Chapter 17 There Is, Probably, No Need for Such an Institution—The Freudenthal Institute in the Last Two Decades of the Twentieth Century



Jan de Lange

Abstract In the 1970s, IOWO became well-known in the mathematics education community. IOWO was an institute for the development of mathematics education, with Professor Hans Freudenthal as flag bearer and source of inspiration. For purely political reasons the government decided that there was no need for such an institution in the 1980s, and that all collaborators should move to SLO, the institute in the Netherlands that is responsible for curriculum development. Most people refused to accept this offer. Many letters were written by our international colleagues in order to let IOWO survive. The politicians found a very creative solution: five people were allowed to carry on within the university as researchers (only). In this chapter, I describe how the remaining people took back what was 'stolen' from them. Within ten years the government found that a new very successful institute had been established, and even was 'proud' of this institute for its innovative ideas, and practical uses, based on developmental research.

17.1 Introduction

In 1980 the institute named IOWO¹ was threatened in its existence. The Ministry of Education had concluded that "there was no need for such an institution." Many letters from colleagues all over the world convinced our government that they should insert the word 'probably'. So, five researchers, supported by three administrative staff, were relocated in a small institute, OW&OC,² as part of the Faculty of Mathematics of Utrecht University.

Faculty of Science, Freudenthal Institute, Utrecht University, Utrecht, The Netherlands e-mail: j.delange@uu.nl

¹Instituut voor de Ontwikkeling van het Wiskunde Onderwijs (Institute for the Development of Mathematics Education).

²Onderzoek Wiskundeonderwijs en Onderwijs Computercentrum (Mathematics Education Research and Educational Computer Centre).

J. de Lange (⋈)

Hans Freudenthal, who was instrumental in forming IOWO, was extremely disappointed. His expectations were low, at best. But he decided to stay with the 'sinking ship', as long as there was hope. He lived long enough to see how the institute, in 1991 re-named as Freudenthal Institute (FI), not only survived, but blossomed and grew reaching more than eighty mailboxes at its highpoint early in the 21st century.

Just two years ago, I was invited to reflect on my career as an educational designer. It resulted in a talk titled 'There is, probably, no need for this presentation' (De Lange, 2016). It took me lots of reflection, but I doubt whether the effort was worth the case. So, it must be because of my ripe age and wisdom that again I am invited to reflect. This time on my 'leadership' of the FI. That leadership started in 1981 with being appointed as a coordinator and culminated in becoming professor/director of the institute in 1989, ending with my 'retirement' in 2005.

The reflection in this chapter will be rather impressionistic, but with the best intentions. I will address:

- The mission: innovation in mathematics education.
- By means of connecting research and practice (developmental research).
- In teams of talented people, 'organised' in ways that let them shine.
- Working in a flat, informal maybe even somewhat chaotic, organisational structure.
- Connecting all players, politicians, scientists, practitioners, textbook authors, using a variety of dissemination methods.
- By powerful and relevant new ideas.
- Provocative and innovative with vision.
- Reaching out internationally to validate theories.
- Having fun.

17.2 The Mission: Innovation in Mathematics Education

Even before IOWO was founded, Freudenthal did not hesitate to formulate its mission 'to teach mathematics as to be useful' (Freudenthal, 1968). This was a very relevant question at that time, because of the rise of New Math.

In 1959 a seminar was held in France (Royaumont) with a great impact on mathematics education over the following decades. According to the report of this seminar, insight into the structure of mathematics is of fundamental importance for systematically directed education. Dieudonné, the famous French mathematician, was very influential, and proposed to offer the students a completely deductive theory, starting right from basic axioms. Freudenthal later admitted that not attending Royaumont was one of the two big mistakes he made in his professional life. The other mistake he refused to mention.

Many people in and outside the Netherlands had similar feelings about the mission of the FI and contributed to it. We name a few.

The structure of mathematics is a beautiful edifice, but I do not think there was one student who shared that opinion (Vredenduin, in Goffree, 1985).

Vredenduin made this remark after designing a course as intended by Royaumont, which failed in the classroom.

To know mathematics means to be able to do mathematics: to use mathematical language with some fluency, to do problems, to criticize arguments, to find proofs, and, what be the most important activity, to recognize a mathematical concept in, or to extract it from, a given concrete situation (Ahlfors et al., 1962, p. 8).

The problem is not what kind of mathematics, but how mathematics has to be taught. In its first principles mathematics means mathematizing reality, and for most of its users this is the final aspect, too (Freudenthal, 1968, p. 7).

What didactical phenomenology can do is prepare the following approach: starting from those phenomena that beg to be organized and from that starting point teaching the learner to manipulate these means of organizing (Freudenthal, 1983, p. 28).

If we look back, it is clear for me (joining the institute in 1976) that the mission can be phrased as:

To develop theories about how to teach mathematics as to be useful, and develop materials that fit these theories and allow teachers and learners to learn mathematics in this way.

17.3 By Means of Connecting Research and Practice (Developmental Research)

As a consequence of the just mentioned mission, it seems logical that the methodology should be the approach of developmental research (Freudenthal, 1991; Treffers, 1987). It is research with an important development component. It is not merely established how things are in existing education, but much more how things should be, and one develops education that suits these findings in a theoretical and practical sense (Treffers, 1993). One can also change the order; if one wants innovations in education, the process starts somewhere with design fitting the existing theoretical basis, but also ready to adjust these theories as experiments and experiences dictate. In both cases development and research take place in an integrated, iterative cyclic process (Gravemeijer & Cobb, 2006).

Educational design and development and research is a genre of research in which the iterative development of solutions to practical and complex educational problems also provides the context for empirical investigation, which yields theoretical understanding that can inform the work of others (McKenney & Reeves, 2012). At the FI the big problem in this respect is the place of correct and fitting methodology. As a developmental researcher, you often know 'for sure' that something is really happening. And you have at least proof of its existence. And there is political pressure as well.

When the new discipline Mathematics A was being developed, a careful experiment was designed within the strict boundary restrictions of the Ministry of Education. The project started with pre-experimental design experiments at classroom scale, went into the next phase with two schools, and thereafter to 10 and 40 schools.

It may have been declared a success already when working with two schools. The professionalisation based on these experiments formed the start for the teachers of the 40 and all remaining schools. Experimental teachers became teacher-trainers, co-designers and sometimes ended up at the Freudenthal as colleagues, or with Cito, the Netherlands national institute for educational measurement. Research was carried out on assessment issues and attitudes of students, but whether or not existing methodological criteria were met, remains a bit vague.

We tried to make the experiments more serious by disseminating results often and transparently to the teachers' media, and the commercial publishers. The *Nieuwe Wiskrant*, a journal for mathematics teachers was the rather glossy magazine of the FI, that at its peak reached a large percentage of the target group.

In 1989, as a New Year's gimmick, I wrote a story *De Kamerronde* (De Lange, 1989) for our Freudenthal people, by describing a virtual walk through the institute, and peeking into some rooms. In the remaining part of this chapter I will return to this walk to illustrate the work at the FI. I will start in Room 5.

Room 5

Four gentlemen, varying in age from medium aged to really very old. The old man distinguished himself from the rest by wearing a butterfly tie and looking like a real professor in every aspect. Almost without saying it seems natural that he is the centre of the discussion. The subject of discussion is a new article written by him for his newest (and latest) book with the working title China Lectures. The discussion has two points as its focus: what is common sense and in which respect is mathematics distinct and different from physics?

Iron feels colder than wood. The earth is flat. Is that common sense? The sun sets and rises again. Common sense, or bare reality seen from the perspective of the observer? According to the present text of the draft article, mathematics education needs to be built on common sense, while in physics education you often have to battle common sense because it is an obstacle in the conceptual growth of physical concepts. And what to think about chance and probability in this respect? That is mathematics as pure as it gets, right? But it often collides with common sense. Although, what actually is common sense?

Is common sense a set of generally accepted agreements and trivialities that makes any further discussion unnecessary? If this is the case, then this is only valid within a certain group or at a certain time. Or is it more complex? What about the reasoning part of common sense? You literally say "the sun sets", but you know that is not really true. But it is true for young children.

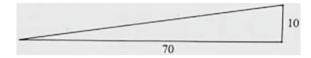
Mathematics is rooted in common sense, the professor dictates. As an example, he mentions the natural number. Kids can acquire this concept within the overwhelming stream of physical and mental activities. Mathematics: just a sniff of common sense, some organisation, and the development continues, resulting in a better organised common sense. Your common sense reasons that 2+3 is 5 and the area of a rectangle is $h \times b$. Mathematics, without physics, gives security, trust your common sense.

The discussion continues. Does 2+3=5 really constitute common sense? And area is length \times width as well? And the theorem of Pythagoras? Is spatial orientation based on common sense?

At the end, the participants make a sub-conclusion: common sense is local, both in time and place, and it includes reasoning.

The professor mumbles something. He will rewrite the draft. Will be continued.

Fig. 17.1 Drawings by the average students



This discussion in Room 5 represents a snapshot of a discussion about the theory of mathematics education. The practice follows next: another snapshot, slightly longer, about a teacher, working also at the FI as a designer-researcher, who teaches rather 'lower-achieving children'.

Her question was simple: "Can you design something in the area of trigonometry?" The resulting booklet was *Vlieg Er Eens In: Goniometrie en Vektoren*³ (De Lange, 1980a). We had tried it out with 'average' level students, but this promised to be something different. A student's reaction was: "You have to think, quite often." So, based on this experience our expectations were rather low.

Soon it became clear that we were too pessimistic. The so-called 'low achieving' students, who are often regarded as not being able to think, read or learn, were soon completely involved in the problems. The difference with the average group became quite clear. These students stayed within the context much longer than their higher scoring friends. An almost trivial example is the reaction of the students to the following problem: "Somebody jumps with a hang glider from a rock at 10 metres high and reaches 70 m horizontally. Draw this situation at scale." The average students drew something like what is shown in Fig. 17.1, while the lower achiever made more often drawings about the rock. Only later on, did these latter drawings become 'naked' triangles.

Another difference between the two groups was interesting as well. There were more students at the lower level who were 'willing' to think. They were very answer-oriented and only after discussions with their peers they accepted 'thinking' may be a part of the learning process. And in this respect, we noticed, quite surprisingly, that the lower achievers outperformed the average students, especially on more complex problems.

One aspect deserves special attention. How is the transfer from one context to another? It was one of those hot days in the past summer. The students wanted a lesson outside, of course. The teacher reacted as desired: "Okay folks, we're going outside to measure the height of buildings, towers, signs, lampposts etc." She gave the students simple angle measurement instruments, some paper and sandwiches and told them: "You have to be back in half an hour!"

My first thought was an exclamation "Good Lord", thinking of my own experiences as a student with outdoor lessons which were not very successful from the knowledge acquisition point of view. Although we certainly enjoyed eating lots of ice-cream, pushing girls in the pond, catching ducks and furthermore embarrassing our female teacher as much as we think was possible.

The teacher invited me to make a stroll around the school to observe the students in the wild. After a slight hesitation, I accepted the invitation. The small park alongside

³Published in English as *Flying Through Math: Trigonometry and Vectors* (De Lange, 1991).

the school building proved, not surprisingly, to be a popular area for investigation. The height of the school, a tree, a lamppost, everything was measured and if the results were not according to expectations, a discussion followed. What a well-educated company!

A couple of streets onwards we found a girl, with beautiful blond curly hair, lying on the ground, neglecting the fact that her blouse was very white indeed. Her girlfriend was taking care of traffic around the girl with the white blouse. "Fifty-three degrees", this latter girl told her friend. "And the distance was ten metres", said her girlfriend, carefully watching cars passing by. From down under came the response: "Then we know the height." She jumped up again, and asked if they were right. We both agreed with them. "Okay, let's do another building or object then." The teacher suggested: "Why don't you measure the fire-brigade ladder." 'Oh', reacted the girl immediately, "but then we need the cosine."

At that moment, I almost became emotional. The girls went to the ladder, we to the school. The ladder measured 12 m. The firefighters had confirmed that the length was actually 15 m. "Right", said the girls, "but we have measured to the edge of the roof, ignoring the piece that was above the edge."

I am still thinking, after all these years, of the girl in the white blouse (De Lange, 1980b).

17.4 In Teams of Talented People, 'Organised' in Ways that Let Them Shine

When the institute was reinvented in 1981 there was an extremely small team, selected carefully by the successor of Freudenthal, Frederik van der Blij. A careful balance between primary and secondary education, between somewhat younger and older, between more theoretical and practical, between more mathematics and social sciences, to name a few. So, in one way or another these people were considered talented. But it was also clear that there was no clear scenario on how to make this handful into a driving force in mathematics education. Reflecting on this starting phase it was clear that the connections for the pre-1980 years were invaluable. There was almost no institution left, but the people were still out there, somewhere.

This network, including many teachers, was kept alive, including continuation of magazines and newsletters to let people know that 'something' was still alive and kicking. That the remaining people were talented, was taken as assumed. They were the ones that were 'selected' to continue the good work, albeit it under very different circumstances. The institute was now really part of Utrecht University. But it was still in the same boring office building out of reach of the university. The battle to stay out of the university's bureaucracy was fought successfully for a very long time. The crown on this battle was the building next door that was really of the very best quality, and where we moved right after the completion of that building. Of course,

this was much later when we really needed three floors for all those working at the FI.

The 'organisation' was not much of an organisation: primary, secondary and new technology. Not a breath-taking structure, but it reflected what we did. There was a lot of freedom and right from day one we had our first big project for five years: the design of a new Mathematics A curriculum. New money, new people hired, based on known and proven talents. This team had a daunting task as school experiments had to start in August 1981, just eight months after the reinvention of the institute.

Luck may have played a large role here. The bureaucratic work on the new project, the foundations, had been laid since 1978, including the participation of the 'old' pre-1980 institute. So, the announcement of the Ministry of Education that experiments would be carried out starting in 1980 and resulting in a new curriculum to be introduced in August 1985, nationwide came exactly at the right time (a detailed description can be found in De Lange, 1987).

It seemed that at least for some time the new institute, suffering under the name of OW&OC was alive and guaranteed a lifespan of at least five years.

Because of the perceived success of this project an extension became reality. A similar curriculum for a different student population followed, extending the lifespan to ten years. We follow the discussion in the starting phase of this project in Room 1.

Room 1

A buzzing room. Almost heated discussion. Four excited people in a small room. The subject of all the excitement: the content of the new curricula Mathematics A and B.

As almost usual for experiments with materials for new curricula the big problem is, that there is more than fits in a curriculum. To cut in an ideal program is difficult. And they agree on only one thing: there need to be cuts. The teachers say so, the development group (FI members with teachers), the experts (and there are many of those), and all others involved. So, the task is simple: what and where to cut?

The experiments started in 1987, which in itself is a small miracle as the Ministry of Education did not consider these experiments necessary as the new curriculum should be similar to the just newly introduced Mathematics A. But the people of FI knew better. To develop student materials, to professionalise teachers, to write articles to acquire ownership, to design high stake tests, to carry out attitude research are essential activities.

The discussion heats up even further. Matrices out? Not a good idea. It is a prime example of showing modelling aspects of mathematics. Maybe exponential growth out? No, that is not very wise, give the famous report of the Club of Rome. And how about statistics?

There is no agreement in sight. The discussion remains heated. The time pressure can be felt.

At the primary level, some very talented people in Room 3 were able to continue activities in which the institute was very instrumental in a facilitative way.

Room 3

Somewhat concerned, the gentleman with the red-rimmed glasses looked ahead. He mumbles a bit hopelessly that he has no clue how to continue the Panama⁴ project. In the chair opposite him sits a somewhat young lady who agrees that the situation is a bit foggy. A discussion at the Ministry of Education had brought clarity, but no transparency, a feeling that many people have after visiting the department.

The Panama project started in 1981 as a collaboration between different parties with the goal of professionalisation of those who are working in and for primary school. The whole project was to be carried out by one person, with some administrative support. The way this project developed over time show what talented people, given the opportunity to shine, can do.

A newsletter developed into a leading professional magazine. The conferences that were organised were always sold out and played a very important role in developments in primary education in the Netherlands. It was a truly national platform and offered the staff of the institute a platform to really shine and inspire.

The gentleman with the trendy glasses knows that the institute and the other collaborators want the project to survive. But there are so many things to do, most of them of a complex nature. And there is so little money. And politics is so difficult. The Ministry of Education has been ordered not to fund 'outside' projects any more. The budget is going down, the partners in the project are in reorganisation, the institute is looking for possibilities and money. The new law on how to organise education-related institutions seems to make matters even more complex, but that, people say, was one of its intentions anyway.

The shaking of heads makes place for frowns on the foreheads. But the sparkling eyes tell another story. The upcoming Panama conference has sold out once more.

17.5 Working in a Flat, Informal, Maybe Even Somewhat Chaotic, Organisational Structure

It's far better to rely upon a broad base of individuals and leaders who share a common set of values and feel personal ownership for the overall success of the organization. These responsible and empowered individuals will serve as much better watchdogs than any single, dominant leader or bureaucratic structure (Terri Kelly, cited by Kastelle, 2013).

The luxury that reflection offers is that it may make you clearer about what you did in your past, more or less intuitively, and just because there seemed to be no other way. If you start with a handful of people that you know very well, it seems a waste of time to think of a structure at all. But soon you will find out that however horizontal or flat, there needs to be a person who is somewhat central. In those years this person was called 'coordinator'. Leen Streefland was the first, but after a year he decided that this suited me more, not realising that this would become another challenge in itself. How to handle the monotone growth that is so typical for the first 25 years of the FI since 1980?

⁴Panama stands for Pabo Nascholing Mathematische Activiteiten (Pedagogical Academy Training Mathematical Activities). Panama is the Dutch network of mathematics teacher educators for primary education. One of the activities of Panama is organising the annual Panama Conference.

To be honest, it never occurred to me that this was a problem. My focus as coordinator was on new bright ideas and opportunities, and putting talented people in charge of the actual execution of these plans after we found money. Somewhere. I think, looking back, that I agreed very much with Kevin O'Connor who stated in a blog: "All organisational structures are evil; but when you have to, align your organisation around markets" (O'Connor, 2012).

Well, the markets were our colleagues in research, but even more the teachers and students. Indeed, there was, with some good will, a weak organisational structure honouring the tri-partition primary, secondary and new media, but other partitions played a role as well: from practice to theory and vice versa, to name an important one. Only much later, when the institute had many more people involved in projects one could see the first steps to middle management. This of course was in part due to the fact that the institute was part of the university. And universities cannot be accused of embracing flat organisational structures. So, the university structure forced us to 'unflatten' the institute to at least a certain level.

So, it is comfortable to reflect at the initial very flat and informal structure. Small is flat, especially if you have been working together already for some time. The need for a coordinator emerged from this structure as something 'natural'.

More conscious was the battle within the university structure against the university. Let me explain this in a bit more detail. Our building in the early ages was shabby at best. And the rather chaotic (remember: flat) way we worked fit perfectly with that building. There were paper and boxes everywhere, and for visitors it was unclear at which moment they actually entered our offices. All of a sudden, they were in, if they had not returned already. But we liked our offices very much, because we were out-of-sight of any university office. We were very much aware that some of our salaries were taken care of, which was very nice indeed, but for the rest we looked more like a start-up business, as they are called today 'free as a bird'.

If we really did something based on an agenda, it was staying out of the bureaucracy. And until the end of my directorship, we succeeded quite well. Of course, we invested heavily in good contacts with the Faculty of Mathematics (trying to stay away from Social Sciences) and the Rector of the University. Especially after reaching out internationally, the executives at that level actually started to like us. Of course, our building was horrible. But just when the owner decided on restoration, another brand-new office building was erected next to the old building. The connections and appreciation with the rector and others ensured us a place in this new, fancy and very representative building. And renewed independence from the university.

We never forget the remark of one of the Secretaries of State for Education after a visit: "Jan, I really appreciate and am fond of the work of all of you, but you need a more representative building and entrance!"

Although we had a terribly good time working in these times, my fear of being eaten by the university was well grounded. Not only has this been proven true after retiring in 2005, but there is plenty of evidence from other sources. The growing bureaucracy, more middle management, more vertical structure, accountability, irrelevance of much work, make universities not really a sparkling, innovative, risk taking environment. I know, reflecting sometimes colours the image. All the better.

Room 9

The phone rings. The man in the room, seemingly deep in thought with stretched legs on his office table covered with lots of paper, awakes and grabs the telephone. "Bolivia", he mumbles. His legs sweep the table, cleaning it from all papers. His attention is on the phone. The line is garbled and the Bolivian English does not communicate well with Dutch English. They, on the other side, want more computers. We were warned by colleagues from the University of Agriculture in Wageningen: there are always needs for having some hardware added. For them it was not computers, but Jeeps.

In the meantime, the line before the open door was waiting patiently. They look around the corner, ever so friendly, but with the signal: we need you. The telephone call ended quite abruptly after promising some more computers.

The room was quickly taken over by the whole team of the project Being in Charge that is about how to become in charge of computers. There is a problem. The problem is simple, the professionalisation course and project are too successful. And now the question is how to deal with this luxury. We will contact the Ministry of Education.

The discussion switches to the Fair Share programme which is based on an intelligent tutoring system. This experimental half-product has been tried out successfully. And the question of how to proceed next is on the table.

The phone rings. All people look at the man in the room picking up the phone, standing this time. He listens and says: "Okay." "The Ministry of Education. They want to talk about the future of Fair Share".

17.6 Connecting All Players—Politicians, Scientists, Practitioners, Textbook Authors—Using a Variety of Dissemination Methods

"What chaos." Famous first words exclaimed by yet another Secretary of State for Education on entering the office building of the institute. She and her company crawled their way to the director's office, which was in a similar style, although the three chairs were made available for seating. Of course, an apology and explanation were required and offered. But the coffee helped a lot, and soon the discussion was about mathematics education and the expanding role of the institute in the world.

She was proud of the growing international role and projects in the United States. Even funds from the National Science Foundation came to the Netherlands. The question was, of course, whether we should accept it. One could easily argue that there was still more than enough work to be done in the Netherlands. But on the other hand, if one really wanted to validate the domain-specific instruction theory of Realistic Mathematics Education, one should reach outside our small country. And the higher regions at the university were very supportive as well. An institute like the FI needed to reach out internationally. Freudenthal himself expressed this point of view repeatedly when traveling across the borders in his favourite mode of transportation, a Land Rover Defender. He loved the Spartan jeep quite a bit, although comfort was lacking. But the discussion about the need for international contacts, and the desirable and fierce discussions resulting from these, was inspirational indeed.

As indicated before, teachers were also engaged in many aspects of the work of the FI. There were repeated interesting discussions with the university about the lack of academic qualifications of many of the people working at the FI. But the institute included teachers in more ways than only as colleagues.

There were professional magazines, conferences (Mathematics for All), regular conferences (Panama Conference, Nationale Wiskunde Dagen⁵), competitions (Alympiad, B-day), school activities (Grote Rekendag⁶), professionalisation, key positions in organisations (CIEAEM, ICMI, Mathematical Sciences Education Board, PISA, the ISTRON Group on mathematical modelling, and so on) and scientific magazines, to name a few. The hundreds of small applets and the electronic work environment for mathematics education, and software can also be mentioned.

Commercial textbook publishers and writers were often suggested to use the material developed by the Freudenthal Institute, free of copyright. In the 1980s, De Jong (1986) published a report about the success achieved by this dissemination strategy for primary education. Ten years later it was not difficult to conclude that similar results were also found in secondary education.

Room 4

An empty room. The New Media project team that should do its work here according to the note on the doorpost, is on the road. The whole team, including a teacher is at another office, at the Ministery of Education, then housed in Zoetermeer.

The room is not too big, and is occupied in large part by a huge, circular, white table. In the corner opposite the entrance is a construction suggesting that we are dealing with new media here. Around the blessings of the advances in technology sits a team that looks at least at ease and relaxed. Three quarters of the full circle are occupied by people from the ministry, and other experts.

The atmosphere is tense. For the team the meeting is very important. Will the department be satisfied, at least, and maybe even excited? That would bring in more money of course.

The ultimate goal is to carry out professionalisation using new media in a way that would later became fashionable as blended learning.

You can feel the tension and excitement in the room. Now and then a simple small nod, sometimes even an affirmative slight smile. "Quite interesting indeed", says the obviously most important person at the three-quarter of a circle part of the table. Exactly on time the session is finished. Other meetings are waiting. Good sign. The most important person stays in the room a little longer to talk a bit more in detail. The hardware is being dismantled. New media are a handful indeed. That causes problems at the school level. Should every school have a system? Plenty of ideas for further development. But they need to leave quickly now, on to a school to implement and experiment with new media at school level.

⁵National Mathematics Days.

⁶Big Mathematics Day.

17.7 By Powerful and Relevant New Ideas

Figure 17.2 shows a number of problems Freudenthal (1968) presented at the colloquium Why to Teach Mathematics as to Be Useful, held in Utrecht in 1967, to prove the fact that it is not so easy to learn that in all these situations the same arithmetical operation applies. He opened the panel discussion at this colloquium as follows:

Ladies and Gentlemen. I open the panel discussion. First of all, I'll give a summary of what I learned these past two days about the ideas of those who dealt with the general theme. I got the impression that we all agree about fundamentals. We all are convinced, I suppose, that mathematics has to be taught in way that people can apply it. We are convinced that this goal cannot be reached by simply teaching applications of mathematics, but that mathematics has to be related to its applications in a much earlier state, in a closer and more fundamental way, and that the ability to apply mathematics can only be acquired by starting with students from situations that have to be mathematised. (see Freudenthal et al., 1968, p. 61)

This can be seen as expressing the philosophy of the very first years of IOWO (1971–1980), the successful institute that was considered not be useful anymore. It barely survived the early 1980s with a small team, as described before. It led eventually to the idea of Realistic Mathematics Education.

This idea was developed in a powerful way by the Wiskobas⁷ group in the 1970s already, later to be followed by similar development in secondary schools, and parallel the development of a more computer-oriented approach in mathematics education. The plea for more applications in education starting in reality was facilitated through the projects mentioned earlier (the Mathematics A project), facilitated by politics and the Ministry of Education.

Another powerful idea that really 'made' the FI, was the developmental research approach. In this way, the research was fed by practice, brought on a higher scientific level and validated before 'descending' again to the practical level. In the meantime, all kinds of material were developed from complete curricula to beautiful micro designs, from software to applets, from tasks and tests for classroom assessment to high-stake tests.

The communication that is part of developmental research also had some relevant instruments, which were mentioned before.

If I have ten marbles and I give three away, how many are left? If I have ten marbles, and john has three less, how many does he have? If there are ten students in the room and three are girls, how many are boys? If I'm ten years old now, how old was I three years ago? If B is between A and C, B is at a distance of 7 miles from A, and C is at a

Fig. 17.2 Problems presented by Freudenthal

distance of 10 miles from A, how far is B from C?

⁷Wiskunde op de Basisschool (Mathematics in Primary School).

Room 7

A heavily gesticulating woman, speaking with an accent from down south in the Netherlands, tries to convince her more business-like colleagues. Not that the emotions rise particularly high, but the devil is in the details. Tomorrow an important presentation will take place, and the project needs to be brought out into the open in a very good way.

The research is about comparing what teachers think their students at the beginning of primary school are able to do, and what they actually can do.

The teachers were quite conservative in their expectations. The students fared much better than expected. This finding has great and important consequences, especially from the perspective of connecting to the reality of students. Do not start at zero if you know your students are much farther on the 'number line' of learning.

The woman is still gesticulating heavily. She is worried about her presentation. Is there time enough to present everything clear and transparent? Her counterpart, the gentleman, is less worried. His slides are well laid out, and not too many. He tries to calm down the lady. In vain, however.

17.8 Provocative and Innovative with Vision

Room 2

Three comfortable chairs, a lady and two gentlemen. They look as if they are thinking seriously about the task at hand. Indeed, the three have been invited to do a whole morning presentation at a large conference in the United States. Important points come up in the discussion: what kind of people will attend (mathematics educators), what do they want (just about everything, especially concrete examples), how deep will we go into the more theoretical aspects (not too deep), on which points are we different from U.S. society (in many more ways than 'they' think). But more down to earth matters, like plane tickets, also come along

They talk a minute or two about a side trip of one of the three to Princeton. There will be a brainstorm about a favourite subject: higher-order thinking skills. A remarkable initiative given the tradition of, and love affair with, the multiple-choice format. Multiple choice and the new experimental Mathematics A exams seem to be light years apart. Lots of text and context, visuals, open-ended questions, even long-answer-format questions. Okay, looks nice, maybe higher-order thinking skills, but what about validity and costs?

It is a recurrent theme: tests and tasks, both for primary (Van den Heuvel-Panhuizen, 1996) as well as for secondary education (De Lange, 1987). Consider the discussion about the end test in primary schools. Highly valued, taken by almost all children as the direction where to go in secondary education. Multiple choice. In a half-page interview in one of the quality newspapers in the Netherlands, the person at Cito who is responsible for this test, battles it out with me, the director of the FI. I argue that the test is misused as the sole measure of where to place a child within the Dutch tracking system. Research has shown that the teacher's advice is a better indicator for future success. There should be more real-world problem solving in the test and fewer fanciful illustrations to cheer up the kids. Moreover, I criticise the side effect

that the last year at primary schools is dedicated to preparing the students to pass the test: "A whole year lost for test preparation!"

In yet another major article, the discussion focuses on the new Mathematics A curriculum. I argue: "The test tasks of Mathematics A connect to the student's real world, and show mathematics as to be useful". I also made it clear that I am concerned that the quality of the experimental exams will be difficult to maintain. The downward trend has already started. According to me: "The exam fails to meet the principles and philosophy we developed when developing Mathematics A. Insight in concepts and creative thinking are rarely present."

A quotation from another interview makes clear that I am not the only one who is concerned: "The State Secretary for Education shared our concerns about the development of the high-stakes tests. She invited us at the Ministry of Education. 'What to do?' was her obvious question." The problem was purely political. A new law, the same one that declared the FI 'not needed', made it virtually impossible to change practice. The State Secretary was not allowed to fund any project in that vein, because that task belonged to the Cito. That same law played a huge role in the first years of the FI as creativity of a purely political nature was the game to play.

It was deemed necessary to distinguish the institute by being different: sometimes provocative, often innovative, seeking free publicity excelling in communication. And having excellent ties with different levels at the Ministry of Education, that was carefully bombarded with an array of novel ideas: new media, new tests for primary school, comparing textbook results, new curricula (as to be useful), new software, computer science at school, mathematics for all, A-lympiads, collaboration with the Dutch association for mathematics teachers, graphing calculators (ready for the museum right now), cutting edge conferences, international collaboration. Or, in short: never a dull moment.

And the institute was very lucky to find the press on its side. A whole page article about the institute stated the philosophy quite well, quoting Freudenthal about the usefulness of mathematics, and quoting a statement of myself: "What makes us different and innovative is developmental research. In this way, we try to develop a new educational reality by doing research. Not researching 'what is', but 'what ought to be'". And of course, the reporter was happy to quote the well-known American professor Romberg: "We are carrying out a 'robbery' on the Freudenthal Institute." This was his way of announcing a big cooperative project between FI and the University of Wisconsin at Madison.

17.9 Reaching Out Internationally to Validate Theories

Three people played a major role in the internalisation of the institute, one of my personal key issues. The first one was Hans Freudenthal himself. Of course, he was well-known and famous all over the world. Less well known is the fact that he was very much in favour of internalisation, while many people within the institute insisted on 'completing' the national agenda, whatever that means.

There is the story, playing in the late 1970s, about an invitation to make a presentation in Brazil, an invitation from Ubiratan d'Ambrosio (no need to say more). As I had already travelled as far as Norway at the time, people were looking at me. I immediately said yes. There was a slight problem. The then administrative director did not approve at all. The coordinator of secondary education talked to Freudenthal and they concluded I should go anyway. But how to make that secret operation work?

It happened to be February, so snow abounded in the Alps. So officially I went skiing. But what about the money? A solution that we all found quite elegant was to give me more local travel expenses for as long as needed, until my 'skiing' costs were covered. So, I went 'under cover' to Brazil, a turning point in internationalisation.

But Freudenthal did more than just carry out and cover travel under the radar. Quite often when he was invited, especially often to Germany, he invited me to accompany him, and show examples of our design work. The problem was, in my opinion, that my car happened to be a crude Land Rover, that was used for a Sahara trip. For Freudenthal it seemed more like an attraction.

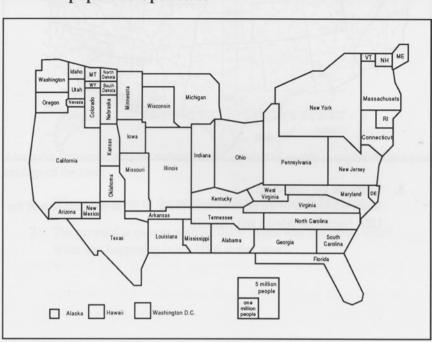
The other two very important people in establishing international projects, were two famous American scholars: Tom Romberg and Tom O'Brien. Almost at the same time they approached me for different activities. Tom O'Brien thought that what happened in the Netherlands was worth spreading in the United States. He acted as my impresario for an east-west coast tour of the United States, doing many presentations in places unknown to me.

It was at least quite interesting, and very enjoyable. I did presentations at schools, at universities, a school boards, at universities, for students and staff, superintendents and enjoyed diners just to get know important people like Marge Cappo. There were workshops, lectures, discussions and other formats. They lasted at least 45 min, but in one case 1.5 days. This was in Montana, where a real reception committee waited for me at the airport, and I barely survived with the couple of hundred slides that were to my avail. Rick Billstein and Johnny Lott were in charge of that incredible event.

Tom Romberg approached us more carefully. Not really the 'artists entrance' as the TO'B tour. Just exploring slowly and carefully to find out if it was worth investing in the Dutch. He was especially interested in developmental research and 'mathematics as to be useful'. He challenged us to design a little unit for use in an American High School, not far from Madison (meaning one hour by car). The teacher (Gail Burrill, later president of NCTM) selected the topic of data visualisation (see Fig. 17.3).

For design, flying tickets and observations we got \$3000. We realised of course that this was almost nothing, but ... The experiment was declared a success. So, this time Tom (and Gail) offered \$6000 if we designed another unit. No surprise. It worked quite well. The next phase was a dinner with Tom at his golf club in Madison (my first time ever). He leisurely informed me that he was happy with the two tiny experiments: "How about participating in a multi-million dollar National Science Foundation project?"

It can be considered as the start of many projects, run by many people in a variety of countries. Not just in Madison, but all over the United States. In Bolivia, South Africa, Indonesia, Malaysia, to name a few, and participating in studies like TIMSS,



The U.S. population per state

Population space: United States in proportion to population, July 1, 1967 (courtesy, Division of Research and Statistics, Ohio Bureau of Employment Services, Chart E-500).

- 1. Estimate the populations of New York and Nevada.
- 2. Explain why Wisconsin lies so far to the west on this map.

Fig. 17.3 Page from the unit data visualization

NAEP and PISA. And participating in the Mathematical Sciences Education Board (MSEB) of the National Research Council, being Secretary of CIEAEM, organising PME Conferences and an increasing number of colleagues in boards of prestigious journals.

The argument we used for internationalisation was to validate the theory of Realistic Mathematics Education. What came free with the ride was the richness of the different cultures. And what there was to learn abroad. Little did we realise at that time that while the way we carry out discussions in our institute (seen as rather vibrant) may have been a bit frank within the Netherlands, it was sometimes a real culture shock to see the differences in culture in general and in education. Hopefully we learned, both ways.

17.10 Having Fun

What I liked most about the Freudenthal Institute, where I spent most of my professional life, was the fact that, in reflection, every day seemed to be a fun-day. During this actual writing activity, one could have seen me smiling most of the time. When Marja van den Heuvel-Panhuizen asked me to reflect on how the institute worked in the last two decades of the past century, it did not take much effort to convince me. Better even, at my ripe old age I decided to attend ICMI 13 in Hamburg. So, I will confront my reflections with reality of the present state of art on mathematics education. It will again be FUN!

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