The image Cyan-Magenta

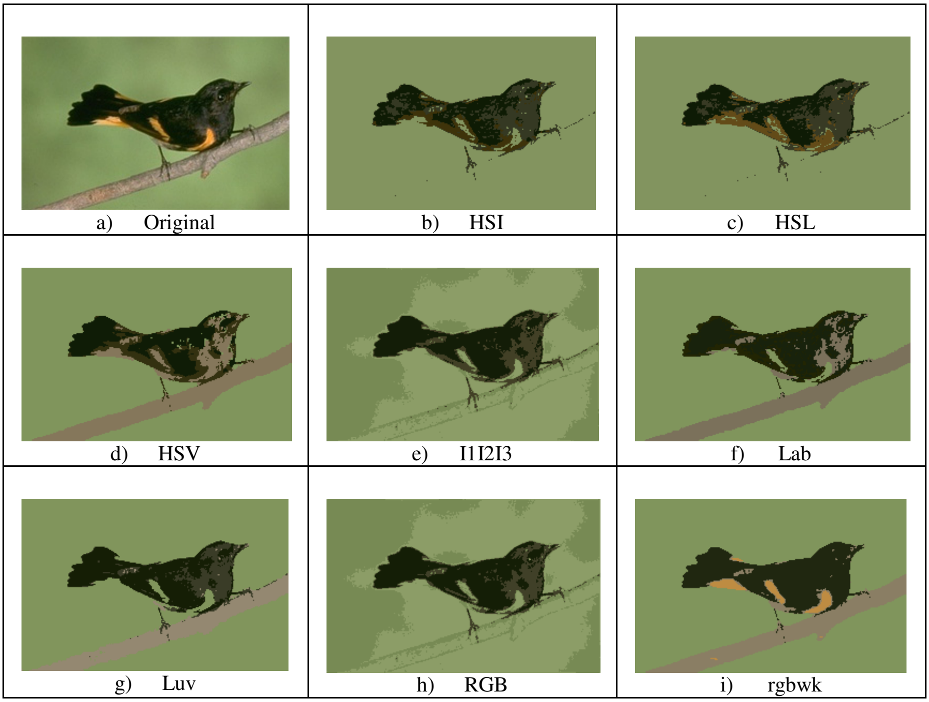


Fig. 4. The image bird

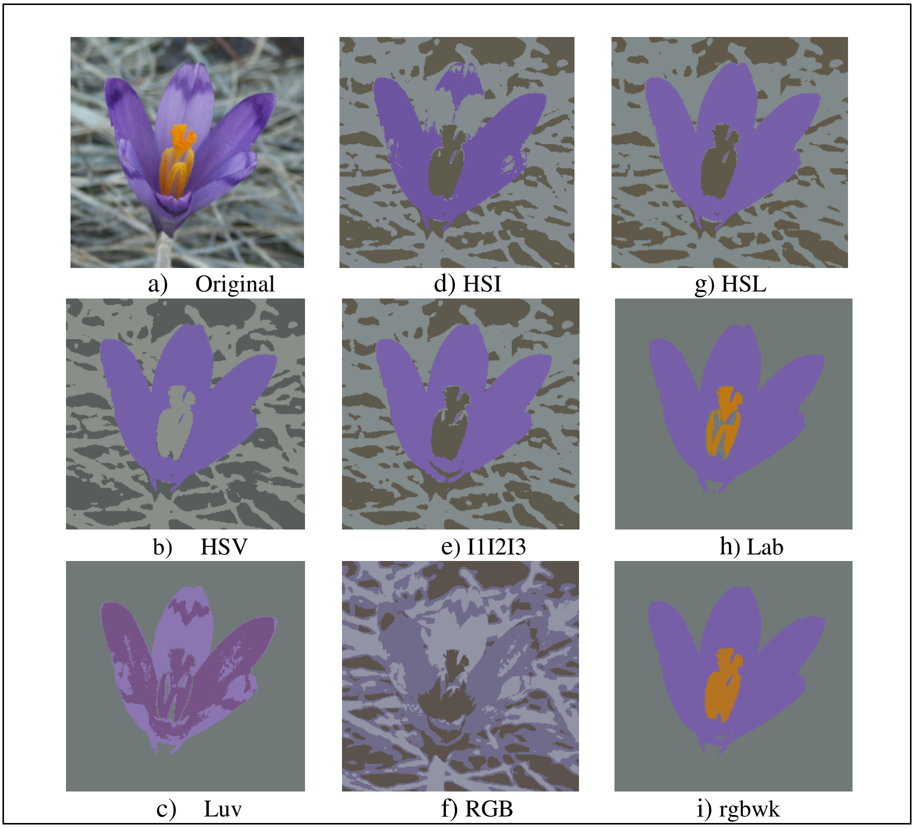


Fig. 5. The image flower

4 Conclusions

A fuzzy color space, *rgbwk* which is useful in the color image analysis is introduced. The semantic of the five values defining a color in this space is the amount of *red, green, blue, white* and *black* necessary to provide the color. The transformation from *RGB* to *rgbwk* turns out to be very simple.

The similarity/dissimilarity formula using the five parameters *r,g,b,w,k* is also introduced and also two new unary operators are defined: the *dual* and the *complement*.

The hue and saturation can be retrieved from the chromatic components *red, green* and *blue* while the luminosity can be retrieved from the achromatic components *white* and *black*. Experimental results verify the efficiency of *rgbwk* fuzzy color space for color clustering.

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