Chapter 1

Notation

Throughout this text, we will be regularly referring to the space-time norms

$$\|u\|_{L_t^q L_x^r(\mathbb{R}\times\mathbb{R}^d)} := \left(\int_{\mathbb{R}} \left[\int_{\mathbb{R}^d} |u(t,x)|^r \, dx\right]^{\frac{q}{r}} dt\right)^{\frac{1}{q}}, \tag{1.1}$$

with obvious changes if q or r are infinity. We will often use the abbreviation

$$\|f\|_r := \|f\|_{L_x^r} \quad \text{and} \quad \|u\|_{q,r} := \|u\|_{L_t^q L_x^r}.$$

We write $X \lesssim Y$ to indicate that $X \leq CY$ for some constant C, which is permitted to depend on the ambient spatial dimension, d, without further comment. Other dependencies of C will be indicated with subscripts, for example, $X \lesssim_u Y$. We will write $X \sim Y$ to indicate that $X \lesssim Y \lesssim X$.

We use the 'Japanese bracket' convention: $\langle x\rangle := (1+|x|^2)^{1/2}$ as well as $\langle\nabla\rangle := (1-\Delta)^{1/2}$. Similarly, $|\nabla|^s$ denotes the Fourier multiplier with symbol $|\xi|^s$. These are used to define the Sobolev norms

$$\|f\|_{H^{s,r}} := \|\langle\nabla\rangle^s f\|_{L_x^r} \quad \text{and} \quad \|f\|_{\dot H^{s,r}} := \||\nabla|^s f\|_{L_x^r}.$$

When $r=2$ we abbreviate $H^s = H^{s,2}$ and $\dot H^s = \dot H^{s,2}$.

Our convention for the Fourier transform is

$$\hat f(\xi) = (2\pi)^{-\frac{d}{2}} \int_{\mathbb{R}^d} e^{-ix\cdot\xi} f(x)\, dx,$$

so that

$$f(x) = (2\pi)^{-\frac{d}{2}} \int_{\mathbb{R}^d} e^{ix\cdot\xi} \hat f(\xi)\, d\xi \quad \text{and} \quad \int_{\mathbb{R}^d} |\hat f(\xi)|^2 \, d\xi = \int_{\mathbb{R}^d} |f(x)|^2 \, dx.$$

Notations associated to Littlewood–Paley projections are discussed in the Appendix (Chapter 11).

H. Koch et al., *Dispersive Equations and Nonlinear Waves: Generalized Korteweg–de Vries, Nonlinear Schrödinger, Wave and Schrödinger Maps*, Oberwolfach Seminars 45,
DOI 10.1007/978-3-0348-0736-4_15, © Springer Basel 2014