Chapter 9
Subdynamics in Knowledge-Based Systems

Using a set of six equations, I propose to model "interactions," the "organization of meaning," and "self-organization" as three coordination mechanisms among expectations; three further equations can be derived to operationalize "double contingency," "identity," and "reflection." One can expect that the subdynamics update one another in co-evolutions as feedbacks and feed-forwards. Interfaces among two (sub)dynamics can be expected to operate with time differences (Δt). Interactions among horizontal and vertical time differences can generate hyper-incursivity in interhuman communications. Hyper-incursion enables us to reconstruct expectations. The social system is probably the only system which can be expected to carry "strong" anticipation while being reproduced as expectations. A system of expectations is not alive, is not constrained by a life-cycle, and does not need to "exist" otherwise than as expectations. The dynamics against the arrow of time are "cultural": they rest on codes as the pillars of discursive knowledge driven upward into horizons of meaning.

In addition to his many discoveries about mental illnesses, Freud carved out the epistemological boundaries of the psychological domain with biology on the one side and sociology on the other. Using the metaphor of *Ego* sitting as a rider on a horse (Freud 1933), *Ego* is not to be considered as an energy system but as a "cybernetic" (Parsons 1958, p. 88, note 16). In relation to sociology, Freud commented (at a workshop in Vienna in 1926) "that he felt like the skipper of a barge who had always hugged the coast, who had now learned that others, more adventurous, had set out for the open sea." He wished them well, but he could no longer participate in their endeavor (Waelder 1958, at pp. 243f.). Parsons (1968), however, argued that Freud himself—approximately at the same time as Durkheim (e.g., 1894, 1912)—had *discovered* the social as the proper subject of sociology. He summarized Freud's demarcation of sociology from psychology, as follows:

> Relatively early, Freud gained the insight that the expression of instinctual need was regulated by the society's moral standards—often, but in no simple sense always, in conflict with instinctual needs—and that these standards were *introjected* into the personality itself, becoming components of its structure. The final form of this conception crystallized about the famous idea of the superego. Later this basic mode of conceptualization was extended to

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> the social environment, conceived of as an environment much in the Cartesian-Durkheimian sense. The famous "reality principle" came to focus on "object relations," which for Freud meant relations to other persons, especially the parents, considered as agents of socialization. But these human objects were not only "adapted to" in the sense true for physical objects; they were also introjected—or, as we now usually say, internalized—to form part of the personality structure, particularly of the ego, in Freud's sense. (p. 432).

Why had Freud himself become reluctant to investigate the social at the above-individual level. Parsons (1952) formulated a begin of an answer to this question, as follows:

> The inescapable conclusion is that not only moral standards, but all the components of the common culture are internalized as part of the personality structure. Moral standards, indeed, cannot in this respect be dissociated from the content of the orientation patterns which they regulate; as I have pointed out, the content of both cathectic-attitudes and cognitive-status definitions have cultural, hence normative significance. This content is cultural and learned. (p. 23)

Parsons saw a possibility to relate Freud's concept of internalization to central tenets of American pragmatism. "Society," as Cooley (1902) argued, exists inside the individual in the form of language and thoughts. Action is then based on reflexive selections among options. On this basis, Parsons (1951, p. 94) formulated the concept of double contingency as the cornerstone of social order.

"Double contingency" means that each of us (*Ego*) expects another human being (*Alter*) to entertain expectations as we entertain them ourselves (Elmer 1995; Vanderstraeten 2002). A second contingency among expectations comes on top of the first contingency of empirical processes in the physical and biological domains. In this model, both consciousness and communication develop in substantive and reflexive layers in parallel. The communicative structures are double-layered: they are both actions and pervade actions to various extents. However, the relations between the two contingencies are asymmetrical. The first contingency (*res extensa*) is internalized in the second (*res cogitans*); the second leaves traces (e.g., cultural artefacts at the social level and memory traces at the individual level) in the first.

9.1 "Double Contingency" and Inter-human Interactions

"Double contingency"can elegantly be specified in terms of the theory of anticipatory systems (using Eq. 8.8 in Chap. 8; cf. Dubois 2000, 2003) as follows:

$$x_t = ax_{t+1}(1 - x_{t+1}); \quad 0 \geq x > 1 \tag{9.1}$$

In words: *Ego* (x) operates in the present (as x_t) on the basis of an expectation of her own next state (x_{t+1}) and the anticipated next state of *Alter* $(1 - x)_{t+1}$. Note that the expectation of *Alter* $(1 - x)_{t+1}$ is here defined in terms of *Ego*'s own expectations about *non-Ego*; that is, $(1 - x)$. The expectations constructed in one's mind about oneself and *Alter* precede possible *communication* between *Ego*'s and *Alter*'s expectations about each other. *Alter* is processed in terms of awareness (Husserl, 1931) without necessarily implying externalization into a communication (Nonaka & Takeuchi, 1995).

Not incidentally, Husserl added the word "Cartesian" to the title of his *Cartesian Meditations.* In Descartes' philosophy, *Alter* of the contingent *Ego* is a Transcendency ("God"). *Ego* knows herself to be fallible and uncertain, but *Alter* is Perfect and Infallible from this perspective. In the (Christian) cosmology, God "is" beyond any doubt because absence would be incomplete. Descartes himself thus constructed an ontological proof of God's existence.

More than 150 years later, Kant (1787: B620–B630) refuted Descartes' inference. He concluded that one can remain agnostic about the existence of God. However, one cannot remain indifferent about the secularized Other. The relation to contingent others is discussed by Kant (1788) in the *Critique of Practical Reason* which follows upon the *Critique of Pure Reason.* Unlike *Ego*'s relation to God, its relations with others are empirical.

How can relations in the second contingency be analyzed? The specification of relations among expectations generates empty boxes to be potentially filled by observations—in other words, redundancy. Hesitant about the possibility to study these empty boxes empirically, Ulanowicz (2014) proposed calling them "apophatic": a biologist has no instrument to measure or theorize empty boxes which do not "exist." Can one, alternatively, perhaps specify what they are *not* (Bateson 1934), and thus specify ranges of possible expectations?

Double contingency provides a micro-foundation of the social. The expectation of *Alter* precedes the interaction, but cannot be reduced to individual action. Parsons (1968) concluded that society therefore can be considered as a category *sui generis* (Parsons, 1968). Luhmann (1977, at p. 70) added to Parsons' definition that "double contingency" can also be considered as the auto-catalyst of social processes between reflexive individuals. Furthermore, Luhmann added that reflexive relations are possible not only among human beings, but also among the codes of the communications (1995, pp. 105f.; cf. Künzler 1989; Strydom 1999). In another, but similar formulation, Giddens (1979) denoted a "double hermeneutics" between the analyst's and the participant's level of action and accounting. (The participant and the analyst can be the same person embedded in different exchanges and discourses.) Each perspective may lead to new horizons.

In such a complex dynamic, the algorithmic approach—using simulations—can be helpful (Hanneman 1988). Simulations require less ambiguous and more parsimonious definitions. The time subscripts allow us to follow the developments when the referent is changing with the description. Simulations can serve the analysis by pointing to the unexpected or unintended consequences of interactions among subroutines (Hedström 2005).

9.2 Simulations of the Second Contingency

In this chapter, I elaborate on interaction, organization, and the self-organization of meaning using the set of incursive and hyper-incursive equations developed in Chap. 8. Two further equations will be derived which can be used to operationalize "reflection" and "identity," respectively. As in the case of "double contingency," these two hyper-incursive mechanisms can be expected to operate "genotypically"; that is, as evolutionary dynamics without reference to a specific and historical state.

1. Eq. 9.1 (above) provided us with a model for double contingency. Double contingency in mutual expectations precedes interaction. The term $(1 - x_{t+1})$ in Eq. 9.1 models a selection of *Ego*'s expectations of *Alter* as non-*Ego*.
2. Interactions imply a historical instantiation. However, one can expect each *Alter* (y) to entertain as another *Ego* an analogous selection term $(1 - y_{t+1})$. The selection terms can operate upon each other and thus lead to the quadratic Eq. 9.2:

$$x_t = b(1 - x_{t+1})(1 - y_{t+1}) \tag{9.2}$$

   Equation 9.2 does not contain any reference to a previous state of the system itself (x_{t-1}). In this model, only *expectations* are operating selectively upon each other. Unlike double contingency, however, this equation models the interactions between *Ego*'s and *Alter*'s expectations.
3. Eq. 9.2 can be extended to more complex configurations by adding a third selection environment. One can add this third (or each next) term as either a hyper-incursive or incursive routine, and thus obtain the following *two* equations:

$$x_t = c(1 - x_{t+1})(1 - x_{t+1})(1 - x_{t+1}) \tag{9.3}$$

$$x_t = d(1 - x_{t+1})(1 - x_{t+1})(1 - x_t) \tag{9.4}$$

Equation 9.3 is a cubic equation which models a "triple contingency" of expectations. The third contingency closes the triad operationally. As argued in Chap. 5, triadic closure is the basis of the system's morphogenesis. All higher-order configurations (quadruplets, etc.) can be decomposed into triads. Equation 9.3 is thus constitutive of the social system of supra-individual expectations. As shown above, the interactions among three selection mechanisms can generate redundancies.

In a paper entitled "Triple Contingency: The theoretical problem of the public in communication societies," Strydom (1999) argued that "the increasing differentiation and organization of communication processes eventuated in the recognition of the epistemic authority of the public, which in turn compels us to conceptualize a new level of contingency. A first step is thus taken to capture the role of the public as a code in communication societies. The code mediates and shapes a "triple contingency." According to Strydom, this differentiation of "public" versus "private" as codes in the communication generated modernity. Note that the public is not considered as a sphere (Habermas 1974) or an audience (Latour 1988), but as a code in the communication.

One can derive that Eq. 9.3 has one real and two complex roots. Since a system cannot continue its operations with the complex solutions, Eq. 9.3 would evolve increasingly into a single value ("eigenvalue") for each value of the parameter *C*. The parameter *C* can thus be considered as a representation of the code of the communication. Horizontal differentiation of this code can then be captured by writing lower-case c_1, c_2, c_3, …, c_n, etc. I elaborate this below.

Note that if only a single fixed code-value *C* would operate, the routine would self-organize "closure" into this value of *C*. In a differentiated system of communications, however, a number of values for the codes (c_1, c_2, c_3, …. c_n) can be expected to disturb tendencies to such operational closure. As argued above in Chap. 5, three (or more) contingencies operating selectively upon one another can shape a fractal manifold containing trade-offs between tendencies to self-organizing closure and organizational interruptions (Ivanova & Leydesdorff, 2015).

4. Eq. 9.4 differs from Eq. 9.3 in terms of the time subscript in the right-most factor. Equation 9.4 can be used to model a specific organization of meanings as an instantiation in the present. The reference to the present in the third factor makes this model historical, whereas the self-organizing system modeled in Eq. 9.3 operates hyper-incursively, in terms of interactions among expectations about possible future states. An instantiation, however, requires (provisional) integration and organization at specific moments of time. In Eq. 9.4, the interaction among expectations is instantiated as a specific configuration at time $t = \mathrm{t}$.

In summary, Eqs. 9.3 and 9.4 model algorithmically the trade-off between the evolutionary and historical perspectives in Triple-Helix relations as discussed in terms of redundancy and information generation in Chap. 5 above.

5. Two more hyper-incursive equations follow as possible members of this family of equations. Analogously to Eq. 9.1, one can formulate as follows:

$$x_t = a x_t (1 - x_{t+1}) \tag{9.5}$$

$$x_t = a x_{t+1} (1 - x_t) \tag{9.6}$$

Equation 9.5 evolves into: $x = (a - 1)/a$ (see Eq. 8.11c on p. 157 for the derivation of this equation as a steady state). It follows that *x* is a constant for all values of *a*. I submit, as an interpretation, that this evolution towards a constant value of the system (*x*) through anticipation can be considered as the self-reference of an expected "identity."

In the second contingency, identity is based not on the history of previous states, but on entertaining the expectation of continuity of the "self." The identity in the network "me" can be distinguished from the "I" (Mead, 1934, at pp. 26f.). Like individuals, organizations can be expected to develop a symbolic identity in the second contingency.

Equation 9.6 can be developed as follows:

$$x_t = ax_{t+1}(1 - x_t) \tag{9.6a}$$

$$ax_{t+1} = x_t/(1 - x_t) \tag{9.6b}$$

$$x_{t+1} = (1/a)[x_t/(1 - x_t)] \tag{9.6c}$$

This model can be simulated (Fig. 9.1): when $x_t > [a/(1 + a)]$, a pulse is generated which first overshoots the value of one (in a virtual domain of possible expectations), but then generates a negative value. This negative value leads to a mirror image of a representation at a specific moment in time, and can thus be considered as a reflection. Reflections enable us to bounce a communication between communication (sub)systems via consciousness (Luhmann, 1988, 1991).

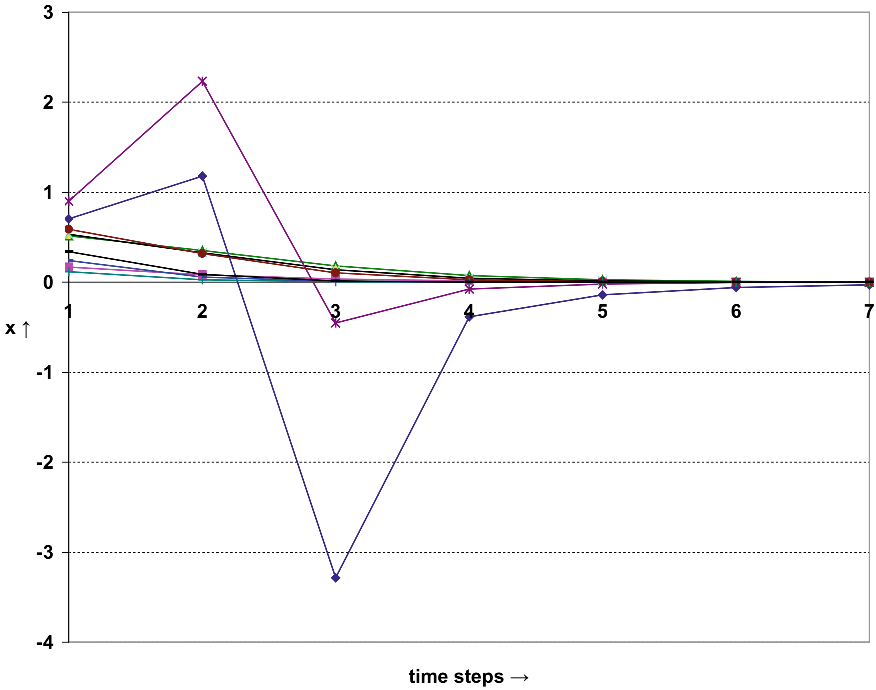

Fig. 9.1 Simulation of Eq. 9.6: the value of x at $t = 1$ is drawn randomly ($a = 4$). (For $a = 4$, the pulse is generated for values of $x_t > 0.8$.)

Dubois (*personal communication*, 16 July 2008)[1] noted that Eq. 9.6 can be derived as the *time inverse* of the incursive (Pearl-Verhulst) equation [$x_t = ax_{t-1}\ (1 - x_t)$]. In Leydesdorff and Dubois (2004), we showed that the sole assumption of social relatedness as a variable among groups of agents provides a sufficient basis for deriving the logistic map as a first-order approximation of the social system. Secondly, I derived the anticipatory formulation of this equation for anticipation in both the interaction term and in the aggregation among subgroups. I will not repeat this argument, but instead follow a more intuitively accessible reasoning based on the *perturbed recursion model* in order first to derive a generalization of the family of logistic equations under discussion here.

9.3 Perturbed Recursions and Incursions

Andersen (2002, at pp. 170 ff.) discussed the logistic equation as a special case of his so-called *perturbed recursion model.* The model can be depicted as follows:

In Fig. 9.2, F is a recursive function that transforms state S_{t-1} into a new state S_t, using a set of parameters $P = p_1, \ldots, p_n$ modelling disturbances. In formula format:

$$S_t = F(S_{t-1}, P) \tag{9.7}$$

Baecker (2002, at pp. 86 ff.) noted that the function F can be considered as the operator of the communication system (S) in Luhmann's model; he used the word "eigen-function" for this recursive loop (cf. von Foerster, 1960). However, more than a single such eigenfunction (i.e., code) can be expected in a differentiated system. Each code structures variation differently. In other words, the recursive selections can be expected to codify different meanings along the main axes of the network.

[1] Dubois added to this communication the following derivation as evidence:

$$x_{t+1} = ax_t(1 - x_{t+1}) \tag{n9.6a}$$

is equal to the following equation for dt = 1:

$$x_{t+dt} = ax_t(1 - x_{t+dt}) \tag{n9.6b}$$

The time reverse of this equation is obtained for dt → –dt, with the negative discrete time –dt:

$$x_{t-dt} = ax_t(1 - x_{t-dt}) \tag{n9.6c}$$

or, with –dt = –1:

$$x_{t-1} = ax_t(1 - x_{t-1}) \tag{n9.6d}$$

So with a time translation of t → t + 1 for the whole equation, one obtains:

$$x_t = ax_{t+1}(1 - x_t) \tag{9.6}$$

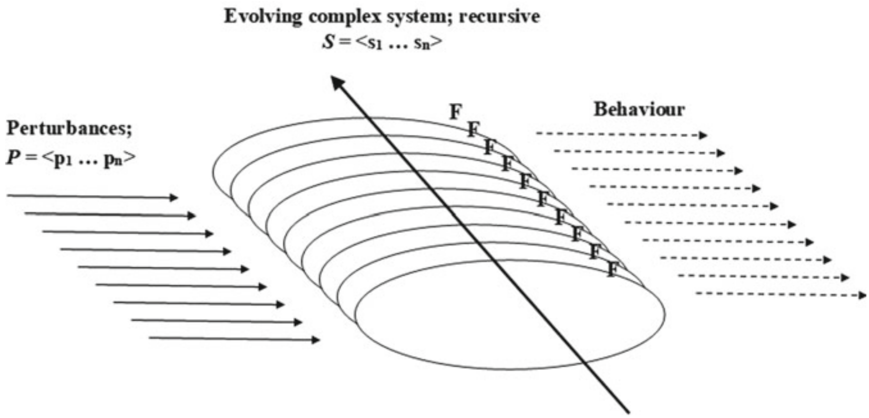

Fig. 9.2 The perturbed recursion model (Adapted from Andersen, 2002, at p. 170 and Leydesdorff, 2001, p. 102.)

Following Baecker (2002), one can add a subscript *c* (of *coding*) to the operation and use the *E* of environment as the source of disturbances; one can then rewrite Eq. 9.7 as follows:

$$S_t = F_c(S_{t-1}, E) \tag{9.8}$$

When differentiation prevails, the environment *E* of each subsystem is composed of the other subsystems, with a remaining term ε as representation of the residual (that is, as yet undifferentiated) environment. Using the lower-case *f* and *s* for the *sub*systems, one can then rewrite Eq. 9.8 as follows:

$$s_{i,t} = f_i\left(s_{i,t-1}, s_{j,t-1}, s_{k,t-1}, s_{l,t-1}, \ldots, \varepsilon\right) \tag{9.9}$$

The windowing of the subsystems upon each other is based on horizontal differentiations. Each subsystem (*i*) codes (using f_i) its own previous development and the development which it finds in its (selection of) relevant environments. However, meaning can be provided to this development by the other (selecting) subsystems from the perspective of hindsight. This adds a vertical differentiation between historical recursion and reflexive incursion, as follows:

$$s_{i,t} = f_i\left(s_{i,t-1}, s_{j,t}, s_{k,t}, s_{l,t}, \ldots, \varepsilon\right) \tag{9.10}$$

The vertical differentiation between recursion and incursion is based on the dynamics of meaning and codification; the horizontal one is based on functional differentiation. The state of a specific subsystem ($s_{i,t}$) can now be dependent both on the previous state of this subsystem ($s_{i,t-1}$), and on the previous and the current states of the other subsystems. Systems and subsystems continue to operate historically (Eq. 9.9) and to provide meanings to one another (Eq. 9.10). In discrete time, however, the recursive and incursive operations are not *ex ante* synchronized and can therefore be expected to differ with a Δt at each interface. One can expect the routines to update one another.

At all these interfaces one can thus expect asynchronicities. Using these asynchronicities from the backward perspective of incursion, meanings can be propelled both horizontally and vertically. The incursions can be formalized in a manner analogous to that of the incursive formulation of the logistic equation provided in Eq. 9.5, but multiplied by references to the selection requirements of other subsystems *j*. In formula format:

$$x_{i,t} = ax_{i,t-1} \prod_{j=1}^{n} (1 - x_{j,t}) \cdot \varepsilon \tag{9.11}$$

In Eq. 9.11, *n* represents the theoretically expected number of subsystems. While this number was analytically restricted in Parsons's structural-functionalism—using his so-called *four*-function paradigm—this number can vary in Luhmann's (1997) theory with the historical development of the media of communication and their symbolic generalization into codes (Distin, 2010; cf. Merton, 1938, 1948). As noted, Simon (1973) conjectured that there may be an alphabet of possible codes in interhuman communications.

On the basis of this general model one can consider, for example, the incursive and recursive version of the Triple-Helix model as special cases. Incursively, three relevant selection environments operate on the development of the resulting arrangements, as follows:

$$x_{it} = ax_{j,t-1}\big(1 - x_{j,t}\big)\big(1 - x_{k,t}\big)\big(1 - x_{l,t}\big) \cdot \varepsilon \tag{9.12}$$

In other words, the evolutionary TH model is incursive; the institutional one focuses on historical recursions, such as trajectories and historical transitions following the arrow of time.

9.4 Transversal and Longitudinal Propagation of Meanings

Because the differentiation can lead to asynchronicities and therefore ΔT-values specific for each interface, some models can be expected to advance more rapidly than others. For example, the market can be expected to operate faster than a research process; the difference in speed may lead to delays at each interface. Interface management in a knowledge-based corporation is meant to align time horizons: "whichtechnology can be introduced on the market at which moment in the future?" The models are synchronized at $t = \mathrm{t}$ by decisions about future options and historical constraints at interfaces. This synchronization is based on an expected time difference at interface.

While vertical differentiation—in terms of interactions, organization, and self-organization—was already available in pre-modern societies (High Cultures, as in the Middle Ages), updates over time provide *modern* societies with another, that is, second mechanism for organizing reflexive communications in an anticipatory mode. The horizontal differentiation is subordinate to the vertical in pre-modern societies; in modern societies, however, the horizontal differentiation is coded, and this coding provides another degree of freedom. The acceleration by organization in a High Culture is replaced by one based on trade-offs between stabilizing organization and globalizing self-organization in the communications.

Each interface—both in the horizontal and vertical dimensions—can contain a time difference Δt. The two incursive terms in the corresponding equations can also operate upon each other. This may lead to hyper-incursion. Hyperincursion at the regime level cannot be historically manifest, but the feedback of hyperincursion on the incursion can leave a footprint. The hyper-incursive regime and the historical trajectory co-evolve in terms of providing (potentially different—local and global) meanings to the events.

Each incursive equation contains a reference to the historical dynamics and another to the evolutionary one. Interactions between vertical and horizontals incursions can generate a quadratic incursion or, in other words, next-order hyper-incursion (Fig. 9.3). Since a hyper-incursive system operates against the arrow of time, *one can expect this routine to generate redundancy instead of information*, since the arrow of time is inverted.

Incursions contain both a reference to the current and the previous state. When historical incursions interact *recursively,* a variation (entropy) is generated. However,

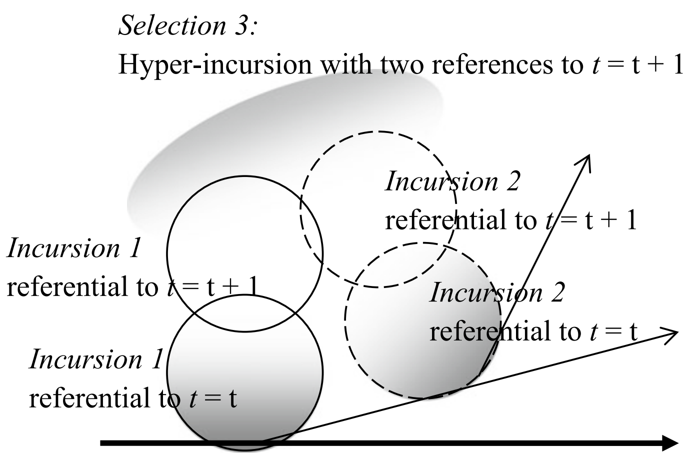

Fig. 9.3 Hyper-incursivity at the interface emerging at $t = \mathrm{t} + 1$ between two incursive routines. *Source* Laydesdorff (2009, p. 21)

when two different incursions are interfaced *incursively*, hyper-incursion can be expected. Each incursion generates both information and meaning, and thus becomes organized both historically (with reference to t = t) and reflexively (with reference to t = (t + Δt)). However, a hyper-incursive equation cannot be organized and materialized at t = t because it only contains references to future states [at t = (t + Δt)]. The resulting hyper-incursion builds on the interactions between references to present or future states in two (or more) underlying incursions. Since this hyper-incursion no longer contains a reference to historical time, *an incursive subdynamic is additionally needed for making the (interactions among) expectations historically relevant* (Eq. 9.4).

Luhmann (2000) hypothesized that self-organization among the differently coded fluxes of communication can be brought under organizational control by decisions. This formulation may sound action-based, but this inference would be too fast. The third contingency can be provided by another code operating incursively; for example, by taking decisions.

9.5 Decisions and Decision Rules

In Chap. 8, the hyper-incursive formulation of the logistic equation (Eq. 8.8) was elaborated into:

$$x_{t+1} = 1/2 \pm 1/2\sqrt{[1 - (4/a)x_t]} \tag{9.13}$$

Depending on the plus or the minus sign in the equation, two future states are generated at each time step. Since this formula is iterative, the number of future states doubles with each next time step. After N time steps, 2^N future states would be possible. For $N = 10$, the number of options is more than one thousand.

Dubois (2003, at p. 115) proposed a decision function u(t) for making a choice between one of two options:

$$u(\mathrm{t}) = 2\, d(\mathrm{t}) - 1 \tag{9.14}$$

It follows that $u = +1$ for the decision $d = 1$ (true), and $u = -1$ for the decision $d = 0$ (false). In a social system, however, more choices than these two extremes may be possible. Social systems operate in a distributed mode with a probability distribution of preferences. In distributed systems, decisions can be organized and codified into decision rules (Bertsekas & Tsitsiklis, 1989). Luhmann (2000) argued that organizations can be considered as the results of codifications in the dynamics of decision-making. The stabilization of decisions in rules, for example, can generate an institutional layer of the social system in which decision-making can develop routines and thus follow trajectories.

Note that the decisions (at the individual level) or decision rules (at the organizational level) do not determine the hyper-incursive dynamics of the self-organizing regime, but only guide these dynamics historically as instantiations along trajectories. *Autopoiesis* controls its own operation (Varela, 1979); decisions do not have to be taken at each step. The distribution of decisions and non-decisions changes the historical conditions by inducing reorganization. Without this historical opportunity to anchor the routines, the interfacing of expectations would remain in an elusive realm of expectations operating on expectations.

9.6 Inter-human Coordination in the Second Contingency

I distinguished three coordination mechanisms above: interaction, organization, and self-organization. I will now discuss these three equations in more detail.

9.6.1 Interactions

As noted, anticipatory interaction can be modeled based on the mutual selections of *Ego*'s and *Alter*'s expectations of each other, leading to Eq. 9.2 (above):

$$x_t = b(1 - x_{t+1})(1 - x_{t+1}) \tag{9.2}$$

Equation 9.2 does not include a term referring to previous states; only expectations are operating selectively upon each other. Unlike "double contingency," interaction is a social dynamic and no longer an individual (mental) one. (I use "*b*" instead of "*a*" for the parameter in order to highlight this difference.)

Equation 9.2. can be elaborated as follows:

$$x_t = b(1 - x_{t+1})(1 - x_{t+1}) \tag{9.2}$$

$$x_t/b = 1 - 2x_{t+1} + x_{t+1}^2 \tag{9.2a}$$

$$x_{t+1}^2 - 2x_{t+1} + (1 - x_t/b) = 0 \tag{9.2b}$$

$$x_{t+1} = 1 \pm \sqrt{x_t/b} \tag{9.2c}$$

This interaction system can be simulated as the following oscillation (Fig. 9.4):

The interactions oscillate in Fig. 9.3 around the value of one. The system reaches its largest fluctuations (between zero and two) for $b = 2$.[2] On each side, the interaction

[2]The system vanishes for $b < 2$ because the term under the root can then become larger than one, and therefore $x_{t+1} < 0$ in case of the (possibly) random choice of the minus sign in Eq. 9.2c.

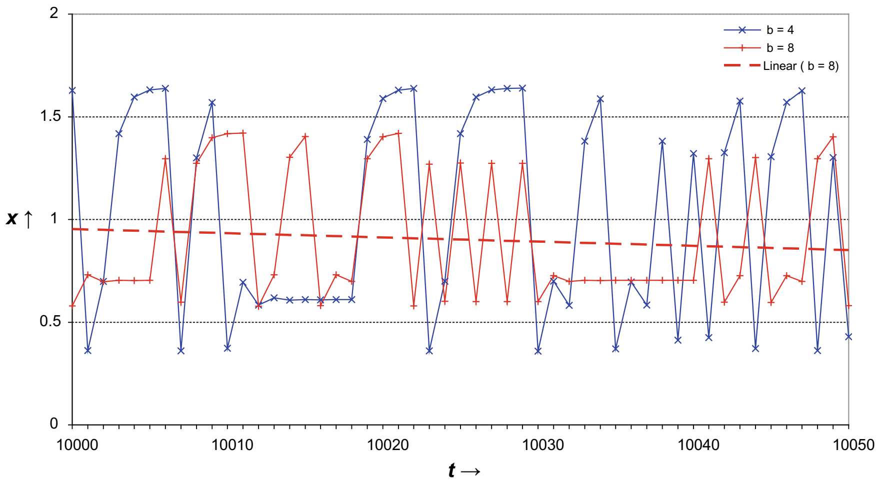

Fig. 9.4 Simulation of hyper-incursive interactions

can be continued for a number of iterations before the alternate resumes its operation. I modeled this here (in Excel) by using a random number to choose the plus or minus sign in the evaluation of Eq. 9.2c. The (potentially random) variation warrants the continuation of the interaction. In other words, these hyper-incursive interactions serve to generate variation in the *cogitatum*, such as, among other things, the communication of newness supporting and inducing the *morphogenesis* of organization and self-organization (Achterberg & Vriens, 2009).

9.6.2 Organization versus Self-organization

Whereas two selection environments are needed for interactions, one can add either a hyper-incursive or an incursive subroutine to Eq. 9.2 (above) for modelling organization and self-organization. At each addition, one obtains two possible equations:

$$x_t = c(1 - x_{t+1})(1 - x_{t+1})(1 - x_{t+1}) \tag{9.3}$$

$$x_t = d(1 - x_{t+1})(1 - x_{t+1})(1 - x_t) \tag{9.4}$$

As noted, Eq. 9.4 differs from Eq. 9.3 in the time subscript of x in the third factor. The reference to the present in this third factor bends the system back to its present state and thus makes this model historical, whereas the self-organizing system of Eq. 9.3 operates hyper-incursively. In the case of Eq. 9.4, however, the interaction among expectations can be instantiated by a specific historical organization at $t =$ t.

The roots of Eq. 9.4 can be derived (analogously to Eq. 9.2) as follows:

$$x_t = d(1 - x_{t+1})(1 - x_{t+1})(1 - x_t) \tag{9.4}$$

$$x_{t+1}^2 - 2x_{t+1} + 1 - x_t/[d(1 - x_t)] = 0 \tag{9.4a}$$

$$x_{t+1} = 1 \pm \sqrt{x_t/d(1 - x_t)} \tag{9.4b}$$

Simulation of this system shows that the organization of communications vanishes after a variable number of steps for all values of the parameter d (Fig. 9.5).

Figure 9.5 shows this development using Excel for the simulations. However, Excel depicts the historical end of the organization of communications as zeros, while these zeros may be based on values of $x > 1$ which lead to a negative value of the denominator of the term under the root in Eq. 9.6b. In this case, the root of this equation is complex and can no longer be evaluated. In other words, the organization does not disappear in the sense of "dying," but its historical development can be insufficiently complex to instantiate self-organization among the fluxes of communication. Note that the self-organizing dynamic does not have to be instantiated at each step.

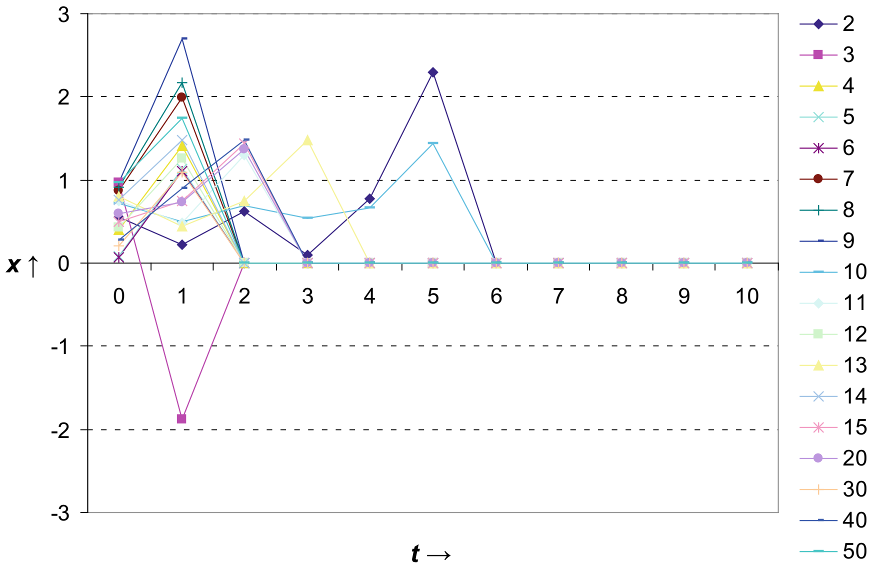

Fig. 9.5 *Organization* of interactions for different values of the parameter d

9.6.3 Self-organization

Equation 9.3 can be developed as follows:

$$x_t = b(1 - x_{t+1})(1 - x_{t+1})(1 - x_{t+1}) \tag{9.3}$$

$$\frac{x_t}{b} = (1 - x_{t+1})^3 \tag{9.3a}$$

$$\left(\frac{x_t}{b}\right)^{1/3} = (1 - x_{t+1}) \tag{9.3b}$$

$$x_{t+1} = 1 - \left(\frac{x_t}{b}\right)^{1/3} \tag{9.3c}$$

Equation 9.3 has three roots of which two are complex (Mike Burke, personal communication, 10 October 2008). The real solution of Eq. 9.3c can be denoted as:

$$x_{t+1} = 1 - \sqrt[3]{\frac{x_t}{b}} \tag{9.18a}$$

and the two complex roots are:

$$x_{t+1} = 1 - \sqrt[3]{\frac{x_t}{c}}\left(\frac{-1 \pm i\sqrt{3}}{2}\right) \tag{9.18b}$$

Since the further operation cannot evaluate the complex solutions in a next time-step, the system in this case can be expected to continue with the real solution. This leads to a single and stable solution for each value of b. At the level of subsystems, these relative constants can be considered as the codes of the communication.

9.7 Discussion and Conclusions

In this chapter, I have related Dubois' incursive and hyper-incursive equations modeling anticipatory systems (Dubois, 1998, 2000, 2003) to Luhmann's (1995 [1984]) social systems theory. I used both theories heuristically; the construction of a relation between Luhmann's theory and Dubois' computations required a translation of the one theory into the other, and therefore minor changes.

Luhmann (1984, p. 605; 1995, pp. 446f.) noted that "[s]elf-referential, autopoietic reproduction would not be possible without an anticipatory recursivity." However, he did not specify the anticipatory mechanisms. In his final and summarizing book, Luhmann (1997) returned to this issue and provided two references to Rosen's (1984) book entitled *Anticipatory Systems*. However, these references were framed in a biological context ([1997, pp. 206 and 820] 2012, Vol. 1, p. 123 and Vol. 2, p. 137). Luhmann added (at p. 821) that in the domain of meaningful processing of information one cannot avoid a reference to the present when defining the relation between past and future. This time-dimension with the present at its origin is elaborated as a degree of freedom in the theory and computation of anticipatory systems. In other contexts, Luhmann (e.g., 1990a, b, p. 98, n.10) developed a semantics for the discussion of "time."

Relevant for this study is furthermore his distinction between social differentiation and systems differentiation (Luhmann, 1997)which accords to a high degree with Simon's (1973) distinction of horizontal versus vertical differentiation. Social differentiation is possible in the communication because communications can be coded in a variety of ways in language, and codes of communication can be generalized symbolically (Distin, 2010).

The different subsystems operate in parallel, but not necessarily synchronously. As Luhmann (1984, at p. 128) formulated: "*Social* differentiation serves as an uncoupling mechanism. It divides the time-orientations in the different systems and therefore accepts that things can be urgent in one system, while another system can take its time" (italics added, L.). *Systems* differentiation, however, organizes the social system at different levels. This differentiation of micro-level interactions, meso-level organization, and macro-level self-organization in the processing of meaning could be elaborated in this chapter in terms of six equations.

Furthermore, the model of recursive perturbations could be used to consider the various formulations of the logistic map as a family of equations. The spanning of the time dimension enabled me to reformulate some "paradoxes" in Luhmann's social-systems theory as questions amenable to empirical research about trade-offs: is positive entropy or redundancy (negative entropy) generated by self-organization prevailing? The duality of structure in social systems (Giddens, 1979, 1984) could thus be considered as a consequence of the difference between the forward and backward arrows represented as time-subscripts of Eqs. 9.3 and 9.4 (above), respectively.

Unlike interaction and self-organization, the *organization* of meaning is historically constrained; specific organizational forms can be replaced with other organizations in relation to (i) the ongoing interactions—generating variation from below—and (ii) the hyper-incursive self-organization of the communication into codes at a relatively global level (Eq. 9.4). Luhmann (1995, at p. 600n.[1984. at p. 551n.]) expressed the relationship among the three coordination mechanisms of expectations as follows:

> […] in all social relations, under all circumstances a difference between society and interaction is unavoidable, but not all societies are acquainted with organized social systems. We therefore exclude organizations, but only from treatment on the level of a general theory of social systems. On the next level, that is, of concretizing the theory, one would perhaps need to distinguish between societal systems, organizational systems, and interaction systems and develop separate theories for each type because these three separate ways of forming systems (i.e., dealing with doubling contingency) cannot be reduced to one another.

Organization structures communication at specific moments of time by using incursion and thus remains rooted in history. As against double contingency as the micro-foundamental operation at the level of the mind participating in communication, organizations can entertain different expectations synchronously because organizations are interfacing expectations (in the first two terms of Eq. 9.4) and looping into the present state x_t (in the third term of Eq. 9.4). From this perspective, the individual *cogitans* might be considered as a minimal form of organization among expectations. Agents can take decisions on the basis of trade-offs between differently coded considerations.

Organization in the communication of meaning is historical and can therefore be expected to develop along a trajectory for a number of time-steps. However, without further variation as input from below or codification from above, any specific organization of communications can be expected to erode in due time because the construction remains a superstructure on an underlying *information* flow generating uncertainty (Schattschneider, 1975). The organization of communication provided us with a basis for measurement. Meaning is historically instantiated in organizations; the imprint of self-organization at the organizational level can be measured as mutual *redundancy* (Chaps. 4–6 above). The suggested calculus of redundancy would enable us to test theoretical expectations empirically—and thus to obtain counter-intuitive results—whereas the calculus of anticipations in terms of incursions and hyper-incursions can be elaborated in terms of simulations which may enrich the intuitive understanding.

References

Achterbergh, J., & Vriens, D. (2009). *Organizations: Social systems conducting experiments.* Dordrecht: Springer.

Andersen, P. B. (2002). Dynamic semiotics. *Semiotica, 139*(1/4), 161–210.

Baecker, D. (2002). *Wozu Systeme?* Berlin: Kulturverlag Kadmos.

Bateson, G. (1972 [1934]). *Steps to an ecology of mind*. New York: Ballantine.

Bertsekas, D. P., & Tsitsiklis, J. N. (1989). *Parallel and distributed computation.* Englewood Cliffs: Prentice-Hall.

Cooley, C. H. (1902). *Human nature and the social order*. New York: Charles Scribner's Sons.

Distin, K. (2010). *Cultural evolution.* New York, NY: Cambridge University Press.

Dubois, D. M. (1998). Computing anticipatory systems with incursion and hyperincursion. In D. M. Dubois (Ed.), *Computing Anticipatory Systems, CASYS-First International Conference* (Vol. 437, pp. 3–29). Woodbury, NY: American Institute of Physics.

Dubois, D. M. (2000). Review of incursive, hyperincursive and anticipatory systems -- foundation of anticipation in electromagnetism. In D. M. Dubois (Ed.), *Computing Anticipatory Systems CASYS'99* (Vol. 517, pp. 3–30). Liege: Amercian Institute of Physics.

Dubois, D. M. (2003). Mathematical Foundations of Discrete and Functional Systems with Strong and Weak Anticipations. In M. V. Butz, O. Sigaud, & P. Gérard (Eds.), *Anticipatory behavior in adaptive learning systems (Lecture Notes in Artificial Intelligence)* (Vol. 2684, pp. 110–132). Berlin: Springer.

Durkheim, E. (1894). Les règles de la méthode sociologique. *Revue Philosophique de la France et de l'Étranger*, *37*, 465–498.

Durkheim, E. (1912). *Les formes élémentaires de la vie religieuse*. Paris: Félix Alcan.

Elmer, J. (1995). Blind me with science: Motifs of observation and temporality in Lacan and Luhmann. *Cultural Critique, 30,* 101–136.

Freud, S. (1933). *New introductory lectures on psychoanalysis*. New York, NY: Norton.

Giddens, A. (1979). *Central problems in social theory*. London: Macmillan.

Giddens, A. (1984). *The constitution of society*. Cambridge: Polity Press.

Habermas, J. (1974). *Strukturwandel der Öffentlichkeit*. Frankfurt a.M.: Suhrkamp.

Hanneman, R. (1988). *Computer-assisted theory building: Modeling dynamic social systems.* Newbury Park, etc.: Sage.

Hedström, P. (2005). *Dissecting the social: On the principles of analytical sociology*. Cambridge, UK: Cambridge University Press.

Ivanova, I. A., & Leydesdorff, L. (2015). Knowledge-generating efficiency in innovation systems: The acceleration of technological paradigm changes with increasing complexity. *Technological Forecasting and Social Change, 96,* 254–265. https://doi.org/10.1016/j.techfore.2015.04.001

Kant, I. 1781, [2]1787). Kritik der reinen Vernunft. Hamburg: Felix Meiner, 1956.

Kant, I. (1788). *Kritik der praktischen Vernunft*. Riga.

Künzler, J. (1989). *Medien und Gesellschaft: Die Medienkonzepte von Talcott Parsons, Jürgen Habermas und Niklas Luhmann*. Stuttgart: Ferdinand Enke Verlag.

Latour, B. (1988). the politics of explanation: An alternative. In S. Woolgar & M. Ashmore (Eds.), *Knowledge and reflexivity: New frontiers in the sociology of knowledge* (pp. 155–177). London: Sage.

Leydesdorff, L. (2001). *A sociological theory of communication: The self-organization of the knowledge-based society*. Parkland, FL: Universal Publishers; at http://www.universal-publishers.com/book.php?method=ISBN&book=1581126956.

Leydesdorff, L. (2008). The communication of meaning in anticipatory systems: A simulation study of the dynamics of intentionality in social interactions. In D. M. Dubois (Ed.), *Proceedings of the 8th International Conference on Computing Anticipatory Systems CASYS'07* (Vol. 1051 pp. 33–49). Melville, NY: American Institute of Physics Conference Proceedings.

Leydesdorff, L. (2009). The non-linear dynamics of meaning-processing in social systems. *Social Science Information*, *48*(1), 5–33.

Leydesdorff, L., & Dubois, D. M. (2004). Anticipation in social systems: The incursion and communication of meaning. *International Journal of Computing Anticipatory Systems, 15,* 203–216.

Leydesdorff, L. (2009). The non-linear dynamics of meaning-processing in social systems. *Social Science Information, 48*(1) 5–33.

Luhmann, N. (1977). Interpenetration – Zum Verhältnis personaler und sozialer Systeme. *Zeitschrift für Soziologie, 6*(1), 62–76.

Luhmann, N. (1984). *Soziale Systeme. Grundriß einer allgemeinen Theorie.* Frankfurt a. M.: Suhrkamp.

Luhmann, N. (1988). Wie ist Bewusstsein an Kommunikation beteiligt? In H. U. Gumbrecht & K. L. Pfeiffer (Eds.), *Materialität der Kommunikation* (pp. 884–905). Frankfurt a.M.: Suhrkamp.

Luhmann, N. (1990a). *Die Wissenschaft der Gesellschaft*. Frankfurt a.M.: Suhrkamp.

Luhmann, N. (1990b). Meaning as sociology's basic concept. *Essays on self-reference* (pp. 21–79). New York: Columbia University Press.

Luhmann, N. (1991). Die Form "Person". *Soziale Welt*, *42*(2), 166–175.

Luhmann, N. (1995). *Social Systems.* Stanford, CA: Stanford University Press.

Luhmann, N. (1997). Die Gesellschaft der Gesellschaft. *Frankfurt a.M*: Surhkamp.

Luhmann, N. (2000). *Organisation und Entscheidung*. Opladen: Westdeutscher Verlag.

Mead, G. H. (1934). *Mind, self, & society from the standpoint of a social behaviourist: Works of G. H. Mead*. Chicago: University of Chicago Press.

Merton, R. K. (1938). *Science, technology and Society in Seventeenth-Century England*. Bruges, Belgium: Saint Catherine Press.

Merton, R. K. (1948). The bearing of empirical research upon the development of social theory. *American Sociological Review, 13*(5), 505–515.
Nonaka, I., & Takeuchi, H. (1995). *The Knowledge Creating Company*. Oxford/New York: Oxford University Press.
Parsons, T. (1951). *The social system*. New York: The Free Press.
Parsons, T. (1952). Superego and the theory of social systems. In *Social structure and personality* (pp. 17–33). London: Routledge.
Parsons, T. (1958). Social structure and the development of personality: Freud's contribution to the integration of psychology and sociology. *Psychiatry, 21*(4), 321–340.
Parsons, T. (1968). Interaction: I. Social interaction. In D. L. Sills (Ed.), *The international encyclopedia of the social sciences* (Vol. 7, pp. 429–441). New York: McGraw-Hill.
Schattschneider, E. E. (1975). *The semisovereign people: A realist's view of democracy in America*. Belmont, CA: Wadsworth Publishing Company.
Simon, H. A. (1973). The Organization of Complex Systems. In H. H. Pattee (Ed.), Hierarchy Theory: *The Challenge of Complex Systems* (pp. 1–27). New York: George Braziller Inc.
Strydom, P. (1999). Triple contingency: The theoretical problem of the public in communication societies. *Philosophy & Social Criticism, 25*(2), 1–25.
Talcott Parsons, (2016) Social Structure and the Development of Personality. *Psychiatry*, *21*(4):321–340.
Ulanowicz, R. E. (2014). Reckoning the nonexistent: Putting the science right. *Ecological Modelling, 293,* 22–30.
Vanderstraeten, R. (2002). Parsons, Luhmann and the theorem of double contingency. *Journal of Classical Sociology, 2*(1), 77–92.
Varela, F. J. (1979). *Principles of biological autonomy*. Amsterdam: North Holland.
von Foerster, H. (1960). On self-organizing systems and their environments. In M. C. Yovits & S. Cameron (Eds.), *Self-organizing systems* (pp. 31–50). London: Pergamon Press.
Waelder, R. (1958). Neurotic ego distortion: Opening remarks to the panel discussion. *International Journal of Psycho-Analysis, 39*, 243–244.