Interactive Visualizations in Learning Mathematics: Implications for Information Design and User Experience

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Abstract. Learning mathematics seems not to be an easy task for many students. One of the reasons why mathematics may be difficult to learn is because mathematical concepts (e.g. numbers and functions) are not intuitive or accessed through everyday experience (Chiappini and Bottino, 1999). One way of trying to facilitate learning mathematics is through the use of interactive visualizations. The aim of this study is to draw attention to the importance of user experience and information design principles in order to design effective interactive visualization in learning mathematics. This article reviews some studies on visualization in learning mathematics, describes some principles both for information design and for user experience, and discusses their relevance in creating effective interactive visualization in learning mathematics.

Keywords: Interactive visualization, learning mathematics, information design, user experience.

1 Introduction

Learning mathematics seems not to be an easy task for many students. People of different ages have difficulties dealing with the rationale behind mathematics [1]. For many children, learning mathematics can cause boredom or anxiety, reducing motivation, which is considered to be very important in learning mathematics, as motivation influences the maintenance of learning processes [2].

One of the reasons why mathematics may be difficult to learn is because mathematical concepts (e.g. numbers and functions) are not intuitive or accessed through everyday experience [3]. Chiappini and Bottino [3] explained that in mathematics, differently from in the physical concrete world, the learning object "can be only conjured up by means of the use of external representations".

One way of trying to facilitate learning mathematics is through the use of interactive visualizations. Following the definition of Ware [4], visualization is "a graphical representation of data or concepts". It is recognized that visualization probably "enhances comprehension, engagement and satisfaction among students" [5].

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Visualization in learning mathematics is considered a key component of reasoning, problem-solving and proving, as well for illustrative purposes [6]. In addition, it is considered to be a powerful tool to explore mathematical problems and to give meaning to mathematical concepts and the relationship between them [7].

Although many researchers consider visualization helpful in learning mathematics, it seems that there is a need for a great number of studies in this area. The learning aspects seem to be more thoroughly investigated than the visual ones. There appears to be a need for a better understanding by designers and educators of how to create effective interactive visualizations in learning mathematics.

This paper review some studies on visualization in learning mathematics, describes some principles both for information design and for user experience, and discusses their relevance in creating effective interactive visualization in learning mathematics.

For a better understanding of the discussion presented in this paper some definitions - in addition to the definition of visualization presented above - are important. For the purpose of this discussion, mathematics is "the abstract science of number, quantity, and space" [8]. User experience "is about creating a meaningful experience through a device" [9]. Finally, information design is defined "as the art and science of preparing information so that it can be used by human beings with efficiency and effectiveness" [10].

2 Research on Visualization in Learning Mathematics

There has been much research on visualization in learning mathematics [7]. According to Presmeg [11], visualization research in mathematics education started slowly, growing from a psychological basis in the late 1970s and early 1980s. However, it seems that the question of "whether visualizations help develop mathematical concepts" is still ambiguous and often contradictory [12]. Phillips et al. [12], in their literature review of 40 articles on visualizations in mathematics classrooms, found that visualization may interfere when the goal is analytic proficiency, and that it works differently for more and less gifted students and for older and younger students. They concluded that the usefulness of visualizations is related to the goals of mathematics instruction.

Investigating young people, Hoffkamp [13] studied how the use of interactive visualizations helps students of 15 and 16 years old in understanding concepts of calculus. The conclusion was that interactive visualizations activate the formation of intuitive access to concepts of calculus.

Focusing on university students, Souto and Gómez-Chacón [14] examined the visualization processes from a cognitive point of view. They conducted an empirical study in order to improve the teaching of mathematical analysis by emphasising visualization processes. They found out, among other things, that choice of representation and register is very important for solving the problem successfully. They concluded that, from a didactic point of view, it is essential to use different types of representations and registers and their coordination.

Like Souto and Gómez-Chacón [14], Gómez-Chacón [15] also investigated university students. However, she focused on investigating the affect (i.e. emotions, values and beliefs) associated with visualization processes in a dynamic geometric environment, finding that student teachers believe that "visual thinking is essential to solving mathematical problems". The participants believed that software tools have an impact on motivated behaviour and enhance a positive self-concept as a mathematical learner.

Also investigating the role of visualization in learning mathematics, Rösken and Rolka [7] conducted an empirical study in order to investigate integral calculus (4 problems) and different aspects of visualization, including: how students deal with visualizations and "the visual images that students use for working on specific problems". They concluded that on the one hand, visualization proved to be a useful tool for working on the problems, helping them to find creative ways to modify the tasks and also to show the underlying obstacles. On the other hand, the findings showed that visualization did not always help the students to solve the problems correctly. They argue that students are still "reluctant to visualize" as they "are cognitively fixed on algorithms and procedures instead of recognizing the advantages of visualizing this problem".

With a different approach to the research described above, Sedig [2] examined the importance and application of flow (i.e. a type of intrinsic motivation) while learning mathematics. Flow is defined as "an optimal experience in which an individual can greatly enjoy an activity". Although flow potentiality in learning activities has been discussed in the literature, there is still a limited understanding of how this can be done. Sedig investigated how to build the characteristics of flow and put them into operatio in computer-based mathematics learnware for children. It was concluded that the design of such systems is facilitated when an "operational or prescriptive model exists that guides the design."

Sánchez-Torrubia et. al. [5] investigated the pedagogical impact of interactive tutorials in visualization and learning of mathematical concepts in computer science curricula. Based on an investigation on tutorial tools in Java play, they proposed a set of characteristics that a visualization tutorial should fulfill. They are: easy to use, intuitive and visual, interactive, platform independent, for many teachers and/or first course students, clear and concise explanations of the theoretical concepts.

The research described above point to many different aspects and characteristics of visualization in learning mathematics. As said, the visual characteristics of such visualizations have been little investigated. However, these characteristics seem to be crucial for the success of an interactive visualization. Below are some principles that can help designers to create quality interactive visualizations in learning mathematics.

3 Principles of Information Design and User Experience Applied to Learning Mathematics

"Hence the central problem of an education based upon experience is to select the kind of present experiences that live fruitfully and creatively in subsequent experiences." (John Dewey, Experience and Education, [16] p. 13)

As mentioned in the section above, learning and teaching mathematics can be seen as a challenge for many people. The new interactive visualizations allow many different approaches to learning mathematics. However, researchers have shown that the success of visualization in learning mathematics is related to the quality of the material presented, among other factors.

In the context of visualization in mathematical education, Jeschke and Manya [17] pointed to different factors that determine the quality of visualizations. They are: expressiveness, effectiveness, adequateness, objectives (i.e. the information which should be extracted from the graphic representation of the mathematics contents), foreknowledge (i.e. the preconditioned foreknowledge of mathematical concepts and objects of the student), perceptibilities (i.e. the perception capacities of the students), usual conventions in the area of application, and characteristics of the representation medium (e.g. the resolution, the number of representable colours). In order to consider all these factors there is a need for knowledge from different areas, such as visualization, perception, interaction design and education.

Information design and user experience principles may be useful to help in the understanding of such factors and to achieve quality in this kind of visualization. This is because these principles deal with certain aspects of visual representation of interactive visualizations that are relevant to a good user experience. Therefore, these principles can guide designers and educators to create successful visualizations that meet the requirements of a learning experience.

There are many principles related to visualizations. For the purpose of this paper, the principles of three authors were chosen in order to illustrate different aspects related to interactive visualization in learning mathematics. They are: the "Golden Rules" by Shneiderman et al. [18], the principles of excellence in graphics by Tufte [19], and the cognition principles of Pettersson [20]. While Shneiderman's principles deal more with user experience aspects, both Tufte and Pettersson focus on information design. These authors were chosen both because of recognition in their areas and the subject of their principles. These principles are briefly described and commented below.

Shneiderman et al. [18] proposed Golden Rules that according to them can be applied to most interactive systems. Shneiderman first proposed these principles in 1985 and he has since refined them. These principles aim to help users to increase productivity by "providing simplified data-entry procedures, comprehensible displays, and rapid informative feedback to increase feelings of competence, mastery, and control over the system." They are:

1. Strive for consistency.
2. Cater to universal usability.
3. Offer informative feedback.
4. Design dialogs to yield closure.
5. Prevent errors.
6. Permit easy reversal of actions.
7. Support internal locus of control.
8. Reduce short-term memory load.

The principles focus on user experience aspects, such as usability aspects (i.e. efficacy, efficiency and satisfaction of users with an interface). In the design of interactive visualization in learning mathematics, it is relevant to observe these principles in order to help student learning. This is because they can improve confidence and control over a system. For example, in order to give confidence to students it is important to create functions in the visualization system that allow easy reversal of actions (principle 6). If this principle is not considered students may feel insecure taking actions and, as a result, they may stop using the interface. As the authors explain, allowing reversible actions "relieves anxiety, since the user knows that errors can be undone, and encourages exploration of unfamiliar options".

Another principle is the need for informative feedback in the learning interface. The system should give feedback for every user action. Feedback on the action is fundamental for the user to get involved with the interface. Visual presentation can help the students to perceive the feedback of the system, which should be dependent on both frequency and importance of the actions. As Shneirderman et al. (2010) explain, feedback can be modest for frequent and minor actions, whereas feedback should be more substantial for infrequent and major actions.

The short-term memory load principle is important for visualization in learning mathematics, as complex actions are difficult to remember. Therefore, the interface should help students to recognize instead of remember a lot of information from screen.

The second group of principles, proposed by Tufte (2001), are related to graphical excellence. Tufte defines practical graphical excellence as "the efficient communication of complex quantitative ideas". Based on an analysis of several examples of graphs (e.g. maps, narratives of space-time, relational charts), he built a theory of graphical data and proposed five principles of graphical excellence. They are:

- Graphical excellence is the well-designed presentation of interesting data – a matter of substance, of statistics, and of design.
- Graphical excellence consists of complex ideas communicated with clarity, precision, and efficiency.
- Graphical excellence is that which gives to the viewer the greatest number of ideas in the shortest time with the least ink in the smallest space.
- Graphical excellence is nearly always multivariate.
- Graphical excellence requires telling the truth about the data.

(Tufte, [19] p. 51)

As Tufte argues, "the principles of information design are universal – like mathematic – and are not tied to unique features of a particular language or culture" (Tufte, [21] p. 10). In order to achieve a universal understanding of mathematical concepts, graphical excellence is fundamental. The first principle proposed by Tufte draws attention to the need for a deep understanding of both content and design in order to create graphics with excellence. Tufte proposed principles based on printed material, and therefore did not consider the digital aspects on an interface. However, he does consider the interface when he mentions the space size and also the time to understand the information. As Hassenzahl [9] defined it, user experience is about meaningful experience. Therefore, in order to achieve that we should communicate ideas with clarity, precision, and efficiency – as Tufte [19] proposed. Clarity, precision and efficiency are important not only in learning mathematics but also in learning any other subject. When we talk about abstract numbers it seems that both the precision with which we deal with the data, and the relation between the content and how it is presented are crucial for the success of an interactive visualization.

With regard to the multivariate characteristic of graphical excellence, Tufte draws attention to the difference between design variation and data variation. A graphic should show only data variation, not design variation. He explains that the "use of two (or three) varying dimensions to show one-dimensional data is a weak and inefficient technique", which can lead to ambiguity in perception (Tufte, [20], p. 71). This aspect is very important when designing an interactive visualization in learning mathematics, as ambiguity in perception can disrupt learning. Designers chose the number of dimensional representations related to – and not exceeding - the number of dimensions in the data [20].

Another important issue to consider when pursuing the creation of outstanding quality interactive visualizations is their graphical integrity. According to Tufte, in order to be truthful and revealing, data graphics must quote data in context – "leaving out data sufficient for comparisons". In addition, he argues that clear and detailed labelling can help avoid distortion and ambiguity. It seems that in addition to the visualizations, clear instructions on how to do some tasks, and what the visual elements mean, should be given, so that students gain a better understanding and are more likely to achieve their learning goals.

Finally, the third group of principles described was proposed by Petterson (2012), in his book "It depends: principles and guidelines" (4th edition). Based on theories of visual and verbal communication Pettersson proposed 16 design principles divided into four groups: functional, administrative, aesthetic, and cognitive. For this paper, the cognitive principles are described, namely:

- Facilitating attention.
- Facilitating perception.
- Facilitating processing.
- Facilitating memory.

Pettersson claims that information materials should facilitate human attention in order to capture and hold attention. One way of drawing students' attention in a message is to emphasize it, for example with different font size, or font style. Captions are important as they can direct attention to specific picture elements. Using arrows or lines in various colours can do the same. Pettersson also claims that other tools to emphasize important details within moving pictures are: "contrast, graphics, shading, split screens, text, voiceover narration and zoom lens movements".

In order to facilitate perception, many aspects should be considered. Pettersson explains that both prior experience and context are very important for the perception of contexts. He argues, “simplicity in a message will result in perception of that message”.

According to Pettersson, the facts that students of different ages have very low “pictorial capability” and also that graphics in learning materials are used inconsistently reduce understanding and learning. He claims that the available study time should be considered when choosing the style of the artwork. Where the learner´s study time is limited, line drawings are more effective. On the other hand, where unlimited study time is allowed, more realistic versions of artwork are more effective. The type of visuals used may also affect comprehension.

Another important aspect that can facilitate mental processing is the fact that “learners are most able to build connections between verbal and visual representations when text and illustrations are actively held in memory at the same time”. This aspect is related to the memory facilitation principle. Pettersson argues, “memory is greater when a verbal and a visual code are activated at the same time, rather than only one of them” (Pettersson, [22], p. 166). Because of this, the relevant text should be put as close as possible to pictures. He also draws attention to the “pictorial superiority effect”, which means that memory for words is inferior to memory for pictures.

Pettersson points to three general prerequisites for learning: motivation, influences (e.g. social, technical, biological, psychological factors), and presentation of information. He explains, “For learning to occur we must be mentally prepared to learn. We must be interested and curious. We must be willing to learn! We learn better when we understand the reasons for learning, when we are motivated to learn, and when we work to achieve an important goal” [22].

4 Conclusion

Both the review of studies on interactive visualization in learning mathematics and the information design and user experience principles point to the need for a better understanding of visualization potentialities and careful knowledge in this area. Understanding of information design, visualization design, interaction design, perception, display technology, and learning theories is crucial to the success of interactive visualizations in learning mathematics.

The principles shown above can help designers and teachers to create interfaces that improve learning capacity and take advantage of the device’s possibilities. As Ferrara [23] claims, the power of interactive technologies should be exploited in ways that “allow controllable linkages between measurable events that are experienced as real by students and more formal mathematical representations of those events”.

Human-centered design seems to be an important method of building interactive visualizations that promote learning. As both the principles and previous research show, the individuals’ characteristics and human capacities, and their cultural background among other characteristics, are important for learning motivation and fixation.

In 1991, Zimmermann and Cunningham [24] already envisioned that visualization in mathematics would be more important than it was at that time and also that more powerful visualizations would be available. They were right. The challenge now seems to be to design more useful visualizations and to improve their quality in a way that helps students to experience it, learn with motivation, and retain the knowledge learned.

References

1. Rapp, W.H.: Avoiding Math Taboos: Effective Math Strategies for Visual Spatial Learners. Teaching Exceptional Children Plus 6(2), Article 4 (2009), http://escholarship.bc.edu/education/tecplus/vol6/iss2/art4
2. Sedig, K.: Toward operationalization of 'flow' in mathematics learnware. Computers in Human Behavior 23, 2064–2092 (2007)
3. Chiappini, G., Bottino, R.M.: Visualisation in teaching-learning mathematics: the role of the computer. In: Proceedings of Graphics and Visualization Education, Coimbra, Portugal, July 3-5 (1999)
4. Ware, C.: Information Visualization: Perception for Design, 3rd edn. Morgan Kaufmann (2012)
5. Sánchez-Torrubia, M.G., Sastre-Rosa, M.A., Giménez-Martínez, V., Escribano-Iglesias, C.: Pedagogical impact of interactive tutorials in visualization and learning of mathematical concepts in computer science curricula. In: Proceedings of Informatics Education Europe Conference, Montpellier (2006)
6. Arcavi, A.: The role of visual representations in the learning of mathematics. Educational Studies in Mathematics 52, 215–241 (2003)
7. Rösken, B., Rolka, K.: A picture is worth a 1000 words – the role of visualization in mathematics learning. In: Novotná, J., Moraová, H., Krátká, M., Stehlíková, N. (eds.) Proceedings 30th Conference of the International Group for the Psychology of Mathematics Education, pp. 457–464. PME, Prague (2006)
8. Oxford Dictionaries. Definition of mathematics in English, http://www.oxforddictionaries.com/us/definition/american_english/mathematics
9. Hassenzahl, M.: User Experience and Experience Design. In: Soegaard, M., Dam, R.F. (eds.) The Encyclopedia of Human-Computer Interaction, 2nd edn. The Interaction Design Foundation, Aarhus (2013), http://www.interaction-design.org/encyclopedia/user_experience_and_experience_design.html
10. Horn, R.E.: What is Information Design? Information Design as an Emerging Profession. In: Jacobson, R. (ed.) Information Design. MIT Press (1999)
11. Presmeg, N.C.: Research on visualization in learning and teaching mathematics: Emergence from psychology. In: Gutierrez, A., Boero, P. (eds.) Handbook of Research on the Psychology of Mathematics Education: Past, Present and Future, pp. 205–235. Sense Publishers, Rotterdam (2006)
12. Phillips, L.M., Norris, S.P., Macnab, J.S.: Visualizations and Mathematics. In: Visualization in Mathematics, Reading and Science Education. Models and Modeling in Science Education, vol. 5, pp. 45–50. Springer, Heidelberg (2010)
13. Hoffkamp, A.: The Use of Interactive Visualizations to foster the Understanding of Concepts of Calculus - Design Principles and Empirical Results. Zentralblatt für Didaktik der Mathematik 43(3), 359–372 (2011)

14. Souto, B., Gómez- Chacón, I.M.: Challenges with visualization. The concept of integral with undergraduate students. In: Proceedings The Seventh Congress of the European Society for Research in Mathematics Education (CERME-7). University of Rzeszów, Poland (2011)
15. Gómez-Chacón, I.M.: Prospective Teachers' Interactive Visualization and Affect in Mathematical Problem-Solving. The Mathematics Enthusiast 10(1&2), 61–86 (2013)
16. Dewey, J.: Experience and education. The Macmillan Company, New York (1938)
17. Jeschke, S., Manya, N.E.: The Role and Potential of Visualization in the Mathematical Education of Engineers. In: Proceedings of the 2009 ASEE Annual Conference. American Society for Engineering Education, Austin (2009)
18. Shneiderman, B., Plaisant, C., Cohen, M., Jacobs, S.: Designing the User Interface: Strategies for Effective Human-Computer Interaction, 5th edn. Prentice Hall (2010)
19. Tufte, E.R.: The Visual Display of Quantitative Information. Graphics Press, Cheshire (2001)
20. Pettersson, R.: It Depends: Information Design – Principles and Guidelines, 4th edn. Institute for Infology (2012), http://www.iiid.net/PDFs/ItDepends.pdf
21. Tufte, E.R.: Envisioning Information. Graphics Press, Cheshire (1990)
22. Pettersson, R.: Information Design 5 – Cognition. Institute for Infology (2013), http://www.iiid.eu/rune-pettersson-information-design-5-cognition/
23. Ferrara, F., Pratt, D., Robutti, O.: The role and uses of technologies for the teaching of algebra and calculus: ideas discussed at PME over the last 30 years. In: Gutiérrez, A., Boero, P. (eds.) Handbook of Research on the Psychology of Mathematics Education: Past, Present and Future, pp. 237–273. Sense Publishers (2006)
24. Zimmermann, W., Cunningham, S.: Editors' introduction: What is mathematical visualization? In: Zimmermann, W., Cunningham, S. (eds.) Visualization in Teaching and Learning Mathematics, pp. 1–8. Mathematical Association of America, Washington, DC (1991)