Development of Teaching Material Volume Calculations Using a Wooden Puzzle

Takamitsu Tanaka[1], Masao Tachibana[1], and Ichiro Hirata[2]

[1] Iwate University, Japan
[2] Hyogo Prefectural Institute of Technology, Japan
{taktak,tatimasa}@iwate-u.ac.jp, ichiro@hyogo-kg.jp

Abstract. This paper explains the development of a wooden puzzle containing wooden joints designed to teach elementary school students how to calculate volume. An experiment was performed with the puzzle, which is a cube with wooden joints, targeting 1st-grade students. The shape of the puzzle is known to be difficult for children to work with. Next, we introduce the results of a test of volume calculation designed for children and describe the methods used to teach volume calculation in Japan. Finally, we compare the concepts of volume calculation and how to create a three-dimensional object in 3 D–CAD to reveal that the ideas are almost identical.

Keywords: Wooden Joint, Wooden Puzzle, Volume Calculation, Design.

1 Introduction

In 2008, the authors carried out a study of a toy with a wooden joint. There are more than 200 different wooden joints used in furniture and architecture in Japan. This research represents the first time a wooden joint has been applied to a toy. As we experimented making toys, we were contacted by an elementary school mathematics teacher who expressed interest in using the toy to teach students how to calculate volume. With this background, the current investigation began in 2013.Children were recruited to participate in the experiment. We used 3D-CAD to design puzzles with five different types of cornerstones and then created a three-dimensional (3D) object using a 3D printer. The concept of volume calculation learned in elementary school and the design process of the 3D-CAD puzzles are similar. Therefore, this study describes the basic idea for studying volume calculation using digital learning materials and calculating volume through stereolithography.

2 Experiments Puzzle

The toys can be easily assembled due to the simplicity of the joints and learning to calculate the volume of a solid cube or rectangle can be done by 5th or 6th grade pupils in Japan. The first step is to learn the theory of volume and assembling a solid

S. Yamamoto (Ed.): HIMI 2014, Part II, LNCS 8522, pp. 109–117, 2014.

cube or rectangle can help. Then, the calculation of the volume of a rectangular parallelepiped several associated therewith to be learned how to calculate the volume of basic. There are a number of ways elementary school teachers can teach volume calculation in math classes. In current textbooks, figures are only shown in two dimensions making complex volume calculations difficult. Therefore, elementary school teachers must either teach volume calculation in the classroom, or develop a model of teaching materials for children to learn on their own. Currently, few wooden toys are available for this area of teaching. Of the types that are available, some are color-coded and can be built to show volume from 1㎤ up to1000㎤. These teaching materials are sufficient when dealing with simple volume calculations but inadequate in slightly more complex computational problems. Because of this problem, using a blackboard illustration to explain a complicated shape does not provide the necessary information. Teachers can instruct students on how to calculate a line as depicted in Fig. 1.

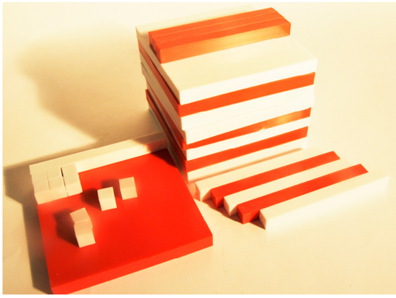

Fig. 1. Materials volume calculations are currently used

Fig. 2. Experimental observation of students

The process of this calculation method is considered similar to the process of assembling a wooden puzzle. The research was conducted in the hope that students may be able to learn volume calculation. The first puzzle discussed is a wooden cube. This type of puzzle is freely available in many countries and is a popular game. When assembling the components, the user can understand how the cube goes together. This type of product is easy to manufacture and looks attractive. As the shape of the prototype of the three-dimensional object is easy to understand, the production of a wooden cube puzzle was examined. In addition, an observation and experiment model for the experimental observation is described. When designing the puzzle for this study, 3D-CAD was used in order to save time. The puzzle was designed with 7 parts, each side measuring 6 cm after assembly. The design was based on the knowledge of how children play and it was decided to use only a small number of parts so that volume calculation would be easier. Elementary schools' create a modeling many kinds of component count is large, but it is expected that variations of modeling is extremely reduced component count is less. On the other hand, it can be inferred that assembling the cube and is easy number of parts is small. The students observed in this study were from 2nd grade and 5th grade and given 15 minutes to complete the puzzle. The results from this observation were as follows: no students from 2nd grade were able to complete the puzzle. However, one 5th grade student was able to complete the puzzle without any help and a further two students finished when given some instructions.

3 Current Status of Teaching Methods of Volume Calculation in Japan

3.1 Teaching System Volume Calculation

Japanese education on system volume calculation presents two calculation methods: "Measure the volume using an instrument" and "Use other ways to determine the volume." The strategy "Use other ways to determine the volume" consists of two sub-strategies: learning formulas for the volume of shapes such as cuboids, cubes, pillars, cones, spheres, etc., and calculating the volume of complex figures that represent a combination of cubes or rectangular solids. This study addresses volume calculation for complex figures.

3.2 Teaching about Volume

This section describes the methods used to teach about volume and volume calculation in Japanese schools, which starts in first grade.
As with length or area, students are taught to measure volume in four stages:

(A) Direct comparison
(B) Non-direct comparison
(C) Measurement in non-standard units
(D) Measurement in standard units

1st-grade students study (A) to (C), and 2nd-grade students study (D). Specifically, 1st-grade students participate in activities such as comparing volume and receive guidance from the teacher to enrich their learning experience, which forms the basis of their understanding of measurement and volume. 1st-grade students also engage in learning activities to compare methods (A) to (C).In 2nd grade, instead of using method (C) to measure volume, students learn method (D). That is, they learn how to take accurate measurements using a standard unit. Further, the volume of the water in the handle can be obtained by using subtraction or addition to calculate the sum or difference of the volume of water entering the two containers. The main purpose of this lesson is that the amount of storage stability holds for volume. Then, students will also understand that addition and subtraction may be applied. This learning forms a basis for determining the area and volume of a complex figure. Thus, in 1st and 2nd grade, students learn primarily through direct experience and activities, for example, by measuring and comparing in detail the amount of liquid in containers. Finally, they learn what represents the standard unit of measurement for liquid volume.

Next, I will describe how students learn to measure volume. In 3rd and 4th grade, students are taught to measure volume using fractions. Beginning in 3rd grade and continuing in 4th grade, they learn to use one and then two decimal places. In 4th grade, they also practice using liters (L) and deciliters (dL) as the standard units for measuring volume. They use a measurement instrument and express their measurements in a standard way. However, instead of measuring volume in 5th grade, students are taught to obtain the volume using calculation. Prior to this, in 4th grade, they learn about calculating the size of rectangles and squares. They also study the standard units used to represent area. In 5th grade, the students learn the concepts of unit volume. At this point, they study how to quantify the volume of many cubes (e.g., 1 cm3) and then progress to learning the formulas for calculating the volume of a cube or rectangle. Thus, in 5th grade, students clearly understand the idea of volume, volume measurement, and the use of standard measurement units. This allows them to capture the ideas used in integrated learning and volume calculation. Then, to learn how to quantify volume using an activity to investigate the volume of a cube or rectangular solid, the students are shown building blocks that are each a 1 cm cube and asked to consider how many sides there are. Through this study, students learn the formulas “volume = side × side × side of the cube" and "volume = length × width × height." Further, in the 6th grade, students are taught to determine the volume of cylindrical or prismatic shapes. In this case, they apply their knowledge of the formula "volume = base area × pillar of the body. "The volume of a cone is one-third that of a pillar with the same height and bottom area. This I learned for the first time in a middle school mathematics department. The volume of a sphere is two-thirds of the volume of the cylinder. In junior high school, as confirmed by this experiment, students learn the formula for the volume of a sphere and cone. To teach students how to calculate the volume of a cuboid, cube, pillar, cone, or sphere, teachers guide them in advanced thinking with the goal of enabling the students to “create a formula for the volume of their own shape." However, in the current way of teaching about volume, there is no formula for the volume of an original figure. Therefore, students need to learn how to determine the volume of complex figures. The most important method for obtaining the volume of a complex figure involves utilizing previously learned methods.

3.3 The Basic Complex Figure

As shown above, by 5th grade, students should learn about rectangles and cubes and know how to find the volume of a complex figure. However, our investigation of students’ actual performance shows a tendency in regard to the only complex figure given. According to the survey results, the children have difficulty finding the volume of basic shapes. In 2013, the Iwate Prefectural Board of Education conducted a survey of 5th grade students. In the survey, the students were asked about how they could determine the area of a complex figure.

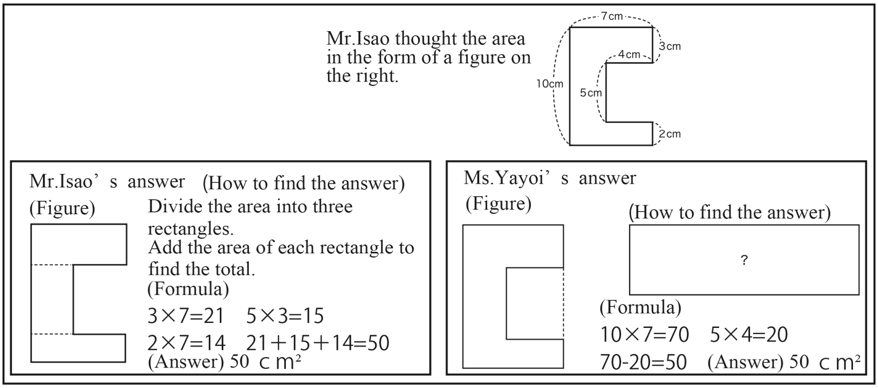

Fig. 3. Iwate Prefectural Board of Education conducted a survey of 5th grade

Ms. Yayoi determined the same area using a different method from Mr. Isao's.

Please write out Ms. Yayoi's method of solving the problem.

The children should answer, "Consider the large rectangle. Subtract the missing area."

The percentage of correct answers was 71.4%, meaning that about 30% of the students answered incorrectly. In 2001, the National Institute for Education Policy Research surveyed 6th grade students. The students were asked to write an equation for the volume of a stereo. The question is as follows. The percentage of correct answers for this question was 79.5%.

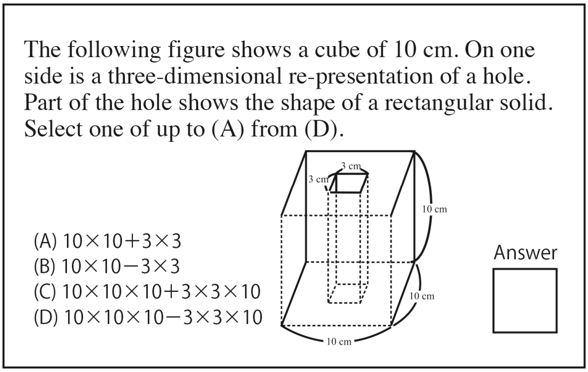

Fig. 4. National Institute for Education Policy Research surveyed

The results of these two questions, which are about a basic figure and complex figure, respectively, show that the children have weak performance in this area.

According to a research report on the curriculum, "Children will consider various rectangular cubes, and it is expected that you [the teacher] will provide a 3D shape composed of these units, and practice with it repeatedly. In addition, the children will actively incorporate the concepts of arithmetic into their calculation of volume.

Through this instruction on ingenuity, the children will understand the meaning of measurement units and volume, so as to enrich their sense of size and volume. "The International Association for the Evaluation of Educational Achievement conducted the "Trends in International Mathematics and Science Study 2011,(TIMSS2011)" which posed the following question to 4th-grade students.

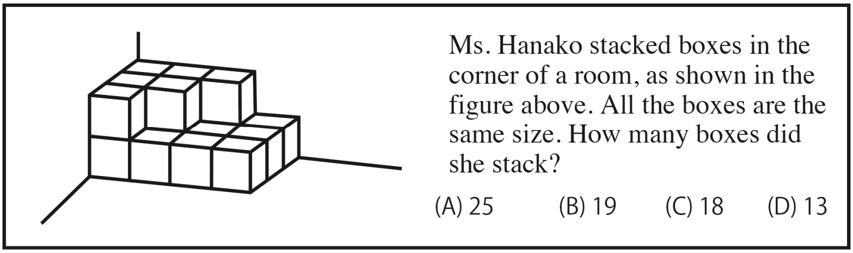

Fig. 5. Surveyed to 4th-grade students (TIMSS2011)

Among the 4th -grade students, 83.7% answered the question correctly (this number is 62.7% for international students).

In addition, the test administrators gave the following question to students in their 2nd year of junior high school.

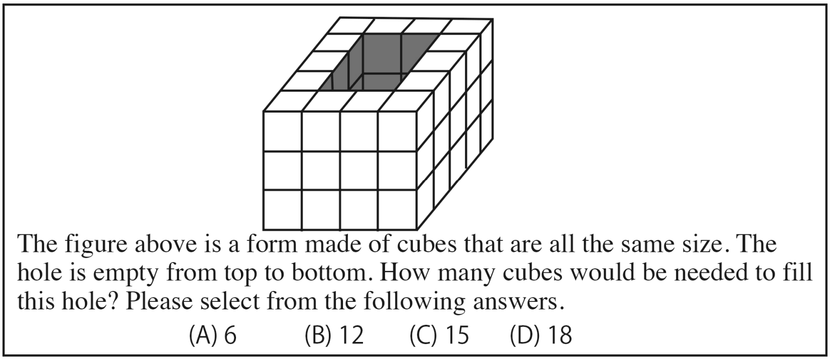

Fig. 6. Surveyed to 2nd year of junior high school students (TIMSS2011)

The correct answer rate for the Japanese students was 80.1% (46.8% for international students).As these results show, in learning about volume calculation, it is important that the children touch the molded products that are actually complex figures. Also, teachers should point out that the figure is a combination of a basic figure and complex figure to show the children how volume is important.

4 Relationship of Volume Calculation and 3D-CAD

Chapter 3 described the method of teaching volume calculation. By manipulating 3D objects in education on volume calculation, children can form hypotheses about how to calculate volume. In this chapter, we describe the common points shared by the design process of 3D-CAD and the concept of volume calculation.3D-CAD is a software program that allows the user to create a 3D shape on the computer. It recognizes two types of shape: a “solid type" and “surface type. "The design process for a “solid type" shape is similar to the process of creating a graphic in volume calculation. A solid 3D shape is created by combining the following two methods:

(1) Creating the shape using one-to-one geometry
(2) Cutting unwanted parts from a single mass

Fig. 7 is a diagram of a 3D shape that is used in a math textbook in Japan. We describe the design process of 3D-CAD using this shape as an example.

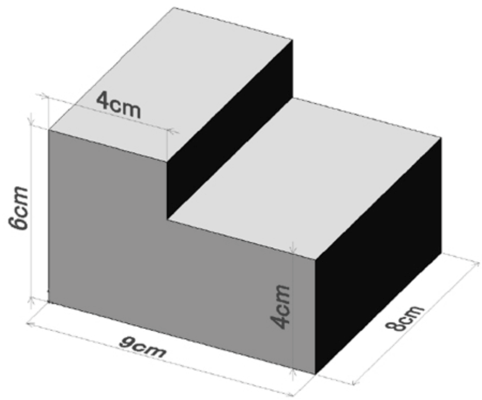

Fig. 7. Diagram of a 3D shape that is used in a math textbook in Japan

The design process of the shape shown in Fig. 8 may be any of the following three methods:

1) After creating a 4 cm x 6 cm x 8 cm cube, we add a 5 cm x 4 cm x 8 cm cube (Fig. 8a).
2) After creating a 9 cm x 4 cm x 8 cm cube, we add a 4 cm x 2 cm x 8 cm cube (Fig. 8b).
3) After creating a 9 cm x 6 cm x 8 cm cube, we delete the 5 cm x 2 cm x 8 cm cube (Fig.8c).

Even for the volume calculations for Fig. 7, the process is similar to the one described above. Based on the above description, we can say that the design process of

3D-CAD is effective as a method of volume calculation. Next, we will consider a complex 3D shape such as the one shown in Fig. 9 and discuss whether the same process applies.

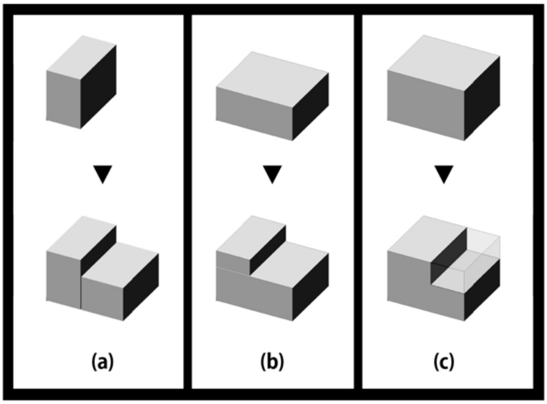

Fig. 8. Three ways of calculating the volume of a 3D shape

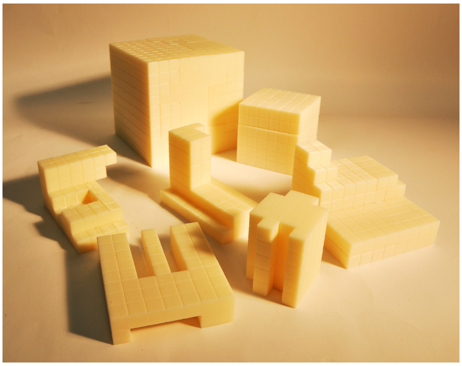

Fig. 9. 3D-Printer's model

Fig. 9 shows examples of complex 3D models created by a professional engineer using cubes in 3D-CAD. He used the methods "Creating the shape using to one-to-one geometry" and "Cutting unwanted parts from a single mass" as described above.

Therefore, it seems that the concept of volume calculation learned in elementary school is the same process used in 3D-CAD.This demonstrates that the process of creating complex 3D shapes in 3D-CAD (Fig. 10) can be applied to issues such as volume calculation in the near future.

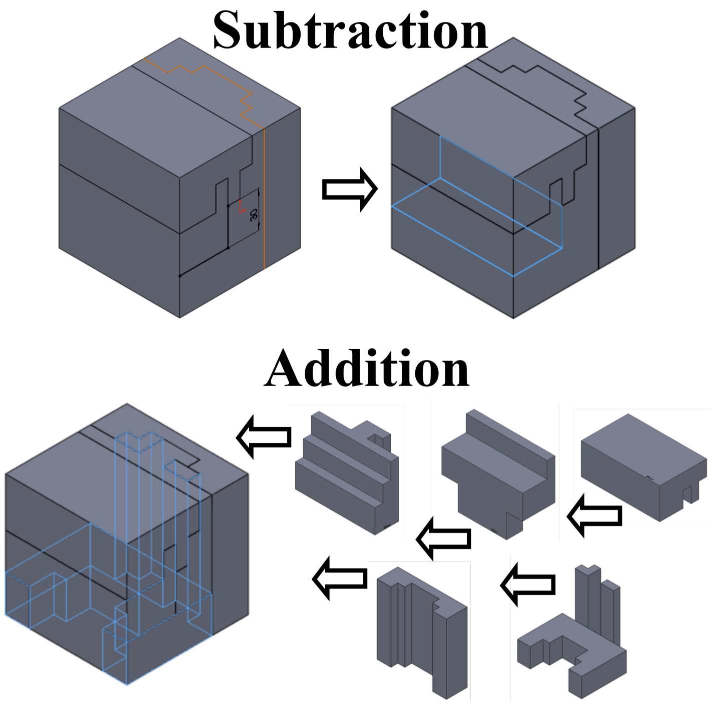

Fig. 10. Process of creating a complex 3D shape in 3D-CAD

5 Conclusion and Future Plan

This paper described the toy with wooden joints that we designed and discussed its application to educational materials on volume calculation.In considering the problems and teaching methods of volume calculation in Japan, we hypothesized the importance of children learning to handle 3D models.The investigation showed the concept of volume calculation can be applied in 3D-CAD.This indicates the possibility of teaching the concept of volume calculation using 3D-CAD.We also believe it may be possible in the future to teach materials on volume calculation using tablet PCs.

References

1. Tanaka, T.: The trial of the woodwork toy development for the small children using a wood joint. Society of Art Educa-tion in Univercity (45), 215–222 (2013)
2. Takagi, S.: Play Puzzle. Heibon. Ltd. (1981)
3. Fuji, T., Iitaka, S. (40 auteurs other), Atarashi Sansuu 5-I. Tokyo Shoseki Co., Ltd. (2011)