Modal Analysis of Rocks and Ores in Thin Sections

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Abstract. The article is devoted to the history and justification of the modal analysis of rocks and ores with a microscope. It is shown that the Delesse-Rosiwal-Glagolev ratios do not follow from the Cavalieri principle. They do not allow one to find the exact volume of the minerals in rocks or ores, but give only their average estimates. It is also shown that the volume fractions of convex mineral grains in rocks and ores, taken equal to the fractions of their flat sections, are always underestimated if compared with the matrix. Due to the wide variety and complexity of forms of mineral grains, the methods of stereological reconstruction lead to integral equations with a difficult to define form factor. Most likely, tomography methods should come to replace the modal analysis of rocks and ores in thin sections.

Keywords: Rocks and ores · Modal analysis · Stereological reconstruction

1 Introduction

Modal (quantitative mineralogical) analysis of rocks and ores in thin sections is one of the first fundamental quantitative methods of mineralogy (including technological mineralogy), petrography (in classifications of rocks and ores, and petrological reconstructions), and lithology (i.e., petrography of sedimentary rocks). That is why its rigorous justification is of fundamental importance owing to the Delesse-Rosiwal-Glagolev ratios, as well as to stereological reconstruction. It makes sense to summarize the history of these methods in Russia and abroad, and to formulate the conclusion about their prospects.

2 The Cavalieri Principle and the Delesse-Rosiwal-Glagolev Ratios

First of all, we point out that the relations suggested by Delesse (1848) $dV_i = dS_i$, Rosiwal (1898) $dS_i = dL_i$, and Glagolev (1932) $dL_i = dN_i$, decreasing the dimension of space (namely, equating the volume fractions of minerals to areal, areal to linear, linear to point-like), have no relation to the Cavalieri principle: $S_{1i} = S_{2i} \rightarrow V_1 = V_2$ (if the areas of all arbitrarily close parallel sections of two bodies are pairwise equal, then their volumes are also equal, Fig. 1).

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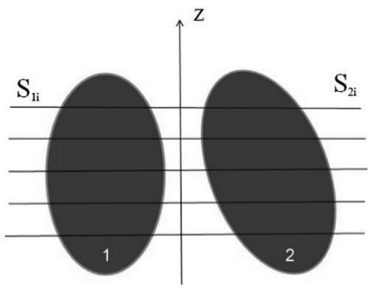

Fig. 1. To the justification of the Cavalieri principle

In recent notation, this principle, historically preceding integral calculus, has a clear meaning: $V_1 = \int S(z)\, dz = V_2$, where S(z) is a continuous function of the areal fraction of a certain mineral along the z axis being normal to the sections. But, the modal analysis of rocks and ores in thin sections, which accumulates the statistics of areal, linear or point fractions of minerals from section to section, has nothing to do with the integration procedure. It only leads to an assessment of their average values. In this case, it can be argued that the volume of any mineral in a rock or ore is within the certain interval:

$$S(z)_{min}\Delta z = \int S(z)_{min} dz < V < \int S(z)_{max} dz = S(z)_{max}\Delta z$$

where Δz is the thickness of rock sample under study.

Despite of this contradiction, which was also considered in the works (Krumbein 1935; Chayes 1956), the method became firmly established in practice because of its apparent simplicity and was step by step automated (Shand 1916; Wentworth 1923; Hunt 1924; Dollar 1937; Hurlbut 1939) up to the use of modern computers for image analysis of thin sections. The list of parameters characterizing the cross-section of minerals, and the speed of processing have grown many times. But, in terms of the reconstruction of the true metric characteristics of mineral grains from those of their flat or even linear sections, the ideology remains the same.

Companies that produce computer structure analyzers offer software packages without discussing the fundamental problems. The analysis of 2D images does not use the available chapters of mathematics. For example, the distances between the mineral grains in thin section are replaced by the Euclidean distances between the points taken within the grains, whereas there is a more complicated, but easily programmable Hausdorff's metric which allows one to do this procedure correctly. In turn, it gives us the possibility of calculating space covariograms between the mineral grains of different species and their various clusters in the rock.

3 Stereological Reconstruction

A new line of research, i.e. stereological reconstruction, arose from the obvious observation that an arbitrary cross section of a spherical shape is always less than its characteristic cross section (Fig. 2, above). And it follows from this that the volume fraction of the convex mineral phase in the rock and ore, equal to the fraction of its flat sections, is always underestimated if compared with the host matrix. The corresponding general problem – finding the distribution of true particle sizes from the size distribution of their random sections – belongs to the inverse problems typical for geophysics. It is analytically solved only for spherical and ellipsoidal particles due to relatively simple description of these forms (Wicksel 1925, 1926). But, even in this case, the practical use of the theory requires the selection of the best solution and an estimate of the errors (Fig. 2, below). For more complex forms of mineral grains this can't be done without mathematical modeling on powerful computers.

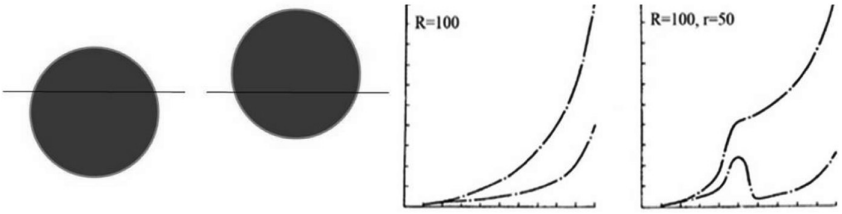

Fig. 2. Left: the size of the cross section of a convex grain is always less than the characteristic one, in the modal analysis its volume fraction is underestimated. Right: to recognize the true size of spherical particles by the size of circular sections; R = 100 – particles with a radius of 100 arbitrary units; R = 100, r = 50 – two sets of particles of the same type, indistinguishable in sections (for example, two generations of one mineral); horizontal scale – section radii from 0 to 100 bits per 10 classes; vertical scale – frequencies by classes (lower curves) and accumulated frequencies (upper curves).

The history of this area in Russia can be found in the following incomplete list of works: Zhuravsky A.M. Mineralogical analysis of thin section in terms of probabilities. Moscow-Leningrad: Gosgeolizdat, 1932. 20 p.; Glagolev A.A. Quantitative mineralogical analysis of rocks with the microscope. Leningrad: Gosgeolizdat, 1932. 25 p.; Glagolev A.A. On the geometric methods of quantitative mineralogical analysis of rocks. Moscow-Leningrad: Gosgeolizdat, 1933. 47 p.; Glagolev A.A. Geometric methods for quantitative analysis of aggregates with a microscope. Moscow-Leningrad: Gosgeolizdat, 1941. 263 p.; Chayes F. An elementary statistical appraisal. New York: John Wiley & Sons, Inc., 1956; Shvanov V.N., Markov A.B. Granulometric analysis of sandstones in thin sections//Geology and exploration. 1960. N 12. P. 49–55; Ivanov N. V. A new direction in testing ore deposits. Moscow: Gosgeolizdat, 1963. 179 p.; Chernyavsky K.S. Stereology in metallurgy. Moscow: Metallurgy, 1977. 375 p.; Ivanov O.P., Ermakov S.F., Kuznetsova V.N. Improving the accuracy of determining the weight particle size distribution of ore minerals from measurements in thin

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4 Conclusions

Thus, due to the extraordinary diversity and complexity of the forms of mineral grains in rocks and ores, the methods of stereological reconstruction lead to integral equations with an analytically difficult-to-define form factor. The practical application of the theory is drowning in the selection of the best solution to the inverse problem and complex estimates of measurement errors. It seems that the modal analysis of rocks and ores in thin sections should be replaced by tomography methods. Standardizing modal analysis of rocks and ores in thin sections by creating their artificial counterparts with previously known volume fractions of mineral grains and a wide range of petrographic structures can serve as an inter-laboratory comparison of the accuracy of the method. But it does not solve the problems in essence.

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