Chapter 6
Back to the Future: Richardson's Multilateral Arms Race Model

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Abstract Lewis Fry Richardson was a groundbreaking scholar, not only in modern meteorology but also in world affairs. His two major books, both published posthumously in 1960, were harbingers for what was to follow in scholarly international relations. In one, he collected wide-ranging, detailed quantitative information on disaggregated conflict processes in a variety of historical contexts. In the other, he showed two basic innovations for the social sciences: (a) the power of mathematics for understanding complex social systems and (b) the importance of understanding the interdependence of things that are typically studied separately. That latter insight is the focus of this chapter wherein I show that the Richardsonian insight on coupled behaviors leads to a network perspective on social interactions at the global scale. We call these coupled interactions *networks*. I trace the development of Richardson's thinking about coupled phenomena to the development of network thinking in the social sciences. I conclude with some recommendations for the arms race research program as applied to the current era.

6.1 Introduction

Based on his path-breaking work in meteorology (Richardson, 1922), Richardson imagined a future in which his models were implemented for real time forecasting. The illustration is shown in Fig. 6.1. He imagines a spherical structure which is mapped to a globe and painted correspondingly inside. People calculate the prediction equations corresponding to which part of the map they are assigned to.

The original version of this chapter was revised: The name "Gregory D. Hess" has been corrected to "George D. Hess" in Reference list. The correction to this chapter is available at https://doi.org/10.1007/978-3-030-31589-4_12

Thanks to Phil Schrodt five decades of inspiration and to Nils Petter Gleditsch for his stewardship.

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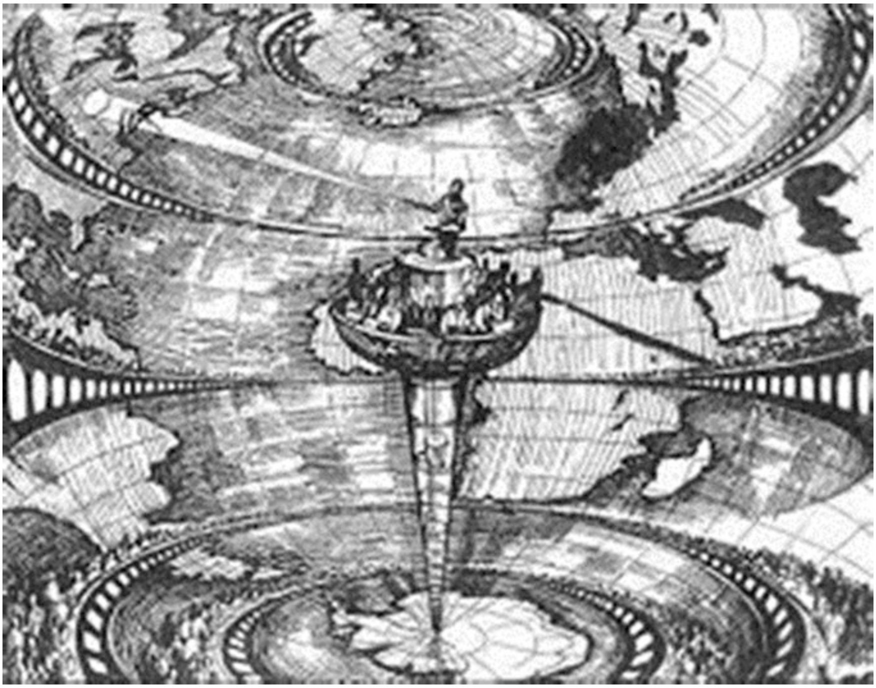

Fig. 6.1 Richardson's visualization of a laboratory for real-time weather forecasting, based upon his mathematical models. *Source* Richardson (1922: 219)

He imagined 64,000 people – which he called calculators – would be required. A team would distribute the calculation of each part of his equation system, and it would be coordinated and monitored by supervisors. At the top of a pillar inside the sphere, a conductor is in charge of all teams. The musicians in this sense are individuals playing slide-rules and calculating machines.

Richardson's work in meteorology continues to be important to this day. Indeed, they have played a role in the discovery of global warming (NOAA National Weather Service, 2019; Weart, 2008, 2018) through their role in global circulation models of weather.

However, when Lewis Fry Richardson learned that his weather models could be useful in military applications, he quit working in this domain and reportedly destroyed his unpublished results. Richardson was a Quaker and would eventually spend part of the First World War as an ambulance driver (Section Sanitaire Anglaise 13) from 1916 to 1920 (Wilkinson, 1980). During this time, he wrote his first offering on the causes of war, but at the time there was nowhere to publish it. During his time in France, his location was under frequent bombardment and not only were the ambulances quite busy, they also had to run the gauntlet to deliver the wounded to medical attention. This reportedly had a large impact on Richardson.

When he turned from weather prediction, he focused on wars and conflicts, notably in two volumes. One of these concentrated on analyzing data (Richardson, 1960b) and the other focused on mathematical analysis of the insecurity of nations (Richardson, 1960a). In that latter effort, he drew upon his development of mathematical equations in meteorology and applied them to human behavior. His initial focus was on the arms race that preceded the First World War. In terms of his modeling, this is his best-known work.[1]

Although he was not the first person to use the term arms race, he was the first to formalize what it meant precisely. Richardson begins with a linear model for two nations, expressed as a pair of differential equations (1960b: 16, Eqs. 7 & 8):

$$dx/dt = ky - \alpha x + g \tag{1}$$

$$dy/dt = lx - \beta y + h \tag{2}$$

where x represents the military spending of one nation and y the military spending of its main rival. The drag on increasing military spending at ever higher levels is represented by the term $-\alpha x$ for the first country (aka country x) and $-\beta y$ for the second country (referred to as country y); g and h are historical constants reflecting the respective hostility of x and y toward each other.

In the 1960s when this idea was introduced into the study of politics and economics, it was very difficult (for social scientists) to solve differential equations. Even Richardson's initial attempts to use data to look at these equations were quite simplistic. Basically, everyone translated these into a set of difference equations

$$\Delta x = ky_t - \alpha x_t + gt \tag{3}$$

$$\Delta y = lx_t - \beta y_t + h_t \tag{4}$$

which could be dealt with via straightforward mathematical tools and could (if you squinted) be examined empirically via linear regression – possibly via a two-stage least squares estimator. Richardson analyzed this analytically in his volume and several scholars worked in this domain quite successfully. Hess (1995) provides a good overview.

During the Cold War, there was widespread empirical work on arms races, frequently using a statistical approach to estimating and validating the underlying action-reaction equations developed by Richardson in *Arms and Insecurity*.[2] Following Richardson's lead most of these studies looked at pairs of countries, or groups of countries aggregated into pairs (NATO versus Warsaw Pact). Readers will notice that most of these studies were conducted in the last century during the (first?) Cold War. Dissolution of the Soviet Union led to a subsequent scholarly focus on the putative peace dividend. In the meantime, global military spending has grown from about one trillion dollars per annum to a current total of around 1.7

[1] Although he made contributions to geography (e.g., the theory of compactness) (see Gleditsch & Weidmann, 2020, in this volume) and to other disciplines as well, such as criminology.

[2] For an extensive bibliography of such works, see Gleditsch & Njølstad (1990: 384ff). For references to more recent work, see Smith (2020) and Diehl (2020), both in this volume.

trillion dollars per annum (SIPRI, 2018). But a search for recent articles on arms races leads to a mountain of research on biological interactions from the microscopic to the species level, but little contemporary social science (Smith, 2020, in this volume).

6.2 Multilateral Arms Races

The two-nation arms race was actually the toy model through which Richardson introduced his basic ideas. He quickly moved beyond that, though most scholarly work has not. A sterling exception is Schrodt (1981), who focused on a multi-polar world with more nuanced distributions of armaments (and therefore spending).[3]

The basic structure of a multi-nation arms race, in Richardson's terms, much like weather systems is given as a system of ordinary differential equations as shown in Eq. 5, where $\mathrm{x_i}$ is the military spending for nation i, and $\kappa_{i,j}$ has the action-reaction coefficients off the diagonal, and the economic constraints on the diagonal and g_i portrays the hostility terms.

$$\frac{dx_i}{dt} = g_i + \sum_{j=1}^{j=n} k_{ij} x_j \quad \forall \quad i \in \{1, 2, 3, \ldots, n\} \tag{5}$$

This equation is the crux of the multilateral system of equations. Instead of x and y there is now a vector of countries stored in x_i, where i is an index of all the countries to be included (including the previous y from the bilateral case). κ is an $i \times i$ matrix. The off-diagonal elements collect the action-reaction terms, linking each i to each other i with a coefficient that conveys the reaction of a single country to each other countries' military spending. For example, in the first row of Table 6.1, there is a weak reactivity between Czechoslovakia and Germany (2). These effects are asymmetric as shown by the coefficient of 36 between Germany and Czechoslovakia. Germany is threatened by small changes whereas Germany is more reactive to changes in the military spending of Czechoslovakia. Collected on the diagonals are the economic constraints, wherein higher spending tends to dampen subsequent spending in the same country (thus the negative sign). Turning to the first element, there is a strong economic constraint in Czechoslovakia (−30) which constrains military spending increases. The country with the greatest economic constraint is Great Britain and Northern Ireland (−45). The overall hostility terms from the two-nation model are collected in the vector g for each country; these are not shown. In the end, this representation is equivalent to writing out a Richardson model for each country; what is unique is that it generalizes for any number of countries.

[3] There are a few exceptions to this generalization, notably Wallace (1979).

Table 6.1 The κ matrix from Richardson's study of the 1935 multi-nation arms race

	Czech	China	France	Germany	GBNI	Italy	Japan	Poland	USA	USSR
Czech	−30			2				1		
China		−30					12			18
France			−54	4		4				
Germany	36		36	−30	18			3		72
GBNI				4	−45	6	2			
Italy			18		36	−15				18
Japan		12					−30		36	36
Poland	9			3				−30		9
USA				2	6	2	4		−21	6
USSR		2		8	6	2	4	1		−30

Values are multiplied by 30 for presentation purposes, as in the original. The determinant (unscaled) of this matrix is −0.37. *Source* Richardson (1960a: 202, Table 51)

The κ coefficients were calculated by Richardson, but not necessarily using any statistical methodology. Instead, he coded them from subject matter expertise, where Richardson was the expert having closely followed current events in the news and radio reporting of the day. If the determinant of the κ_{ij} is not zero, the equilibrium conditions can be easily solved.

6.3 Military Spending

Richardson's initial study focused on military spending. His Table 1 (1960a: 6–7) portrays a variety of data about countries over the pre-war period from 1913–15, but he also collected information on the number of war dead and population. These data were taken from various almanacs, historical sources, and Parliamentary documents. Subsequent uses of the Richardson arms-race model largely focused on military spending data – often normalized by population or total governmental expenditures.

The data from his study of the multilateral arms race in 1935 (Richardson, 1960a: 202ff) are given in Table 6.1. These $\kappa_{i,j}$ are basically the linkages or reactivity of each nation to each other nation. A value of zero indicates that the two nations essentially disregard each other's military spending. Diagonals convey a negative value that reflects the economic constraints faced by each country.

Many have argued that the arms race leading to the onset of the First World War was actually a competition over military equipment, notably Navies (Lambelet et al., 1979), and my own position is that it really makes more sense to think of arms races in terms of stock-flow models (Ward, 1984a). However, for the purposes of this exposition, I focus on military expenditure data, eschewing the obligation of developing a coherent capital stock measure for military technologies in the 21st

Table 6.2 Military spending in thirteen countries, in million US $ 2016

Country	2016	2017
US	600,106	597,178
China	216,031	228,173
Russia	69,245	55,327
Saudi Arabia	63,673	69,521
France	57,358	56,287
India	56,638	59,757
UK	48,119	48,383
Japan	46,471	46,556
Germany	41,579	43,023
South Korea	36,934	37,560
Israel	14,783	15,501
North Korea	13,000	13,000
Pakistan	9,974	10,378

Century. Data have been taken from the Stockholm International Peace Research Institute (SIPRI) which has been the gold standard for military spending data for several decades.

Military spending in 2016 US constant dollars is from SIPRI military spending database (accessed 28.6.2018). For North Korea data are based on an estimate of one-third of the budget. GNP is estimated in the CIA *World Factbook* at 40 billion current dollars. Thus, I have estimated military spending to be about 13 billion dollars per annum.

Table 6.2 portrays military expenditure data for thirteen countries taken from the SIPRI database. These data illustrate that the US and China each have an order of magnitude more military spending than any other country, though current US expenditures are not as high as they were at the height of the so-called Cold War. The US current spends over twice what China spends on an annual basis, though factors of production vary widely between these two countries, as they do for many others in this list. It also serves to point out that a variety of activities undertaken by the military are accounted in different ways by different countries. Some countries do not include space exploration and satellite activities as military ones. Both intelligence and financial activities sometimes come under a military purview and other times do not. Thus, an exact comparison is hard to justify in a nuanced way. Sometimes keeping aging weapons systems is more expensive than buying new ones, but procurement may dependent upon legislative oversight and always takes time. As a result, military expenditures are a complicated mixture of procurement of new systems as well as parts and services for aging systems. Obviously, the cost for a battalion of infantry varies substantially across these countries, spanning Pakistan, North Korea, Israel, Saudi Arabia, and many western European countries. Suffice it to say, however, that the US and China are outliers in the distribution of military spending for contemporary countries.

The annual changes in these expenditures are not extreme. Saudi Arabia has the greatest growth in military spending, about 9% per annum, with China, India, and Israel close behind at about 5−6% growth per annum. All the other countries are within a percent or two of exhibiting a rate of change that is close to zero. Russia is negative, but not at any substantial level. I return to these data below.

6.4 Reactivity

Richardson had a complicated way of estimating the coefficients for his model. He describes his approach:

> In September 1938 the author made the assumptions shown in Table 45, remarking, with apologies to all concerned, that it was an act partaking not only of science but also of art or perhaps, alas, of caricature. For several years he had been attending to the profuse comment on the friendships and animosities of nations offered by various publications but especially by the British radio and the *Glasgow Herald*. This matrix was intended to be a general estimate and summary of such common information relating to the year 1935. (Richardson, 1960a: 193)

This can be considered as a network of interactions among these thirteen countries (ignoring for the moment the diagonals). This is shown in Fig. 6.2.

6.5 The System of Equations

Richardson never got to see his room full of weather calculators, nor to see how his application of ordinary differential equations might play out when examined with data. Solving the differential systems of equations by hand was possible analytically, but deriving real trajectories was a technology then yet to come. Fortunately, since then the science and engineering of dynamical systems has made enormous progress.

How much progress? In the course of a weekend eight decades after Richardson's work, I encoded Richardson's κ_{ij} matrix for 1938, and with military expenditure data taken from Richardson's own source (*The Literary Digest*, 23 February 1935: 42) was able to implement this 10-national differential equation system.[4] Figure 6.3 shows the trajectories of four major countries in this ten-country arms race: France, Great Britain and Northern Ireland, Soviet Union, and Germany. There is explosive growth in these trajectories. This shows, as claimed by Richardson (1960a: 12), that 'The equations are merely a description of what people would do if they did not stop to think.' Richardson was ambivalent

[4]I am unaware of anyone else having done this.

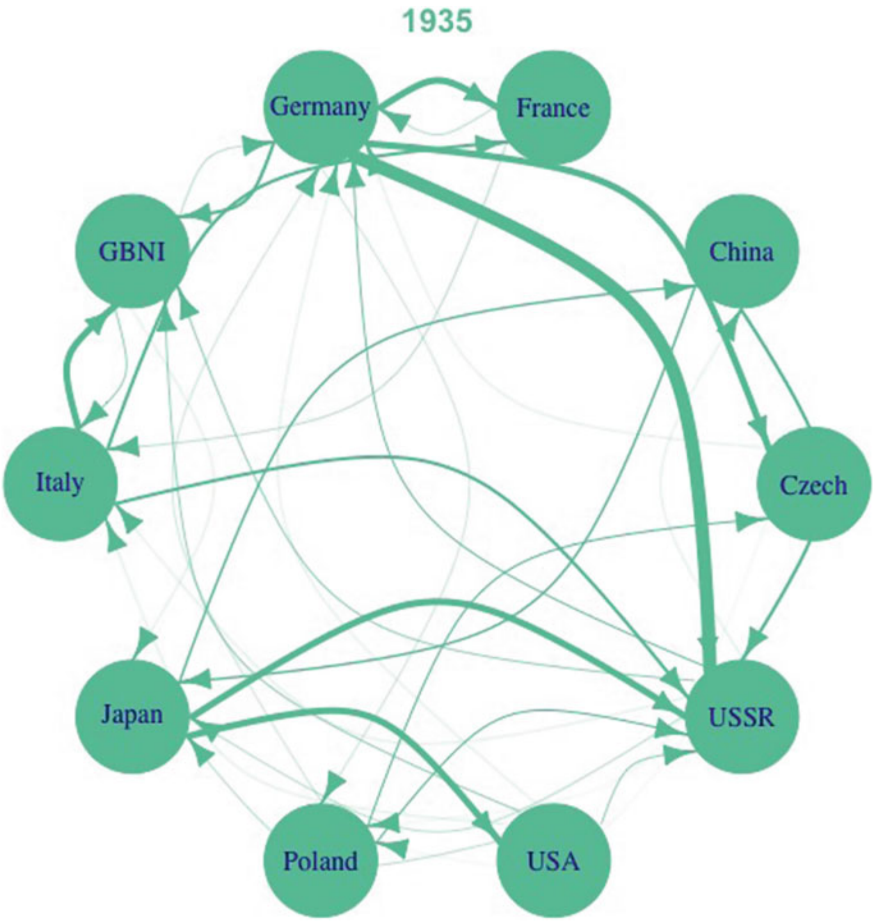

Fig. 6.2 Richardson's $\kappa_{i,j}$ visualized as a weighted network

about whether arms races brought about wars, but it should be obvious that this system was embroiled in the Second World War by September of 1939, and the trajectories of spending by these countries did, indeed, grow exponentially.

Richardson clearly underestimated the damping effect – in peace time at least – of high levels of spending on subsequent increases. This system probably overestimates the explosive nature of the arms expenditures because of their reactivity to spending in a large number of potential rivals (and allies). Several scholars have worked to point this out in empirical studies of arms races, beginning with Caspary (1967).

In addition, scholars have pointed out that it is not really all about military spending, but rather what the spending is buying that should be the central focus of competitive arms processes (Luterbacher, 1974; Ward, 1984, 1985). In economics, these are called stock-flow models. Richardson's differential equation system can

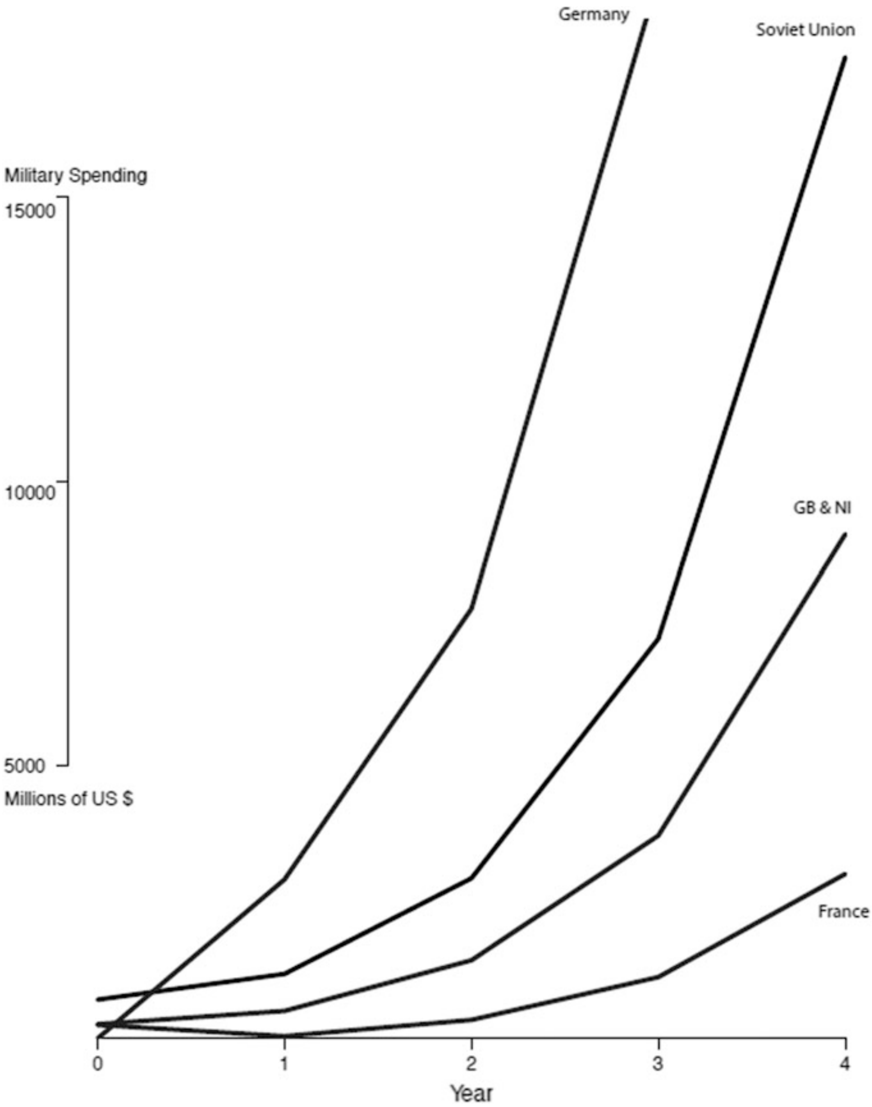

Fig. 6.3 Trajectories of four countries in Richardson's 10-nation arms race. *Source* Specified by his estimates of κ_{ij} in Richardson (1960a: 202)

easily handle both emendations – though collecting accurate and comparable data on stockpiles of weapons has proven challenging. Even the US Central Intelligence Agency had great difficulty of this in the 1970s and 1980s.[5]

[5]See the description of the Team B project, led by Richard Pipes (Pipes, 1986).

If we wanted, we could go back and collect more data to better examine the system. Someone should.[6] But I turn, instead, to an agenda for looking at contemporary arms races.

6.6 What About the Present?

How can Richardson's encodings be 'replicated' today? Fortunately, a lot has changed since 1938 to make this feasible. Providentially, content analysis began with the release of the documents that led up to the First World War and work thereon by Robert North and colleagues at Stanford University (Choucri & North, 1972; North et al., 1963). The event data movement in international relations also was founded by work that was started by a historian, who introduced systems theory into political science, McClelland (1961). His World Event Interaction Survey (McClelland & Hoggard, 1969) led in turn to CAMEO (Schrodt et al., 2009) and many others. A recent version has been used in the ICEWS project (Boschee et al., 2015; O'Brien, 2010) with event data based on coding that has been tested with subject matter experts. The database on which it is based, has data from 1990 to last week, and at present is based on over 45,000,000 news stories from more than 300 local, national, and global news sources. The dictionary of actors is frequently updated, but the verbs that are used in the National Language Processing algorithms are specified from a defined list of twenty major actions, that are further disaggregated into 360 categories. The top-level CAMEO codes and an example of one disaggregation are provided in Table 6.3.

Frequently, the categories are divided into four broad groupings, reflecting a cross tabulation of verbal and nonverbal events with conflictual and cooperative ones. These categories are referred to as the quad codes, though there is some disagreement about whether the comments should be a separate category (some scholars delete all comments) and result in pentacodes (OEDA, 2016; Schrodt, 2015).

Herein, I focus on material conflicts among nations which occur for event types 16–20 in the CAMEO codebook. This includes types of coercion, assaults, fights, and unconventional mass violence. These are conflicts that are not just verbal spats between nations, but rather are those that involve some use of material resources. This variable is typically called material conflict. I gather the counts of bilateral material conflict events from the ICEWS database. Herein, I use an annual aggregation, though any sensible temporal aggregation greater than a day is possible. The data for 2017 are shown in Table 6.4.

[6] This, as they say, is left to the reader as an exercise.

Table 6.3 The CAMEO Codes, and a disaggregation of category 03

Cameo Codes for Event Types

01 MAKE PUBLIC STATEMENT	11 DISAPPROVE
02 APPEAL	12 REJECT
03 EXPRESS INTENT TO COOPERATE	13 THREATEN
04 CONSULT	14 PROTEST
05 ENGAGE IN DIPLOMATIC COOPERATION	15 EXHIBIT FORCE POSTURE
06 ENGAGE IN MATERIAL COOPERATION	16 REDUCE RELATIONS
07 PROVIDE AID	17 COERCE
08 YIELD	18 ASSAULT
09 INVESTIGATE	19 FIGHT
10 DEMAND	20 USE UNCONVENTIONAL MASS VIOLENCE

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03    EXPRESS INTENT TO COOPERATE
030      Express intent to cooperate, not specified elsewhere
031      Express intent to engage in material cooperation
0311        Express intent to cooperate economically
0312        Express intent to cooperate militarily
0313        Express intent to cooperate on judicial matters
0314        Express intent to cooperate on intelligence
032      Express intent to engage in diplomatic cooperation
033      Express intent to provide material aid, not specified elsewhere
0331        Express intent to provide economic aid
0332        Express intent to provide military aid
0333        Express intent to provide humanitarian aid
0334        Express intent to provide military protection or peacekeeping
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Adapted from the CAMEO Codebook by the author, http://eventdata.parusanalytics.com/cameo.dir/CAMEO.09b6.pdf

Table 6.4 Annual counts of material conflict events sent by the country on the row toward the country on the column in 2017

	US	Ch	Rus	Fr	Ger	Jap	NKor	SKor	SArabia	Isr	India	UK	Pak
US		1	42		6	1	1	8	1	1	9	6	6
China	15						6				9		
Russia	42				1	2	1						
France	8		13									2	
Germany	6		2							4			
Japan	7	1					3	1					
N. Korea	3												
S. Korea	5					1	5						
Saudi Arabia													
Israel					43						1	1	
India	6	3										1	47
UK	8		1								1		
Pakistan	14										174		

Diagonals are empty. ICEWS Dataverse https://dataverse.harvard.edu/dataverse/icews, extracted by author

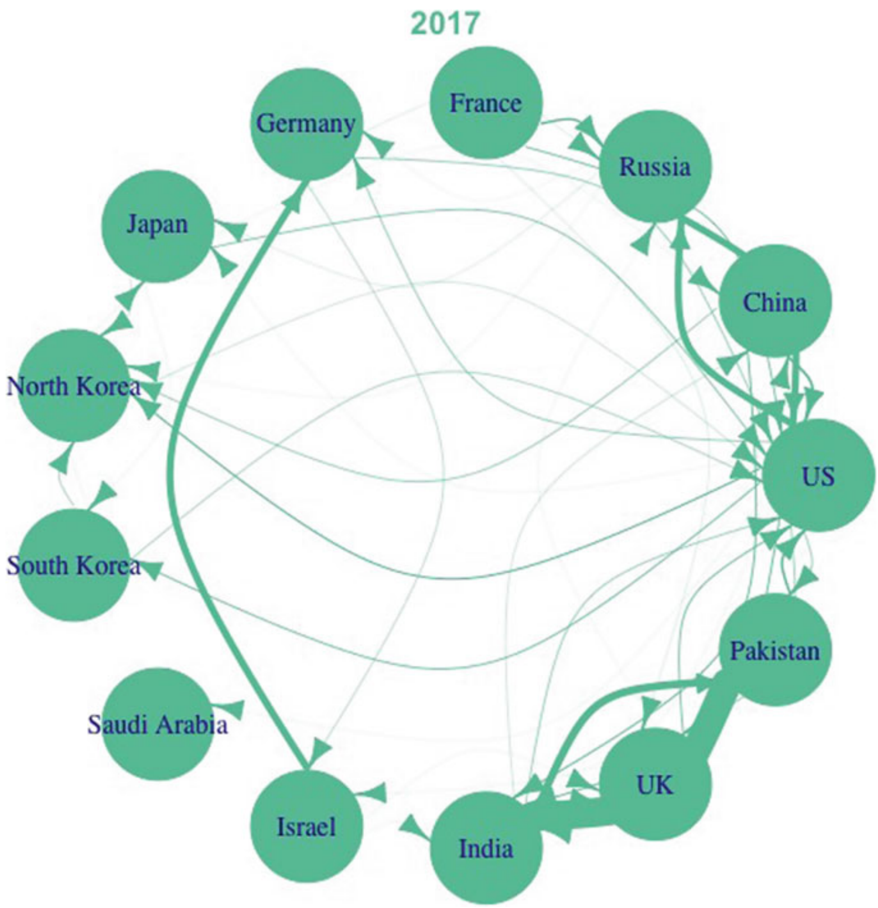

Fig. 6.4 Richardson's $\kappa_{i,j}$ visualized as a weighted network using ICEWS material conflict data for 2017

In these data, Pakistan receives the most material conflict, followed by the US, India, Russia, and China. In terms of sending material conflict, India, followed by the US, far outpaced any of the other countries, though Russia and Pakistan also have sizable material conflict patterns. I display these data in Fig. 6.4 as a network, similar to Fig. 6.1.

This shows reactivity among India, Pakistan, and the UK that is quite strong, as well as among the US, China, Russia clustering. Also, there is quite a bit of conflict from Israel to Germany. All the other reactivities are at a low level.

In order to make this representation compatible with the Richardson $\kappa_{i,j}$ matrix, two things need to be accomplished. First among them is to provide diagonal elements. Then, the matrix needs to be appropriately scaled. Turning to the first task we have already established that the growth rate of military spending varies between 9% for Saudi Arabia to about zero percent for the lowest countries.

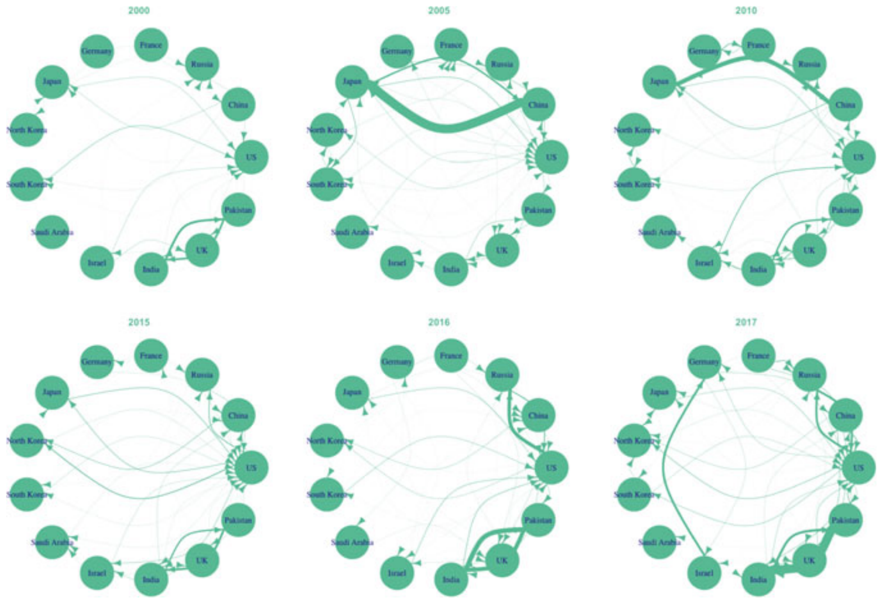

Fig. 6.5 Different annual $\kappa_{i,j}$ representation

This suggest that the diagonals need to off-set, more or less, the sum of the row reactivities. But for now, I set aside this task, for which I suggest a solution below. I focus herein on the year 2017, but it should be obvious that these will fluctuate from year to year, as shown in Fig. 6.5 which has representations for various years from 2000 to 2017.

It is straightforward to implement this 13-nation arms race for 2017 using either Python or R computer libraries that do all the heavy lifting.[7] Because we have access to the SIPRI data for all these countries (more or less), we have enough information to estimate the parameters of a differential or difference equation system, with embedded dependencies. There are two approaches to this, one is to use network methods for the difference equation system (Hoff, 2015; Hoff et al., 2015). The second is to simply use the simulation itself to generate potential parameter values, driving the simulation successively closer to the real data trajectories. This later approach was developed in the 1970s (see, for example, Allan, 1983, or for a Bayesian version, Raftery & Zeh, 1993) and is described more recently in Cellier & Greifeneder (2013).

It is straightforward to use the R packages deSolve and FME to simulate and estimate the parameters for dynamical systems of differential equations.[8] Such

[7]Email me, if you would like a copy, to be available in the fullness of time on my github.

[8]Many tools also exist for this in other modeling/statistical framework, e.g., MATLAB among many others.

systems can be difficult to estimate successfully because of the inherent sensitivity of the systems. But this sensitivity is a feature, not a bug. If the system were not sensitive to minor changes in one part of it, it would fail as an implementation of the design of the system. At any rate, many tools and diagnostics are now available to estimate successfully such dynamic systems, which can then be used to simulate different scenarios.

I end this chapter here, with this suggestion for arms race research in the future. If it is going to carry us forward, it needs first to go back. First, it needs to go back to the careful tradition of the construction of internally consistent dynamical mathematical models like those developed in the 1930s. Second, it needs to go back to the modeling and statistical methods developed in the 1970s. But it also needs to go forward to the wealth of empirical data on arm and armaments now available to scholarly and policy communities. And, it needs to embrace the more recent work-saving devices that sit on our desks and can simulate complicated systems of interdependent processes while we stand at our desks.

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