Designing an Interactive Tutoring Tool for Improving Mathematical Skills

Despina Lepenioti[1], Stella Vosniadou[1], and Christina Alexandris[2]

[1] Department of Philosophy and History of Science,
National and Kapodistrian University of Athens, Athens, Greece
des_lepenioti@hotmail.com, svosniad@phs.uoa.gr
[2] Department of German Language and Literature, School of Philosophy National
and Kapodistrian University of Athens, Athens, Greece
calexandris@gs.uoa.gr

Abstract. Based on the developmental transitions proposed by Stafylidou and Vosniadou (2004), three training sessions were created using the Cognitive Tutor Authoring Tools. The purpose was to compare the effects of two external representations on fraction understanding and interpretation, compared to a control group that received no training. The participants in the experimental groups were trained in creating external and symbolic representations for fractions using pies or number lines. Results indicated that participants in the experimental groups had greater improvement from pretest to post-test than the control group. The number line experimental group had greater improvement in tasks testing fraction equivalence and fraction interpretation as measure than the pie experimental group. Based on the results of the classroom experiment and the resulting User Requirements, the proposed interface was further reinforced with various sessions concerning fraction operations, including addition and subtraction. Special emphasis was placed on user-friendly and ergonomic interaction.

1 Introduction

Students' knowledge of fractions in elementary school has been shown to predict students' overall mathematical ability in high school [1], [2]. This finding indicates that fractions are important. However, fractions have been shown to be difficult for students to understand. Vosniadou and her colleagues [3] have explained these difficulties to result from negative transfer of natural number knowledge and operations.

Stafylidou and Vosniadou [4] described three developmental transitions in the development of the fraction concept. First, students see the fraction as two independent natural numbers. Second, students consider fractions to be always smaller than the unit. Only in the third developmental transition students see fractions as a relation between the numerator and the denominator, but they still face problems understanding the density of the rational numbers [5].

In order to test the effects of training using interactive external representations for fractions on fraction understanding we designed an interactive tutoring tool that included three training sessions where students would create external and symbolic representations for fractions.

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The effectiveness of the tool was tested with a training classroom experiment. We expected that the students who practiced with the tool would demonstrate better learning gains in the post-test compared to the pretest, than the students in the control group, who received no training. We also expected that the students who practiced for fractions with the number line external representation would have larger pretest – post-test differences than the students who practiced using the pie external representation because of the different properties of pies and number lines [6],[7].

We finally hypothesized that different external representations would promote different interpretations of fraction, as has been shown in previous research [8],[9]. The pie would promote the interpretation of fraction as part of a whole and the number line would promote the interpretation of fraction as a measure.

2 Data Collection and User Requirements

For the design and evaluation of the Tutoring Tool, data was collected from input by students participating in experimental interactive training sessions involving a set of tasks determined by a pretest – post-test instrument.

The participants were 80 6^{th} graders, 45 boys and 35 girls, with mean age 12.2 years and they attended three middle class elementary schools in a suburban area of Athens. The participants were randomly assigned to one of two experimental groups (pies vs. number lines) or to the control group. The experimental groups practiced in creating external and symbolic representations for fractions using the Interactive Tutoring Tool. The pretest and post-test were given to all the participants in the school classroom and lasted no more than a school hour each. The training took place in the school computer lab, in three sessions and every session lasted for less than a school hour.

The pretest – post-test instrument, included two equivalent forms of the fraction understanding assessment instrument. The assessment instrument included tasks testing conceptual understanding of fractions and procedural understanding of fractions. Four problems that tested the interpretation of fractions as part-whole relationship or as a measure were also added [6].

For the training sessions we designed our tutoring tool using Cognitive Tutor Authoring Tools [10]. The tool included web based problem sets in three sessions, namely “Choose Symbolic”, “Write Symbolic” and “Make External”. The problems embedded interactive representations of pies or number lines and students could provide answers, get feedback and help and manipulate the representations while the students were interacting with the external and the symbolic representation for the fraction. In the beginning of the “Choose Symbolic” session, the experimental groups were presented with an introduction which created the context for the corresponding representation. In the pie experimental group (a) fractions were introduced as pieces of pizza and in the number line experimental group (b) fractions were introduced as measures of distance. In the beginning of every session, students worked with two examples-trials where they received instructions in bubbles while they tested their answers and manipulations: partitioning and selecting parts on pies (a) and partitioning and selecting points of distance on number lines (b). In the middle and in the end of every session students were presented with a short video of the sequence of fractions they worked on. In every session, fraction sequences were created based on the developmental transitions described by Stafylidou and Vosniadou [4], in order to help students understand that some properties of natural numbers cannot be applied on rational numbers. As a first step, students worked on mathematical problems involving creating representations for unit fractions, followed by problems involving simple fractions that had the same denominator. In addition, students worked on problems for fractions that had the same numerator. As a second step, students worked on problems with fractions consistent with natural number ordering, namely: (1a) fractions that have large components as natural numbers and large numerical magnitude as fractions or (1b) fractions that have small components as natural numbers and small numerical magnitude as fractions. Additionally, students worked with fractions inconsistent with natural number ordering: (2a) fractions that have large components as natural numbers and small numerical magnitude, or (2b) fractions that have small components as natural numbers and large numerical magnitude. Finally, the third step included problems with fractions equal to the unit followed by improper fractions.

2.1 Results

No differences on pretest were found by a two – way ANOVA for Group, [$F(2,71)=2.755$, $p=0.070$, $\eta^2=0.072$], or School, [$F(2,71)=0.756$, $p=0.473$, $\eta^2=0.021$]. A 3 Group (pies-number lines-control) × 2 Time (pretest-post-test) repeated measures ANOVA on final score tested the effects of training and revealed main effects for Group, [$F(2,77)=5.255$, $p=0.007$, $\eta^2=0.120$], due to the fact that the participants who practiced using the tool had a greater mean score on all tasks than the participants in the control condition. Post hoc analysis revealed significant differences only between the group that practiced using the number line and the control group, ($p=0.005$). The same analysis revealed the same main effect for Group, [$F(2,77)=5.432$, $p=0.006$, $\eta^2=0.124$], regarding only tasks testing conceptual understanding of fractions, where post-hoc analysis revealed again significant differences only between the group that practiced using the number line and the control group, ($p=0.004$). Group performance from pretest to post-test was tested for every task. The number line group had significantly better scores from pretest to post-test in more tasks than the pie group, especially in the tasks Fraction Equivalence and Fraction Interpretation as measure, where the pie group demonstrated no significant improvement.

3 Design Parameters for Fraction Operations

In order to test more hypotheses concerning how external representations can support conceptual and procedural fraction understanding, the tool was redesigned to address also fraction operations such as fraction addition and fraction subtraction.

3.1 Tutoring Tool Content

The determination of the content of the Tutoring Tool involves the handling of two issues, the Symbolic and External Representation for fractions (1) and the Order of Problem Sets (2).

Symbolic and External Representation for Fractions. In every problem in the existing training sessions, either the symbolic or the external representation for fractions were presented to students and students had to choose, insert or create the corresponding one. The sixth graders that participated in the classroom experiment had little prior experience in interacting with an educational tool. These Users were familiar with the pie external representation and they were able to perform simple actions after being instructed to, for example, inserting numbers, choosing from multiple choice items, pressing buttons for help or submit and test their answers. Students had no prior experience with the number line for representing fractions. In the beginning of the training sessions, students indicated that using and manipulating the number line external representation was a difficult task for them.

After the students became familiar with the representations, they were presented with sessions designed for fraction addition and fraction subtraction involving both the symbolic and the external representation for fractions. Students had to change both representations at every step in order to proceed and find the resulting fractions.

Order of Problem Sets. The sequence of the problem sets is based on Stafylidou and Vosniadou, [4].

Set-1 To help students test their perceptions in the first developmental transition, in fraction operations, the first problem sets include operations with a unit fraction and a simple fraction with same denominators, followed by operations with two simple fractions with same denominators. Students then solve problems for operations involving fractions with equal numerators.

Set-2 In the second step, students solve fraction operations with fractions that are both consistent with natural number ordering, followed by problems with a consistent and an inconsistent with natural number ordering fraction. These problems target to help students comprehend that some properties of natural numbers cannot be applied to rational numbers.

Set-3 To challenge students to a more advanced perception of fractions, the last problem set includes operations with one equal to the unit fraction and a simple fraction, followed by operations with one or two improper fractions. Students may thus perceive that fractions can represent a quantity greater than the unit.

Set 2 and 3 aim at helping students see the fraction as a relation between the numerator and the denominator, which is described in the third developmental transition, where students have gained a better understanding of the fraction concept.

3.2 Interface Design

The User Interface design is aimed to address the above-presented issues in respect to User requirements. The interaction with the Tutoring Tool is targeted to young students and should, therefore, be easy to use, without too many additional features which may tire and discourage the User or may not be easily understood. Strategies such as verification questions are used and clarifying and explanatory elements should be present or directly available for a successful interaction. Keeping the interface features consistent throughout the interaction can help avoiding distractors or any additional cognitive load on Users which may result to an unsuccessful interaction.

Ergonomic Features of Interface (a). Specifically, the external representations, namely, pies or number lines, were presented on the left side of the screen and the symbolic representations, the fraction numerical components, were presented on the right side of the screen. The Interaction – Dialogue framework on the bottom of the screen and the Help button on the top right side of the screen were constantly presented. Feedback and help are available and consistent throughout the interaction. Students are able to move to the next problem only after the problem they are working on is solved. Every step is assessed and immediate feedback is provided. The "Done" button tests final answers and, if correct, moving to the next problem is enabled.

Verification Questions and Explanatory Elements (in Form of Bubbles) (b). The example-trials aimed at guiding students on the way of interacting with the tool such as the manipulation of the representations and the actions required to provide answers. The example trials had the purpose also of showing to the users the way of thinking using the external representations in the corresponding to the fraction interpretation context. In fraction operations, explanatory elements are added in order to explain to students the required procedural steps in order to find the result of the operation and the corresponding fraction conceptual aspects: For example, in steps required to make equivalent fractions, two fractions can represent the same magnitude.

3.3 User Feedback

In the end of the training the users stated that they were satisfied with their performance and that they would like to use the tool again in the future. They could use the tool and the representations with ease and they completed with success more than 95% of the problems right after the example-trials. The results from the classroom experiment confirm the learning gains for the students that practiced using the tool.

4 Conclusion and Future Work

The purpose of the designed tool is to support students understand the conceptual aspects of the fraction concept while avoiding misconceptions emerging from their prior knowledge about natural numbers. The tool is designed to avoid any distractors or additional cognitive costs. Furthermore, the proposed tool can be used to test hypotheses about fraction understanding and the effect of external representations on the fraction concept development. Creating representations for fractions helps students understand the properties of external and symbolic representations and build a mental model for the representation where they can test their perceptions for the fraction concept. Future work and increased application of the tool includes extending its application to address natural number representation on the number line for younger users and supporting the transition from natural numbers to rational numbers.

References

1. Siegler, R.S., Thompson, C.A., Schneider, M.: An integrated theory of whole number and fractions development. Cognitive Psychology 62, 273–296 (2011)
2. Siegler, R.S., Duncan, G.J., Davis-Kean, P.E., Duckworth, K., Claessens, A., Engel, M., Ines, M.: Early predictors of high school mathematics achievement. Psychological Science 23 (2012)
3. Vosniadou, S.: Conceptual change in learning and instruction: The framework theory approach. In: Vosniadou, S. (ed.) The International Handbook of Conceptual Change, 2nd edn., pp. 11–30. Routledge, New York (2013)
4. Stafylidou, S., Vosniadou, S.: The development of students' understanding of the numerical value of fractions. In: Verschaffel, L., Vosniadou, S. (eds.) Conceptual Change in Mathematics Learning and Teaching, Special Issue of Learning and Instruction 14(5), 503–518 (2004)
5. Vamvakoussi, X., Vosniadou, S.: How many decimals are there between two fractions? Aspects of secondary school students' reasoning about rational numbers and their notation. Cognition and Instruction 28(2), 181–209 (2010)
6. Lamon, S.J.: Teaching fractions and ratios for understanding: Essential content knowledge and instructional strategies for teachers. Routledge, New York (2012)
7. Bright, G.W., Behr, M.J., Post, T.R., Wachsmuth, I.: Identifying fractions on number - lines. Journal of Research in Mathematics Education 19(3), 215–233 (1988)
8. Kieren, T.E.: On the mathematical, cognitive, and instructional foundations of rational numbers. In: Lesh, R. (ed.) Number and Measurement: Papers from a Research Workshop, pp. 101–144. ERIC/SMEAC, Columbus (1976)
9. Charalambous, C.Y., Pitta-Pantazi, D.: Drawing on a theoretical model to study students' understandings of fractions. Educational Studies in Mathematics 64(3), 293–316 (2007)
10. Aleven, V., McLaren, B.M., Sewall, J., Koedinger, K.R.: The Cognitive Tutor Authoring Tools (CTAT): Preliminary Evaluation of Efficiency Gains. In: Ikeda, M., Ashley, K.D., Chan, T.-W. (eds.) ITS 2006. LNCS, vol. 4053, pp. 61–70. Springer, Heidelberg (2006)