Chapter 21
Uncomplexity as a Resource

In discussing computational work it is helpful to separate what can be done by reversible (unitary) operations from what can be done by applying measurements. Starting with a maximally complex state, or any other state, the complexity can be eliminated by measuring all Pauli Z operators. The outcome will be a unentangled product state which can be brought to the state $|000\ldots0\rangle$ by applying no more than K single qubit gates. This would have the effect of restoring the uncomplexity to its maximal value.[1]

But measurements are not reversible operations, and necessarily dump heat into the environment. By computational work I will mean the part of a protocol that can be achieved reversibly. For example we may want to reversibly transform the state from some input to a target state that has some information that we can later extract by making a measurement. The computational work by definition is associated with the *reversible transformation only*, and not with the *final measurement*.

For most purposes maximally complex pure states cannot be distinguished from maximally mixed density matrices. Both will give random results for almost all measurements. Since a reversible operation on a maximally mixed state does nothing, maximally mixed and maximally complex states are useless as a starting point for doing computational work.

Thus to do computational work we need some uncomplexity, i.e., some separation between the actual complexity of a computer, and maximal possible complexity 2^K. Uncomplexity is therefore a necessary resource, but it is not generally sufficient. What I am going to show you, by a concrete example, is how adding a single clean qubit to a maximally complex system of K qubits restores the ability to do a great deal of computational work.

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[1] Notice that we did the same thing by adding a single clean qubit.

L. Susskind, *Three Lectures on Complexity and Black Holes*,
SpringerBriefs in Physics, https://doi.org/10.1007/978-3-030-45109-7_21