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Proportional Fair Information Freshness Under Jamming

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Abstract. The success of a UAV mission depends on communication between a GCS (Ground Control Station) and a group of UAVs. It is essential that the freshness of the commands received by UAVs is maintained as mission parameters often change during an operation. Ensuring the freshness of the commands received by UAVs becomes more challenging when operating in an adversarial environment, where the communication can be impacted by interference. We model this problem as a game between a transmitter (GCS) equipped with directed antennas, whose task is to control a group of UAVs to perform a mission in a protected zone, and an interferer which is a source spherically propagated jamming signal. A fixed point algorithm to find the equilibrium is derived, and closed form solutions are obtained for boundary cases of the resource parameters.

Keywords: Age of information · Jamming · Nash equilibrium · Proportional fairness

1 Introduction

In many Unmanned Aerial Vehicle (UAV) applications, a Ground Control Station (GCS) communicates with a group of UAVs to send instructions to control each UAV's mission. However, when such active communication faces the threat of hostile interference, the result can be delay or even interruption in getting such instructions. Larger periods of delay or interruption lead to reduced freshness of received instructions, and can decrease the probability of mission success. Thus we model the probability of mission success as a function of the age of the received information. Such considerations have been gaining prominence in the research literature lately, as reflected by interest in the age of information (AoI), a system delay performance metric that has been widely employed in different applications [1,13,23,25,26].

The pioneering paper, on the impact of hostile interference on age of information is [17]. Our work is based the model of [17] suggesting a relationship

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between SINR at the receiver and AoI of update packets. We employ this approach to model a scenario where a group of UAVs is cooperating to perform a mission. A transmitter GCS employs directional antennas to control the UAVs while jammer radiates a spherically symmetric interference intended to cause the mission to fail. We model this problem via a game-theoretical approach. An interesting feature of this problem is that the rivals have different structures for their strategies. Specifically, the GCS's strategy is power allocation individually between the UAVs, while the interferer's strategy involves assigning power to jam the whole UAV group. While for most jamming games studied in literature rivals strategies have the same structure: either power allocation for both rivals [16,20,24] or assigning power level [7,12,17,22] for both rivals.

2 Model

We consider a group of n UAVs that, following their route/mission to a target in a protected zone, must communicate with the GCS to get/verify position and mission data. The GCS is equipped with n antennas (or n separate antenna beams) to communicate with these UAVs. This communication can be damaged by active interference that might lead to loss of navigation commands and failure of the mission. An interferer, located in the protected zone, is a source of spherically symmetric interference intended to cause the UAV mission to fail. As a metric for data updating in this paper, we consider AoI which reflects the time that has passed since the last update. We assume that the probability π_S of mission success is a function of the average age of information A, and this function is decreasing with A such that: (a) $\pi_S(0) = 1$, i.e., if the data is up-to-date the mission succeeds with certainty; and (b) $\pi_S(A) \downarrow 0$ for $A \uparrow \infty$, i.e., if data is never updated, then the mission fails with certainty. To model the probability of mission success, we will use the ratio form contest success function. This is commonly used to translate involved resources into probability of winning or losing, and has been widely applied in different economic and attack-defense problems in the literature; see, for example, [5,9,11,19,21]. In our scenario, the metric that dictates whether the mission is successful or fails, is age of information. Specifically, in terms of positive constants a and b, the probability of mission success is

$$\pi_S(A) = \frac{b}{b + aA}. \tag{1}$$

2.1 Age of Information

To model age of information, we will employ a generalization of the model introduced in [17]. For convenience of the readers, we give a brief description:

(i) The GCS can transmit at a rate that is proportional to the signal to interference plus noise ratio (SINR) at the receiver. Following [17], when p_i and q are the powers of the transmitting signal by GCS to UAV i and interfering signal, and h_i and g_i are the corresponding channel gains to the UAV, the packet transmission rate associated with power profile (p_i, q) is

$$\mu_i(p_i, q) = z_i \mathrm{SINR}(p_i, q) = z_i \frac{g_i p_i}{N_i + h_i q}, \tag{2}$$

where N_i is background noise power and z_i is a positive constant.

(ii) Depending on the model for how update packets are delivered to the UAV, the age of information metric A_i takes on the form

$$A_i(p_i, q) = \frac{c}{\lambda_i} + \frac{d}{\mu_i(p_i, q)} \tag{3}$$

for packet arrival rate λ_i and constants $c \geq 0$ and $d > 0$. In particular, when $(c, d) = (1, 2)$ and fresh update packets are generated at the UAV as a rate λ Poisson process, the age metric $A(p, q)$ corresponds to the average peak age of an M/G/1/1 queue [3,10]. This is the age metric employed in [17].

We note that various other age metrics can be modeled by specifying (c, d) in (3). For example, with $(c, d) = (1, 1)$, $A_i(p_i, q)$ is the average AoI of an M/M/1/1 server supporting preemption in service [15]. Furthermore, with $(c, d) = (0, 2)$ and just-in-time arrivals (i.e., a fresh update goes into service precisely when the server would become idle) at a rate $\mu_i(p_i, q)$ memoryless server, $A_i(p_i, q)$ is again the average AoI [14]. Finally, with $(c, d) = (0, 3/2)$, $A_i(p_i, q)$ corresponds to just-in-time updates transmitted with deterministic service times at rate $1/\mu_i(p_i, q)$ [14]. In the following, we refer to $A_i(p_i, q)$ as the AoI for any $c \geq 0$ and $d > 0$.

2.2 Formulation of the Game

To define game we have to describe: (a) the set of players, (b) the set of feasible strategies of each player, and (c) the player's payoff [4]. In our scenario, there are two players: the interferer and the GCS. A strategy of the GCS is a non-negative power vector $\boldsymbol{p} = (p_1, \ldots, p_n)$, where p_i is the power employed to communicate with UAV i, and $\sum_{i=1}^{n} p_i = \overline{p}$ is the total power. Let Π_{GCS} be the set of all feasible GCS strategies. A strategy of the interferer is a power level q of the jamming signal. Let $\Pi_I = \mathbb{R}_+$ be the set of all feasible interferer's strategies. Note that the probability of mission success for UAV i is

$$\pi_S(A_i(p_i, q)) = \frac{b}{b + aA(p_i, q_i)} = \frac{b z_i g_i p_i}{(b + ac/\lambda_i) z_i g_i p_i + da(N_i + h_i q)}. \tag{4}$$

We now introduce the auxiliary notations:

$$\alpha_i = b g_i z_i, \;\; \beta_i = (b + ac/\lambda_i) g_i z_i, \;\; \gamma_i = d a h_i, \delta_i = d a N_i, \;\text{ and }\; \Gamma_i(q) = \gamma_i q + \delta_i. \tag{5}$$

With this notation, (4) becomes

$$\pi_S(A_i(p_i, q)) = \frac{\alpha_i p_i}{\beta_i p_i + \Gamma_i(q)}. \tag{6}$$

As criteria for mission success we consider proportional fairness criteria [6,18] for mission success of each UAV, i.e.,

$$v_{\text{GCS}}(\boldsymbol{p}, q) = \sum_{i=1}^{n} \ln(\pi_S(A_i(p_i, q))). \tag{7}$$

This utility is the payoff for the GCS. For the interferer, the cost function is the sum of the GCS payoff and the involved cost of the effort, i.e.,

$$v_I(\boldsymbol{p}, q) = v_{\text{GCS}}(\boldsymbol{p}, q) + C_I q, \tag{8}$$

where C_I is the cost per unit of jamming power. The GCS wants to maximize its payoff, while the interferer aims to minimize its cost function. So, $-v_I$ is the payoff to the interferer. We are looking for Nash equilibrium. Recall that $(\boldsymbol{p}, q)$ is a Nash equilibrium [4] if and only if:

$$\begin{aligned} v_{\text{GCS}}(\tilde{\boldsymbol{p}}, q) &\le v_{\text{GCS}}(\boldsymbol{p}, q), \\ v_I(\boldsymbol{p}, q) &\le v_I(\boldsymbol{p}, \tilde{q}) \text{ for any } (\tilde{\boldsymbol{p}}, \tilde{q}) \in \mathit{\Pi}_{\text{GCS}} \times \mathit{\Pi}_I. \end{aligned} \tag{9}$$

We denote this game by $\mathit{\Gamma} = \mathit{\Gamma}(v_{\text{GCS}}, \mathit{\Pi}_{\text{GCS}}; -v_I, \mathit{\Pi}_I)$.

Lemma 1. *$v_{GCS}(\boldsymbol{p}, q)$ is concave in $\boldsymbol{p}$, and $v_I(\boldsymbol{p}, q)$ is convex in q.*

Proof. Note that

$$\begin{aligned} \frac{\partial^2 v_{\text{GCS}}(\boldsymbol{p}, q)}{\partial p_i^2} &= -\frac{\mathit{\Gamma}_i(q)(2\beta_i p_i + \mathit{\Gamma}_i(q))}{p_i^2(\beta_i p_i + \mathit{\Gamma}_i(q))^2} < 0, \\ \frac{\partial^2 v_I(\boldsymbol{p}, q)}{\partial q^2} &= \sum_{i=1}^{n} \frac{\gamma_i^2}{(\beta_i p_i + \mathit{\Gamma}_i(q))^2} > 0, \end{aligned}$$

and the result follows. ∎

Lemma 1 and the Nash theorem [4] imply the following result.

Theorem 1. *In the game $\mathit{\Gamma}$ there exists at least one equilibrium.*

3 Solution of the Game

In this section we design equilibrium strategies of the game $\mathit{\Gamma}$. By (9), $\boldsymbol{p}$ and q are equilibrium strategies if and only if each of them is the best response to the other, i.e., they are solutions of the equations:

$$\boldsymbol{p} = \text{BR}_{\text{GCS}}(q) = \text{argmax}\{v_{\text{GCS}}(\boldsymbol{p}, q) : \boldsymbol{p} \in \mathit{\Pi}_{\text{GCS}}\}, \tag{10}$$

$$q = \text{BR}_I(\boldsymbol{p}) = \text{argmin}\{v_I(\boldsymbol{p}, q) : q \in \mathit{\Pi}_I\}. \tag{11}$$

To solve these best response equations we will employ a constructive approach.

3.1 Best Response Strategies

In this section we derive the best response strategies.

Theorem 2. *The best response strategy* $\boldsymbol{p}$ *of the GCS to jamming power* q *is unique and given as follows:*

$$p_i = P_i(\Omega(q), q) \text{ for } i = 1, \dots, n, \tag{12}$$

where for each fixed q, $\Omega(q) = \omega$ *is the unique positive root of the equation*

$$S_P(\omega, q) = \overline{p}, \tag{13}$$

with

$$S_P(\omega, q) \triangleq \sum_{i=1}^{n} P_i(\omega, q), \tag{14}$$

$$P_i(\omega, q) \triangleq \frac{\Gamma_i(q)}{2\beta_i}\left(\sqrt{1 + \frac{4\beta_i}{\Gamma_i(q)\omega}} - 1\right). \tag{15}$$

Proof. Since, by Lemma 1, (10) is a concave NLP problem, to find the best response strategy $\boldsymbol{p}$ to q we have to introduce a Lagrangian depending on a Lagrange multiplier ω as follows: $L_\omega(\boldsymbol{p}) = v_{\text{GCS}}(\boldsymbol{p}, q) + \omega\left(\overline{p} - \sum_{i=1}^n p_i\right)$. Then, KKT Theorem implies that $\boldsymbol{p} \in \Pi_{\text{GCS}}$ is the best response strategy to q if and only if the following condition holds:

$$\frac{\partial L_\omega}{\partial p_i} = \frac{\Gamma_i(q)}{p_i(\beta_i p_i + \Gamma_i(q))} - \omega \begin{cases} = 0, & p_i > 0, \\ \le 0, & p_i = 0. \end{cases} \tag{16}$$

By (16), we have that $p_i > 0$ for any i. Thus, also $\omega > 0$, and

$$\frac{\Gamma_i(q)}{p_i(\beta_i p_i + \Gamma_i(q))} = \omega \text{ for any } i. \tag{17}$$

Solving this equation in p_i implies $p_i = P_i(\omega, q)$ as given by (15).

Since $\boldsymbol{p} \in \Pi_{\text{GCS}}$ the ω is defined by the condition that the total power resource has to be utilized by the GCS, i.e., by Eq. (13).

Note that $P_i(\omega, q)$ given by (15) has the following properties:

(i) $P_i(\omega, q)$ is differentiable in ω and q
(ii) $P_i(\omega, q)$ is decreasing in ω from infinity for $\omega \downarrow 0$ to zero for $\omega \uparrow \infty$.
(iii) $P_i(\omega, q)$ is increasing in q to $1/\omega$ for $q \uparrow \infty$.

Note that (i) and (ii) straightforwardly follow from (15). By (15), for a fixed $\omega > 0$

$$\lim_{q \uparrow \infty} P_i(\omega, q) = 1/\overline{\omega} \tag{18}$$

Also, $P_i(\omega, q) = f_i(\Gamma_i(q))/(2\beta_i)$, where $f_i(x) = x(\sqrt{1 + m/x} - 1)$ with $m = 4\beta_i/\omega$. Since $\frac{df_i(x)}{dx} = \frac{2x+m}{2\sqrt{x^2+mx}} - 1 > 0$, $P_i(\omega, q)$ increases with q. This and (18) implies (iii). Then, (i) and (ii) yield existence of the unique root $\omega = \Omega(q)$ for Eq. (13). While (i)–(iii) and (18) imply that $\Omega(q)$ increases with q to $n/\overline{p}$. ■

Note that, $\Omega(q)$ can be found via bisection method and $\Omega(q)$ is differentiable for $q \geq 0$ and increasing from ω_0 for $q = 0$ to $n/\overline{p}$ for $q \uparrow \infty$, where ω_0 is the unique positive root of the equation:

$$S_P(\omega_0, 0) = \sum_{i=1}^{n} \frac{\delta_i}{2\beta_i} \left(\sqrt{1 + \frac{4\beta_i}{\delta_i \omega_0}} - 1 \right) = \overline{p}. \tag{19}$$

Theorem 2 straightforwardly implies the following result.

Corollary 1. *The inverse function $Q(\omega) = \Omega^{-1}(q)$ to $\Omega(q)$ is defined for $\omega \in [\omega_0, n/\overline{p})$ and increases from $Q(\omega_0) = 0$ to $\lim_{\omega \uparrow (n/\overline{p})} Q(\omega) = \infty$. Moreover, $S_P(\omega, Q(\omega)) = \overline{p}$.*

Theorem 3. *The best response strategy q of the interferer to $\boldsymbol{p}$ is unique and given as follows:*

$$q = \begin{cases} 0, \sum_{i=1}^{n} \frac{\gamma_i}{\beta_i p_i + \delta_i} \leq C_I, & \text{(20a)} \\ q_+, \sum_{i=1}^{n} \frac{\gamma_i}{\beta_i p_i + \delta_i} > C_I, & \text{(20b)} \end{cases}$$

such that, when (20b) holds, q_+ is the unique positive root of

$$\sum_{i=1}^{n} \frac{\gamma_i}{\beta_i p_i + \Gamma_i(q_+)} = C_I. \tag{21}$$

Proof. Since, by Lemma 1, $v_I(\boldsymbol{p}, q)$ is a convex in q, by (11), q is the best response strategy to $\boldsymbol{p}$ if and only if the following condition holds:

$$\frac{\partial v_I(\boldsymbol{p}, q)}{\partial q} = -\sum_{i=1}^{n} \frac{\gamma_i}{\beta_i p_i + \Gamma_i(q)} + C_I \begin{cases} = 0, & q > 0, \\ \geq 0, & q = 0. \end{cases} \tag{22}$$

Since $\gamma_i/(\beta_i p_i + \Gamma_i(q))$ is decreasing in q, the result straightforward follows from (22). ∎

3.2 Equilibrium

In this section we establish threshold value of the jamming cost for the interferer to be active, derive the form the equilibrium has to have and design a fixed point algorithm to find the equilibrium.

Theorem 4. *(a) If*

$$\sum_{i=1}^{n} \frac{\gamma_i}{\delta_i \left(1 + \sqrt{1 + 4\beta_i/(\delta_i \omega_0)}\right)} \leq \frac{C_I}{2}. \tag{23}$$

then $(\boldsymbol{p}, q) = (\boldsymbol{P}(\omega_0, 0), 0)$ *is the unique equilibrium where* ω_0 *and* $\boldsymbol{P}$ *given by (15) and (19) correspondingly.*

(b) If (23) does not hold then $(\boldsymbol{p}, q) = (\boldsymbol{P}(\Omega(q), q), q)$ *is the equilibrium where* $\boldsymbol{P}$ *given by (15) and* q *is the positive root of the equation*

$$F(q) = C_I/2, \tag{24}$$

where

$$F(q) \triangleq \sum_{i=1}^{n} \frac{\gamma_i}{\Gamma_i(q)\left(1 + \sqrt{1 + 4\beta_i/(\Omega(q)\Gamma_i(q))}\right)}. \tag{25}$$

Proof. Let $(\boldsymbol{p}, q)$ be an equilibrium. By Theorem 2, $\boldsymbol{p} > 0$. Thus, only two cases arise to consider: (a) $q = 0$ and (a) $q > 0$.

(a) Let $q = 0$. Then, by Theorem 2, $\boldsymbol{p} = \boldsymbol{P}(\omega_0, q)$. Substituting this $\boldsymbol{p}$ into (20a) implies (20a).
(b) Let $q > 0$. Then, by Theorem 2, $\boldsymbol{p} = \boldsymbol{P}(\Omega(q), q)$. Substituting this $\boldsymbol{p}$ into (21) implies (24) and (25). By (21), q is decreasing in C_I. Thus, by (24) and (25), F also decreasing in C_I, and the result follows. ■

By Theorem 2, we have that $\lim_{q\uparrow\infty} F(q) = 0$. Moreover, if (23) does not hold then $F(0) > C_I/2$. Also, note that (23) establishes the threshold on the jamming cost for the interferer to be active (i.e., for $q > 0$ to be an equilibrium) or non-active (i.e., for $q = 0$ to be an equilibrium). While the GCS is always active in communication with each of the UAVS. This remarkably differs with OFDM jamming problem where some of sub-subcarriers could be not involved in transmission [8] and network security problem where some not might be not protected [2].

Interestingly, the equilibrium q can be found using fixed point algorithm. To do so, note that, by Theorem 2 and Corollary 1, there is one-to-one correspondence between ω and q. That is why first in the following proposition we derive an equation for ω, and, then, in Theorem 5, we prove convergence of the fixed point algorithm to find the ω.

Proposition 1. *Equation (24) is equivalent to*

$$G(\omega) = \omega, \tag{26}$$

with $q = Q(\omega)$, *where*

$$G(\omega) \triangleq \frac{2C_I}{\sum_{i=1}^{n} \gamma_i \left(\sqrt{1 + 4\beta_i/(\omega\Gamma_i(Q(\omega)))} - 1\right)/\beta_i}. \tag{27}$$

Proof. Note that

$$\frac{\gamma_i}{\Gamma_i(q)\left(1+\sqrt{1+4\beta_i/(\omega\Gamma_i(q))}\right)} = \frac{\gamma_i\left(\sqrt{1+4\beta_i/(\omega\Gamma_i(q))}-1\right)}{4\beta_i/\omega}.$$

Substituting this into (24) and (25) imply the result. ■

The following theorem shows that Eq. (26) can be solved by fixed point algorithm.

Theorem 5. *$G(\omega)$ has the following properties:*

- **(i)** *$G(\omega_0) < \omega_0$;*
- **(ii)** *$G(\omega)$ is continuous and increasing on ω;*
- **(iii)** *There is ω_* such that $G(\omega) < \omega$ for $\omega < \omega_*$ and*

$$G(\omega_*) = \omega_*; \tag{28}$$

- **(iv)** *The fixed point ω_* of (28) can be found via fixed point algorithm:*

$$\omega^m = G^{-1}(\omega^{m-1}) \text{ for } m = 1, 2, \dots \text{ with } \omega^0 \text{ is fixed.}$$

The algorithm converges to ω_* for any $\omega^0 \in (\omega_0, \omega_*)$.

Proof. (i) follows from (20b). (ii) follows from Corollary 1 and (27). (iii) follows from (i), (ii), Theorems 1, 4(b) and Proposition 1.

Since $\omega^0 < \omega_*$, by (ii) and (iii), $G(\omega^0) < \omega^0$. Then, (28) implies that there is the unique $\omega^1 \in (\omega^0, \omega_*)$ such that $G(\omega^1) = \omega^0$. Thus, $\omega^1 = G^{-1}(\omega^0)$. Similarly, there is the unique $\omega^2 \in (\omega^1, \omega_*)$ such that $G(\omega^2) = \omega^1$, and so on, i.e., there is the unique $\omega^m \in (\omega^{m-1}, \omega_*)$ with $m \geq 1$ such that $G(\omega^m) = \omega^{m-1}$. Thus, ω^m is increasing and upper-bounded. Thus, there exists $\lim_{m\uparrow\infty} \omega^m$, and, this limit is equal to ω_*. ■

Note that for boundary cases of the jamming cost and total transmission power the equilibrium strategies can be obtained in closed form:

Proposition 2. *(a) Let C_I be small. Then*

$$q \approx n/C_I \text{ and } p_i \approx \overline{p}/n \text{ for } i = 1, \dots, n. \tag{29}$$

(b) Let $\overline{p}$ be small. Then $p_i \approx \overline{p}/n, i = 1, \dots, n$ and

$$q \approx \begin{cases} 0, & \sum_{i=1}^{n}(\gamma_i/\delta_i) < C_I, \quad (30a) \\ q_*, & \sum_{i=1}^{n}(\gamma_i/\delta_i) > C_I, \quad (30b) \end{cases}$$

where q_* *is the unique positive root of the equation*

$$\sum_{i=1}^{n} 1/(q_* + \delta_i/\gamma_i) = C_I. \tag{31}$$

(c) Let $\overline{p}$ *be large. Then* $q = 0$ *and*

$$p_i \approx \frac{\sqrt{\delta_i/\beta_i}}{\sum_{j=1}^{n}\sqrt{\delta_j/\beta_i}}\overline{p} \text{ for } i = 1, \ldots, n. \tag{32}$$

Proof. Let C_I be small. Then, by (20b), $q = q_+$. While, by (21), q_+ is large. Then, Eq. (21) can be approximated by $n/q_+ \approx C_I$. Thus, $q = n/C_I$. Substituting this q into (15) implies that $P_i(\omega, q) \approx 1/\omega$, and (a) follows. Let $\overline{p}$ be small. Then p_i also is small for any i. Substituting these p_i into (20a), (20b) and (21) and taking into account that $\boldsymbol{p} \in \Pi_{\text{GCS}}$ implies imply (b). Let $\overline{p}$ be large. Then p_i is large for at least one i. Then, by (20a), $q = 0$. Then, by (13) and (15), ω is small, and $p_i = P_i(\omega, 0) \approx \sqrt{\delta_i/\beta_i}/\sqrt{\omega}$. Then, since $\boldsymbol{p} \in \Pi_{\text{GCS}}$, (32) follows. ■

If background noise can be neglected, then equilibrium strategies also can be found in closed form.

Proposition 3. *If* $\delta_i = 0$ *for all* i, *then,*

(a) if

$$\sum_{i=1}^{n} \sqrt{\gamma_i/\beta_i} \le \sqrt{\overline{p}C_I} \tag{33}$$

then $q = 0$ *is the unique interferer strategy, while there is a continuum of the GCS equilibrium strategies, namely, any strategy* $\boldsymbol{p} \in \Pi_{GCS}$ *such that:*

$$\sum_{i=1}^{n} \gamma_i/(p_i\beta_i) \le C_I. \tag{34}$$

(b) if

$$\sum_{i=1}^{n} \sqrt{\gamma_i/\beta_i} > \sqrt{\overline{p}C_I} \tag{35}$$

then q *and* $\boldsymbol{p}$ *are uniquely defined as follows:*

$$p_i = P_i(\omega) = \frac{\sqrt{(\overline{p}\omega/C_I)^2 + 4\overline{p}\beta_i/(\gamma_i C_I)} - \overline{p}\omega/C_I}{2\beta_i/\gamma_i} \text{ for } i = 1, \ldots, n, \tag{36}$$

$$q = \omega\overline{p}/C_I, \tag{37}$$

where ω *is the unique positive root of the equation:* $\sum_{i=1}^{n} P_i(\omega) = \overline{p}$.

Proof. Since $\delta_i = 0$ for all i, if $q = 0$ then $\boldsymbol{p}$ is any feasible strategy such that $\sum_{i=1}^n \gamma_i/(\beta_i p_i) \le C_I$. Such an equilibrium strategy exists if and only if

$$\min_{\boldsymbol{p}\in\Pi_{\rm GCS}} \sum_{i=1}^n \gamma_i/(\beta_i p_i) \le C_I. \tag{38}$$

It is clear that left-side of (38) is a convex NLP problem, and straightforward applying the KKT theorem implies that its solution is

$$p_i = (\overline{p}\sqrt{\gamma_i/\beta_i})/\sum_{j=1}^n \sqrt{\gamma_j/\beta_i}.$$

Substituting this strategy into (38) implies (33), and (a) follows. While, if $q > 0$ then by (21) and (33) we have that $\sum_{i=1}^n \omega p_i/q = C_I$. This and the fact that $\boldsymbol{p} \in \Pi_{\rm GCS}$ implies (37). Substituting (37) implies that $\boldsymbol{p}$ is given by (36). Note that $\varphi(\omega) = \sum_{i=1}^n P_i(\omega)$ decreases with ω and tends to zero for $\omega \uparrow \infty$. Then, equation $\varphi(\omega) = \overline{p}$ has the positive root if and only if $\varphi(0) > \overline{p}$, and this condition is equivalent to (35). ■

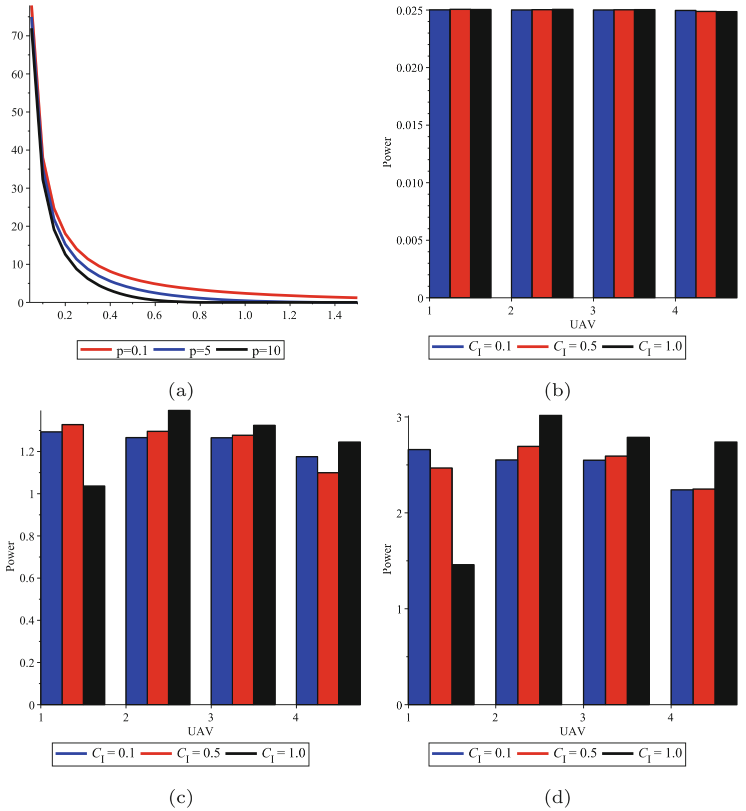

Fig. 1. (a) The equilibrium q for $\overline{p} \in \{0.1, 5, 10\}$, (b) the equilibrium $\boldsymbol{p}$ for $\bar{p} = 0.1$, (c) the equilibrium $\boldsymbol{p}$ for $\bar{p} = 5$ and (d) the equilibrium $\boldsymbol{p}$ for $\bar{p} = 10$.

Figure 1(a) illustrates a decrease in applied jamming power with an increase in jamming power cost and the total transmission power. Figure 1(b) illustrates that for $\bar{p}$ the GCS tends to serve the UAV uniformly. While an increase in $\bar{p}$ allows the GCS to serve in more individual form according to non-uniform power allocation (32). This makes the problem remarkably distinguish from OFDM transmission where uniform strategy arise for large total power resource [8]. This is caused by the fact that OFDM utility can be approximated by a superposition of logarithm and linear function of transmission power for large applied power while proportional fairness utility of the considered game can be approximated similar way for small applied power.

4 Conclusions

The problem to maintain freshness of the commands received by a group of UAVs to succeed a mission under hostile interference was modeled as non-zero game. Proportional fairness in mission success by each of the UAVs is considered as criteria for the GCS. The problem is formulated and solved as non-zero some game The considered game differs remarkably from the conventional jamming games considered in literature [16,20,24] because the structure of the rivals' strategies differ from from each other. In particular, the GCS's strategy is power allocation between the UAVs, while the interferer's strategy is a common power level assignment to jam the whole UAV's group. Moreover, in OFDM jamming game with throughput as transmitter's payoff [8], transmitter's equilibrium strategy is uniform power allocation for large total transmitting power, while, in the considered game, GCS's equilibrium strategy is uniform power allocation for small total transmitting power.

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