Stochastic Stabilizing Control of Networked Control System with Markovian Parameters

Ying Wu, Yanpeng Wu, and Lei Guo

Abstract In order to solve the potential issues caused by network induced delays and dropouts which could arise the performance degradation and system instability, this paper studies the stochastic stability problem of Networked control systems (NCSs) with arbitrary time delays and packet dropouts by using an active time-varying sampling method. The random time delays and successive packet dropouts are driven by two separately Markov chains and NCSs are modelled as a discrete time Markovian jump linear systems. Based on Lyapunov approach, sufficient conditions for the stochastic stability of the networked control system are derived and stabilization controller is designed in terms of linear matrix inequalities (LMIs) correspondingly. Gridding approach is introduced to guarantee the solvability of the LMIs with finite jump modes. A numerical example is given to illustrate the effectiveness of the proposed method which stabilizes the NCS with random time delays and packet dropouts.

Keywords Networked control system • Stochastic stability • Markov chain • Linear matrix inequality • Time delay • Packet dropouts

1 Introduction

Due to the advantages of low installation cost, reduced wiring, easy maintenance and good system flexibility, NCSs have been widely used in manufacturing systems, monitoring system and vehicle highway systems. Despite lots of

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advantages and potentials network brings to the control system, potential issues arise to degrade a system's performance and even cause system instability, such as delays and packet dropouts [1].

Many researchers have studied stability criteria and stabilizing controller design for networked control systems with delays and packet dropouts. Time-based time-delay analysis of the NCS is provided to explain how it affects network systems and an adaptive Smith predictor control scheme is designed [2]. A switched system approach was used to study the stability of networked control systems and optimal gain is calculated for stabilizing controller design [3]. The network-induced random delays are modelled as Markov chains such that the closed-loop system is a jump linear system with one mode [4, 5]. NSCs with packet dropouts are modelled as discrete Markov jump system [6, 7]. So far, the stability synthesis for the NCSs with time delays and packet dropouts as a Markovian jump system with two Markov chains has not been fully investigated.

In this paper, the stochastic stability problem of NCSs with random time delays and packet dropouts is investigated. The closed-loop NCS is modelled as a discrete-time jump system characterized by driving two separately Markov chains. An active time-varying sampling method is proposed to make sure time delay always less than one sampling period [8]. Based on the Lyapunov stability theory, sufficient conditions for the stochastic stabilization of the NCS are obtained and the mode-dependent stabilizing controller for the closed-loop NCS is designed in the linear matrix inequalities (LMIs) formulation via the Shur complement theory. A "gridding" approach is introduced to obtain the finite combination of time delays and packet dropouts which ensures the feasibility of the constructed LMIs [9].

Notation: The notation used throughout the paper is fairly standard. A^T represents the transpose of matrix A, the notation $P > 0$ means that P is positive definite, $\lambda_{\max}(P)(\lambda_{\min}(P))$ denotes the maximal (minimal) eigenvalue of matrix P; $diag\{\cdots\}$ stands for a block-diagonal matrix; $E[\bullet]$ stands for the mathematical expectation; $\|\bullet\|$ denotes the standard norm.

2 Problem Formulation

Consider a linear time-invariant plant described by

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^p$ is the input vector. A, B are constant matrices of appropriate dimensions.

In this paper, the sampling period will be set time varying to make sure time delay is less than one sampling period. In order to achieve this goal, sensor is assumed both time-driven and event-driven. Actuator and controller are event-driven. Suppose time axis is partitioned into equidistant small intervals and the

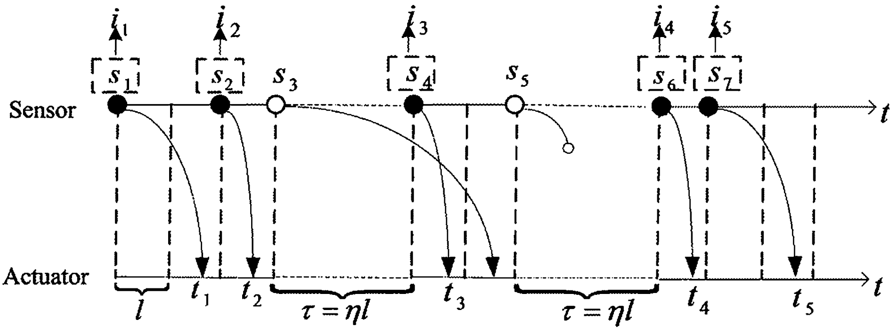

Fig. 1 Sampling and updating conditions of NCS

length of each interval is l. Define t_k as the kth updating instant of actuator, and assume that total transmission delay from sensor to actuator of the updating signal at the instant t_k is τ_k. Then the next sampling instant can be selected as

$$s_{k+1} = \begin{cases} s_k + al & t_k \in [s_k + (a-1)l,\ \ s_k + al) \\ s_k + \tau & t_k \geq s_k + \tau \end{cases} \tag{2}$$

where s_k is the kth sampling instant, τ is the allowable maximum delay from sensor to actuator ($\tau = \eta l$, η is the bound positive integer of delay), a is a positive integer and $0 < a < \eta$.

Figure 1 shows the sampling and updating conditions of NCS. if the transmission time of sampled signal at time s_k is less than τ, the actuator will be updated by the signal and the sensor will be driven to do the next sampling, which is called an effective sampling instant because the signal at this sampling instant is successfully transmitted from sensor to actuator, such as s_1 and s_2 marked in Fig. 1; if the signal sampled at time s_k has not arrived before the maximal allowable updating time $s_k + \tau$, which means the total transmission delay is out of τ, the signal will be discarded and the sensor will adopt time-driven mode, such as s_3 marked in Fig. 1; if packet dropout happened to the sampled signal, which can be seen as a long delay packet, the time-driven mode will adopted by sensor to do the next sampling, such as s_6 marked in Fig. 1.

Suppose h_k as the length of interval between two successive effective sampling instants i_k and i_{k+1}, the discrete time representation of Eq. 1 can be described as

$$x(i_{k+1}) = \Phi_k x(i_k) + \Gamma_0(\tau_k, h_k)u(i_k) + \Gamma_1(\tau_k, h_k)u(i_{k-1}) \tag{3}$$

where $\Phi_k = e^{Ah_k}$, $\Gamma_0(\tau_k, h_k) = \int_0^{h_k-\tau_k} e^{As}Bds$, $\ \Gamma_1(\tau_k, h_k) = \int_{h_k-\tau_k}^{h_k} e^{As}Bds$

Let us introduce a new augmented state $z(k) = \left[\, x(i_k)\ \ u(i_{k-1}) \,\right]^T$. Therefore, we can get the following augmented closed-loop system

$$z(k+1) = \Psi_k z(k) \tag{4}$$

where $\Psi_k = \begin{bmatrix} \Phi_k + \Gamma_0(\tau_k, h_k)K(i_k) & \Gamma_1(\tau_k, h_k) \\ K(i_k) & 0 \end{bmatrix}$

3 Stochastic Stability Analysis and Controller Design

Define d_k as the number of dropped packet between two successive *effective updating instants* i_k and i_{k+1}, then we can get $i_{k+1} - i_k = d_k + 1$, $i_k \in I = \{i_1, i_2, i_3, \ldots\}$. If assume the bound of consecutive dropped packets is d, we can conclude that d_k takes value from a finite set $\Omega = \{0, 1, \cdots, d\}$.

In Sect. 2, we defined the bound of τ_k $(\tau = al)$ is τ $(\tau = \eta l)$ and a takes value from a finite set $M = \{1, 2, \cdots, \eta\}$, and then τ_k takes value from the finite set $T = \{1l, 2l, \cdots, \eta l\}$.

In this paper, we assume that random delays τ_k and packet dropouts d_k are two independent Markov chains that take values in Ω and M with the following transition probabilities

$$\begin{aligned} \omega_{mi} &= \Pr(\tau_{k+1} = il | \tau_k = ml), \quad \forall i, m \in \mathrm{M} := \{1, 2, \cdots, \eta\} \\ \lambda_{nj} &= \Pr(d_{k+1} = j | d_k = n), \quad \forall j, n \in \Omega := \{0, 1, \cdots, d\} \end{aligned} \tag{5}$$

where $\omega_{mi}, \lambda_{nj} \geq 0$, and $\sum_{i=1}^{\eta} \omega_{mi} = 1$, $\sum_{j=0}^{d} \lambda_{nj} = 1$

The transition probability matrixes are defined by Υ and Π.

Based on the above assumptions, the e*ffective sampling period* h_k can be written into: $h_k = \tau_k + \tau d_k$

Therefore, the values of $\Phi_k, \Gamma_0(\tau_k, h_k)$ and $\Gamma_1(\tau_k, h_k)$ are finally determined by τ_k and d_k, system (4) can be seen as a discrete-time Markovian jump linear system with finite jump modes varying in a finite set which is combined by sets T and Ω. Define $\hat{A}(m, n)$ as the jump modes determined by $\tau_k = m$ and $d_k = n$, $K(m, n)$ as the mode-dependent state feedback controller gain, then augmented system (4) can be written into

$$z(k+1) = \hat{A}(m, n) z(k), \quad \forall m \in \mathrm{M}, n \in \Omega \tag{6}$$

where

$$\hat{A}(m, n) = \begin{bmatrix} \Phi(m, n) + \Gamma_0(m, n)K(m, n) & \Gamma_1(m, n) \\ K(m, n) & 0 \end{bmatrix} \tag{7}$$

Definition 1 The system (6) is stochastically stable if for every initial state $z_0 = z(0)$ and initial distributions $\tau_0 = \tau(0) \in \mathrm{T}$ and $d_0 = d(0) \in \Omega$, there exists a finite matrix $Q > 0$ such that $E(\sum_{k=0}^{\infty} \|z(k)\|^2 | z_0, \tau_0, d_0) < z_0^T Q z_0$ holds.

Theorem 1 *If there exists symmetric positive definite matrices $X(m,n) > 0, m \in M$, $n \in \Omega$ satisfying*

$$\begin{bmatrix} X(m,n) & * & \cdots & * \\ \Xi_{01} & \omega_{m0}^{-1}\lambda_{n0}^{-1}X(0,0) & & \\ \vdots & & \ddots & \\ \Xi_{(\eta d)1} & 0 & \cdots & \omega_{m\eta}^{-1}\lambda_{nd}^{-1}X(\eta,d) \end{bmatrix} > 0, \quad (8)$$
$$m = 1,2,\ldots,\eta, \; n = 0,1,2,\ldots,d$$

where $\Xi_{01} = \Xi_{21} = \cdots = \Xi_{(\eta d)1} = A(m,n)X(m,n)$

Then the system (6) is stochastically stable.

Proof Consider the following form of the Lyapunov function:

$$V(k) = z^T(k)P(m,n)z(k) \quad (9)$$

where $P(m,n) = X^{-1}(m,n) > 0$

Then we have

$$\begin{aligned} E(\Delta V) &= E(V(k+1) - V(k)) \\ &= \sum_{j=0}^{d}\sum_{i=0}^{\tau} \lambda_{nj}\omega_{mi} z^T(k)\hat{A}^T(m,n)P(m,n)\hat{A}(m,n)z(k) - z^T(k)P(m,n)z(k) \\ &= z^T(k)V(m,n)z(k) \end{aligned} \quad (10)$$

where $V(m,n) = \sum_{j=0}^{d}\sum_{i=0}^{\eta} \lambda_{nj}\omega_{mi}\hat{A}^T(m,n)P(i,j)\hat{A}(m,n) - P(m,n)$

Define $H(m,n) = diag\{P(m,n), I_{00}, \ldots, I_{\eta d}\}$, and pre-multiply and post-multiply Eq. 8 by $H(m,n)$, we get

$$\begin{bmatrix} P(m,n) & * & \cdots & * \\ A(m,n) & \omega_{m0}^{-1}\lambda_{n0}^{-1}P^{-1}(0,0) & & \\ \vdots & & \ddots & \\ A(m,n) & 0 & \cdots & \omega_{m\eta}^{-1}\lambda_{nd}^{-1}P^{-1}(\eta,d) \end{bmatrix} > 0,$$
$$m = 0,1,2,\ldots,\eta, \; n = 0,1,2,\ldots,d \quad (11)$$

By Schur complement, we can get $V(m,n) < 0$.since $V(m,n) < 0$, then

$$\begin{aligned} E(\Delta V(k)) &= E(V(k+1) - V(k)) \\ &\le -\lambda_{\min}(-V(m,n))\|z(k)\|^2 < 0 \end{aligned} \tag{12}$$

For any integer $M \ge 1$, we have

$$E(V(z(M+1)) - E(V(z_0)) \le -\lambda_{\min}(-V(m,n))E\left\{\sum_{k=0}^{M} \|z(k)\|\right\}$$

Thus, from Definition 1, if $V(m,n) < 0$, the system (6) is stochastically stable.

Theorem 2 *If there exists symmetric positive definite matrices $G(m,n)$ and V(m, n), matrices $R(m,n)$ ($\forall m \in M, n \in \Omega$) satisfying*

$$\begin{bmatrix} G(m,n) & * & * & * & \cdots & * & \\ 0 & V(m,n) & * & * & \cdots & * & \\ \Pi_0 & \Lambda_0 & \omega_{m0}^{-1}\lambda_{n0}^{-1}G(0,0) & * & \cdots & * & \\ R(m,n) & 0 & 0 & \omega_{m0}^{-1}\lambda_{n0}^{-1}V(0,0) & \cdots & * & \\ \vdots & \vdots & \vdots & \vdots & \ddots & * & \\ \Pi_{(\eta d)} & \Lambda_{(\eta d)} & 0 & 0 & \cdots & \omega_{m\eta}^{-1}\lambda_{nd}^{-1}G(\eta,d) & \\ R(m,n) & 0 & 0 & 0 & \cdots & 0 & \omega_{m\eta}^{-1}\lambda_{nd}^{-1}V(\eta,d) \end{bmatrix} > 0, \forall m \in M, n \in \Omega \tag{13}$$

where

$$\begin{aligned} \Pi_0 &= \Pi_1 = \cdots = \Pi_{(\eta d)} = \Phi(m,n)G(m,n) + \Gamma_0(m,n)R(m,n) \\ \Lambda_0 &= \Lambda_1 = \cdots = \Lambda_{(\eta d)} = \Gamma_1(m,n)V(m,n) \end{aligned}$$

Then the system (4) is stochastically stable and the mode-dependent state feedback controller is given by

$$K(m,n) = R(m,n)G^{-1}(m,n), \quad m \in M, n \in \Omega \tag{14}$$

Proof Denote the following matrixes:

$$\begin{aligned} &\tilde{A}(m,n) = \begin{bmatrix} \Phi(m,n) & \Gamma_1(m,n) \\ 0 & 0 \end{bmatrix}, \quad \tilde{B}(m,n) = \begin{bmatrix} \Gamma_0(m,n) \\ I \end{bmatrix}, \\ &\tilde{K}(m,n) = [K(m,n) \quad 0] \end{aligned} \tag{15}$$

Rewrite system (6) into: $z(k+1) = [\tilde{A}(m,n) + \tilde{B}(m,n)\tilde{K}(m,n)]z(k)$, then by applying Theorem 1, we can easy proof this theorem.

4 Numerical Example

Consider the following system

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \tag{16}$$

where u is the control input for the continuous-time linear plant. According to the driven mode and active time-varying sampling method proposed in this paper, the state feedback controller for discrete-time plant in the NCS should be $u(t) = u(i_k) = K(i_k)x(i_k), t_k \leq t < t_{k+1}$

Suppose the length of gridded equidistant small interval l is 0.05 ms, $\tau_k = \{0.05$ ms, 0.1 ms, 0.15 ms$\}$, $d_k \in \{0, 1, 2\}$, the transition probability matrices are given by

$$\Upsilon = \begin{bmatrix} 0.6 & 0.3 & 0.1 \\ 0.5 & 0.3 & 0.2 \\ 0.7 & 0.2 & 0.1 \end{bmatrix}, \quad \Pi = \begin{bmatrix} 0.2 & 0.5 & 0.3 \\ 0.5 & 0.4 & 0.1 \\ 0.6 & 0.3 & 0.1 \end{bmatrix} \tag{17}$$

The state feedback controller gains will be calculated by Matlab LMI Control Toolbox, the results are as follows:

$$\begin{aligned} K(1,\ 0) &= [\text{-}3.9926 \quad \text{-}4.1845]; K(2,0) = [\text{-}4.5639 \quad \text{-}4.9267]; K(3,0) = [\text{-}5.3038 \quad \text{-}5.9421]; \\ K(1,\ 1) &= [\text{-}2.4297 \quad \text{-}2.8355]; K(2,1) = [\text{-}2.4823 \quad \text{-}3.0359]; K(3,1) = [\text{-}2.5342 \quad \text{-}3.2598]; \\ K(1,\ 2) &= [\text{-}1.5838 \quad \text{-}2.1412]; K(2,2) = [\text{-}1.5801 \quad \text{-}2.2609]; K(3,2) = [\text{-}1.5824 \quad \text{-}2.3977]; \end{aligned} \tag{18}$$

The state trajectories of NCS with the feedback control law are proposed in Fig. 2, which shows the networked control system is stochastically stable even if there exist time delays and packet dropouts.

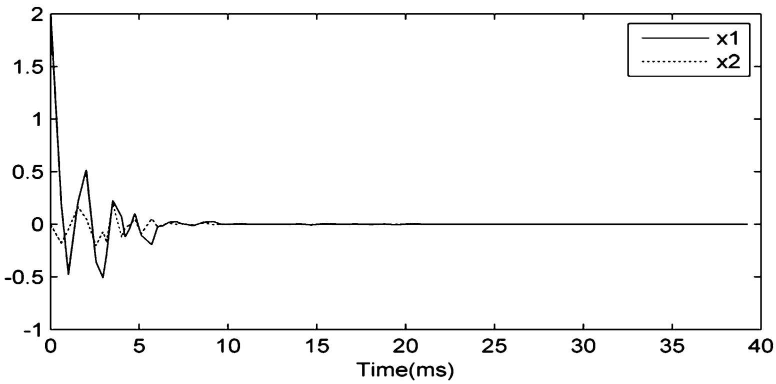

Fig. 2 States trajectory of NCS

5 Conclusion

This paper studies the stochastic stability problem of networked control systems by modeling NCS as a two mode Markovian jump linear system. The random time delays and packet dropouts are driven by two Markov chains. Sufficient conditions of stochastic stability for the jump linear systems are given in terms of a set of LMIs. To solve the LMIs for obtaining feedback gains, the "gridding approach" is adopted to guarantee the LMIs set for the jump linear systems with finite jump modes. Numerical examples illustrate the effectiveness of the proposed strategy for the stochastic stabilizing controller over NCS.

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