

Chapter 8

Low Achievers in Mathematics—Ideas from the Netherlands for Developing a Competence-Oriented View



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Abstract Although in Germany a competence-oriented view on teaching and learning mathematics has been one of the guiding principles for primary mathematics since the early 1990s, this approach was not appreciated for low achievers or for students in special education. Research in special education mostly focused on diagnosis with regard to deficiencies and not considering individual thinking and interpretations of mathematical tasks and problems. Moreover, the usual teaching practice in special education could be characterised by learning step-by-step in a rather mechanistic and reproductive way. Influenced by research papers and encouraging classroom experiences with low-achieving students in the Netherlands, my research focused on the question to what extent competence-oriented diagnosis followed by an inquiry-based learning approach would be appropriate also for students with special needs, or especially for them. Instead of underestimating these students' abilities, it seemed necessary to give them the opportunity to show what they are capable of, for example, by using more open problems that show the ideas students have in mind. In several projects and studies referring to different mathematical topics it could be shown that even low achievers benefited from a competence-oriented diagnosis and from an open approach and that these students were able to choose individual strategies, make use of structures and relations, find patterns and show creative and effective work.

Keywords Low achievers · Tests · Context-related problems · Open problems · Mathematical competencies · Qualitative research

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8.1 Introduction

One will always find in any school learners who have learning problems in general or especially with learning mathematics. One might think that it is possible to use specific methods or materials to avoid those difficulties; however, it has turned out that this is not always sufficient. More fundamental research is necessary concerning the concepts for instruction and the way of planning the teaching and learning processes (see also Scherer, Beswick, DeBlois, Healy, & Moser Opitz, 2016). Moreover, to adapt the teaching to the students' needs, substantial information is needed about the learners' difficulties and about the knowledge the students have available. Research is required to reveal this information. The special focus in this chapter is the influence of the Dutch approach to mathematics education on research carried out with students with special needs in Germany.

8.2 Mathematics Education in Special Education in Germany

In Germany, students with special needs either visit special schools for handicapped students or visit regular schools in inclusive settings. Both settings show extremely heterogeneous groups in classroom and the teacher is confronted with various handicaps, for example deficits in language or visual perception, failure of concentration or reduced memory, which means that a high degree of differentiation is needed. Until now there are two types of teacher education programmes: one for special education and one for the regular school system. Teacher education programmes preparing teachers for an inclusive school system are still under development.

In this chapter, the focus will be on students with learning difficulties and learning disabilities. The corresponding approaches for teaching and learning with relevant concepts for instruction in mathematics will be discussed and the influence from abroad will be reported from the early 1990s.

Looking in more detail at the concepts of teaching and learning mathematics it can be stated that special education followed a more traditional view for a long time. Consequently, the concrete situation in classroom for low achievers was quite different from teaching practice in regular schools. Whereas in regular education one could see an approach to the teaching-learning process in which students are active participants and are offered opportunities for guided-discovery learning, in special education this approach was mostly disregarded. Instead, of having a constructivist view on learning, behaviourism remained the central principle.

Although in 1977 the curriculum for mathematics in special education already pointed out that the students should be able to develop their own methods of problem solving, and the danger of getting stuck in purely schematic thinking through mechanistic drill-and-practice was clearly stated (KM, 1977, p. 7), textbooks and classroom practice did not live up to these high expectations. The generally shared

opinion was that low achieving students cannot cope with more demanding and complex problems, and the mechanistic teaching methods were rarely called into question.

Thus, very often the demands were lowered, and learners' activities were confined to bare reproduction. Textbooks and most teaching proposals and hence the usual teaching practice in special education could be characterised by learning step-by-step in a rather mechanistic and reproductive way. Mechanistic drill-and-practice often replaced insightful learning (Baier, 1977; Grölz, 1983), and low achievers were too often confronted with a 'mathematical diet': problems containing the discovery of patterns and structures were avoided. Regarding context problems, the students were not challenged to mathematise and to really do mathematics.

These findings were not only true for Germany but could be generalised for other countries (Ahmed, 1987; Moser Opitz, 2000; Trickett & Sulke, 1988). In the Netherlands as well, there is a tradition in special education in which mathematics instruction is dominated by the principles of learning step-by-step, isolation of difficulties and giving prescribed and fixed ways of solutions (Van den Heuvel-Panhuizen, 1991).

In Germany, these more traditional approaches can still be found in the majority of textbooks for special education. Although the situation changed over the years, there still is scepticism with respect to using a reformed approach for low achievers in which there is room for students' own contributions to the teaching-learning process. But from the point of view of mathematics education such an approach is necessary for identifying students' existing difficulties and giving them the opportunity to show what they are capable of. In this sense, the research reported below can be understood as a plea for an ongoing change of teaching practice. With concrete examples, the influence of mathematics education in the Netherlands will be illustrated, starting from the competence-oriented view on low achievers' learning processes.

8.3 Looking at the Netherlands: Looking at a Competence-Oriented Approach

8.3.1 Realistic Mathematics Education

The Dutch approach of Realistic Mathematics Education (RME) (Streefland, 1991; Treffers, 1987; Van den Heuvel-Panhuizen & Drijvers, 2014) propagates an approach to mathematics education in which students are given an active role and can come up with their own solutions based on familiar context situations. This use of contexts is one of the main characteristics of RME, but this should not be misunderstood. RME is not only restricted to context-related problems, but also covers inner-mathematical tasks and problems, as the term 'realistic' is derived from 'to realise' in the meaning of 'to imagine' (Van den Brink, 1991, p. 199).

The active acquisition of knowledge and own solutions for problems allow an intellectual and emotional identification (Streefland & Treffers, 1990, p. 315). The

teacher has to offer learning situations that enable the students to make discoveries, but this requires that the student is provided with powerful tools such as (context) models, schemes, symbols (Streefland & Treffers, 1990, p. 313ff.). In the Netherlands, the criteria for a competence-oriented view lead to a critical view on testing procedures formats as well as on learning environments and adequate teaching practices (Streefland, 1990b). As a consequence, different formats for diagnostic procedures as well as teaching and learning situations came into existence.

8.3.2 *Diagnostic Procedures: New Assessment Formats*

Using diagnostic instruments has a long tradition in special education, as those results should give important information for the next learning steps and expected learning processes. The concrete instruments and methods are of major importance, and there are several possibilities for diagnosing learning difficulties. Many tests show what students *do not know*. But they do not show *why* nor do they give information about *what the students are able to do*.

As mentioned above, the German situation in special education in the early 1990s could be characterised by following the paradigm of behaviourism. According to this paradigm, the instruments and procedures for diagnosing mathematical formation were more deficit-oriented than competence-oriented (Scherer, 1999).

At the same time in the Netherlands attempts were made to introduce alternate forms of tests with respect to the test procedures and methods as well as to the construction of problems and items. According to Van den Heuvel-Panhuizen (1990) tests are needed that:

- cover the whole spectrum of the arithmetic/mathematics area concerned (p. 57),
- give children the opportunity to show what they are able to do (p. 61),
- provide information about abilities and strategies (p. 68).

Van den Heuvel-Panhuizen (1991) convincingly presented a Dutch study carried out with low achievers (in Grade 5 and 6) on the topic of ratio—a topic that was usually avoided in special education as it was assumed to be too difficult for low achievers. She developed a written test with context-related items in contrast to the usual formal items. Beyond numerical problems she also integrated qualitative problems that could be solved by estimating or measuring and that allowed individual solution strategies. The results showed a higher rate of correct answers than expected by teachers, school psychologists and school inspectors.

Influenced by this encouraging research, studies have been carried out in Germany as well in which a more competence-oriented approach was taken to assess students than was common in Germany.

Fig. 8.1 Free production
(Streefland, 1990a, p. 44)

$$\begin{array}{cccc}
 3 & 8 & 4 & 20 \\
 3 \times 8 = 28 & & & \\
 8 - 3 = 5 & 20 + 4 = 24 & & \\
 4 + 8 = 12 & & & \\
 3 - 8 = \text{min } 5 & & &
 \end{array}$$

8.3.3 *Students' Own Productions: Open Problems*

One central type of problem—for tests as well as for regular teaching—is that of so-called ‘own productions’ or ‘free productions’ (Streefland, 1990a). Own productions offer various opportunities for own strategies and solutions and support a suitable differentiation (Streefland & Treffers, 1990, p. 315). An example is given in Fig. 8.1, and other illustrations of this format of open problems can be found in Sect. 8.4.2.

Here, students should make their own problems with the given numbers 3, 8, 4 and 20. They were free to choose the operations and could select particular numbers and decide how many numbers they wanted to use in a problem. The worksheet of an eight-year-old student that is shown in Fig. 8.1 shows that within this format mistakes might occur with operations and tasks that have been dealt with in school (3×8 makes 28). But at the same time, the format allows inventions and excursions to new mathematical areas. The worksheet illustrates insights in mathematical operations that have not yet been dealt with in mathematics (subtractions resulting in negative numbers, see also Sect. 8.4.4).

8.3.4 *Making Connections Between Problems: Patterns and Structures*

Mathematics is often named as the science of patterns and structures (Wittmann & Müller, 2008). For all students and especially for students with problems in mathematics making connections and the use of relationships could be a help for understanding (see Scherer, 1997). An example of the Dutch approach for a test item is illustrated in Fig. 8.2. The aim is to use ‘auxiliary problems’ or ‘support problems’ to investigate whether the students have insight into properties of operations and possess the ability to apply them.

Also in Germany, making use of patterns and structures was of great importance for designing learning opportunities for students—probably more emphasis was put on this aspect compared to the Netherlands. In Germany, the operative principle (Wittmann, 1985), that is, the analysis of mathematical objects and operations and their effects, is considered of great importance. The application of this principle

Fig. 8.2 Making use of 'support problems' (Van den Heuvel-Panhuizen, 1996, p. 153)

$$86 + 57 = 143$$

$$86 + 56 = 144$$

$$57 + 86 = 143$$

$$860 + 570 = 1430$$

$$85 + 57 = 142$$

$$143 - 86 = 57$$

$$86 + 86 + 57 + 57 = 286$$

$$85 + 58 = 143$$

can be identified in textbooks in the design of task series and in explicit problems in which students have to look for patterns and structures. For example, students should reflect upon specific modifications for a product when multiplying: What effect can be identified when increasing/decreasing one factor by 1? Is there an effect when changing the order of the two factors? How to reach the same result for a product when the first factor is doubled? This principle is not only true for arithmetic but should also be discussed for geometry. For example, doubling the length of a square side: What effect do you observe for the square area?

8.4 Research in Germany

8.4.1 Competence-Oriented Diagnosis

The guiding principles for diagnostic procedures and ideas for competence-oriented instruments presented above lead to several projects and case studies concerning the diagnosis of existing mathematical knowledge for different mathematical topics: arithmetical knowledge about numbers up to 20 or up to 100, addition and subtraction with numbers up to 20 or 100 (Scherer, 1999, 2005), multiplication and division (Scherer, 2003), understanding of place value (Scherer, 2014). In all studies, the learners were offered a variety of problems for a specific mathematical content with different levels of representation and with context embedded as well as context-free problems. Offering this variety of problems which reflect the competence-oriented view can give deeper insight in the students' competencies while at the same time identifying existing difficulties or misconceptions. The problems were not only administered using a paper-and-pencil test format, but also in interview situations in the sense of Piaget's clinical interview method (Oppen, 1977).

With respect to context-embedded problems, two phenomena can be observed. Firstly, low achievers might solve these problems in daily life but not at school (Carragher, Carragher, & Schliemann, 1985). Secondly, low achievers are often afraid of word problems offered at school which often are too artificial (Scherer, 2009). In Germany, low achievers often struggle with the language and the mathematics (Scherer, 1999). They have, for example, difficulties with the calculations and with understanding the word problems. Also, several studies have shown that especially with regard to mathematics in contexts, low achievers have negative attitudes. Other studies showed that a lot of low achieving students are in fear of failing, and missing self-esteem could be stated (Scherer, 1999, 2009). At the same time, it is clear that all students should be able to solve context problems and as a consequence dealing with context problems at school is of great importance.

Generally, everyday experiences are of great importance for mathematics education for all students (Van den Heuvel-Panhuizen, 2005). On the one hand, context situations can serve as starting points for learning mathematics, for illustrating mathematical ideas. On the other hand, mathematical topics can be used in the field of real-life applications. Although context situations should help to understand mathematics, too often the opposite is true. Due to an inappropriate way of using context problems in mathematics education, students might not see any connection between mathematics at school and in real life, and consequently, they cannot use their experiences from daily life.

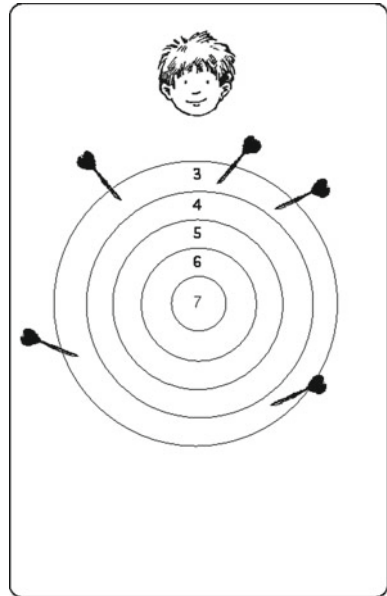
8.4.1.1 Example 1: Solving a Context-Embedded Multiplication Problem

For testing the operations of multiplication and division several items have been designed covering countable and non-countable problems or different ideas for division as quotative and partitive division (for an overview see Scherer, 2003). Figure 8.3 shows an item for solving a context-related multiplication problem illustrated by a dart game. The scored points are visible by means of (thrown) darts and the question was “How many points did the boy score in the dart game?” (Scherer, 2003, p. 13ff.).

The example is taken from a case study with fourth graders of the school for students with learning disabilities (Scherer, 2003). The topic for this grade in special education is multiplication and division with numbers up to 100, whereas this topic is dealt with in second grade in regular school. With these students written tests as well as individual interviews were carried out and the study showed a great heterogeneity within one class (for details see Scherer, 2003).

Vladimir, a 10-year-old boy, classified as a low achiever, was offered a total of 40 items with the paper-and-pencil test (multiplication problems, division problems, context-free and context-embedded problems). Compared to the whole class he was a below average student solving 13 out of 40 tasks correctly. Referring to different items different dependencies became obvious and the interplay of tests and interview could explain some of the phenomena in detail. Especially the interview could reveal his existing competencies.

Fig. 8.3 Context-related multiplication problem



For the context-related multiplication in Fig. 8.3, during the written test Vladimir worked on five similar items with different numbers (the number of arrows and the positions of the arrows varied while the picture of the board with the numbers from 3 to 7 remained the same). For all five items, Vladimir gave the answer 23. Probably, he added all numbers given on the board (calculation error included).

The interview gave more insight in his underlying thinking. Confronted with the item in Fig. 8.3, firstly, Vladimir again added all the numbers on the board ($3 + 4 + 5 + 6 + 7$). He got the result 24, again including a calculation error. The interviewer asked him about the meaning of the arrows. Vladimir then added 5 to his first result and got 29. The interviewer started anew simulating the game: “Imagine that we both play this dart game. One arrow means 3 points.” Vladimir at once calculated the ‘threes’ together, again with a calculation mistake, and finally got the result 16. So, at first sight the context of the dart game did not have any meaning for Vladimir and one might think that this context was not suitable. But reflecting on the context and the interviewer’s stimulation of the real-life connection showed that Vladimir was able to work and argue in the given context correctly.

After working out the new solution, the interviewer reflected on this new result:

I: Why did you do it another way?

Vladimir: Because ... We have played now.

I: Yes. ... And what’s now the correct result? If you want to know how many points the boy scored in this dart game?

Vladimir: Twenty-nine

Obviously, for Vladimir playing a game did mean a specific world, whereas the solution of a mathematical problem took place in another world, probably in a kind of ‘mathematical world’. For mathematics instruction Vladimir’s reduced framing with the first attempt has to be considered: Teachers should try to make those individual framings explicit and encourage students to express further ideas and conceptions.

What could be seen is that especially low achievers have a lot of difficulties. Not only with calculations, but also with understanding a situation, understanding a given text or picture, with finding a correct representation of their ideas etc. At this point, it has to be emphasised that these difficulties are not necessarily to be understood as features of the students themselves, but they can also be consequences of the kind of instruction the students experienced (Scherer, 1999; Van den Heuvel-Panhuizen, 2001). The implicitly critical remarks mentioned above (referring to textbooks) show that the way of teaching and learning has to change. For all students it is absolutely necessary that they are actively involved in the acquisition of mathematics and that they get a genuine understanding of real and modelled situations and of signs and symbols.

8.4.1.2 Example 2: Working Out the Quantity of Dots

The following example is taken from Scherer (1999) from a study performed with third graders in special education. Part of study was the inquiry of the previous knowledge when starting to work with numbers up to 100. Also in this study, written tests as well as individual interviews were carried out and different types of problems were given. Figure 8.4 shows an item for determining the number of dots presented in a structural arrangement. A part of the field of hundred with the structure of five was used with the question whether the children would make use of the given structure. The problem could be solved by counting one-by one and would show if all counting principles are used correctly (see Gelman & Gallistel, 1978). Moreover, the number 48 offers several possibilities for using the structure of fives or tens: the more or less conventional strategy of reading a field horizontally, but also reading it vertically (Scherer, 1999, p. 178 ff.).

The following interview episode illustrates Mary’s (Grade 3) individual strategy:

Mary: [counts the first column of dots tapping with her finger at every dot] One, two, three, four, five ...
[counts with her eyes speaking lowly: ten, twenty, thirty, forty; then one by one] Forty-eight.

The interviewer asked her to explain how she got the result 48.

Mary: I made like this [points at two columns of five], ... those two are ten, then twenty, thirty, forty [points at every two columns of five], and the last I have counted.

For the design of test items in a competence-oriented approach it is also necessary to reflect on concrete numbers and arrangement so that a variety of strategies is possible.

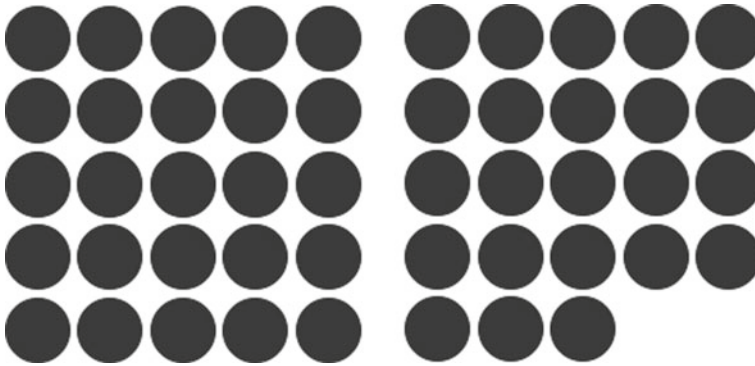


Fig. 8.4 Working out the quantity of dots

8.4.2 *Students' Own Productions: Open Problems*

As mentioned in Sect. 8.3 the competence-oriented view should also be used for designing teaching and learning situations in mathematics. A possible format is that of so-called own productions realised by open problems as illustrated in Sect. 8.3.3, and other examples will be given in the following.

8.4.2.1 **Example 3: Find Problems with Given Numbers!**

The task “Find addition and subtraction problems with the numbers 3, 6, 12, 20!” may help to clarify existing misconceptions, but at the same time it enables the child to show his or her abilities: For example, making use of arithmetic structures and properties or the individual extent of systematic work, etcetera. Moreover, this format covers a ‘natural differentiation’. It allows to include problems at different levels that are neither fixed nor determined by the teacher (Scherer & Krauthausen, 2010). The students are free to choose three or more numbers, although experiences in school showed that many students chose problems with only two numbers.

The worksheet in Fig. 8.5 is from Marc, a third grader who attended a school for learning disabled students and who had been operating with numbers up to 100 in classroom for a few weeks. In total, Marc wrote down 12 problems which he divided into three groups by himself (he did not explain the reason for that). He made subtraction problems as well as addition problems (including all doubles) and many problems with the result 20. No counting or calculating errors occurred. It is remarkable that he wrote down just four subtractions. Moreover, it came to the fore that Marc had difficulties in notating subtractions. In two of them he used the ‘wrong’ notation: $3 - 12 = 9$ and $6 - 20 = 14$. This contrasts with the solution showed in Fig. 8.1, where the student solved this problem by ‘inventing’ negative numbers. An issue that is relevant to note here is that for further diagnosis and figuring out

1) $3-12=9$
 $20-3=17$

2) $20+6=26$
 $6+20=14$
 $20-12=8$
 $20+3=23$
 $20+20=40$
 $12+12=24$
 $6+6=12$
 $3+3=6$

3) $12+20=32$
 $3+6=9$

Fig. 8.5 Open problem with pre-given numbers

the consequences for instruction, the teacher's mathematical background is of major importance (see Scherer, 2007, p. 601ff.).

Open problems can be designed for every mathematical topic. Students can also be encouraged to reflect on the problems they have designed themselves or can be stimulated to produce problems within particular categories such as 'My easy problems' or 'My difficult problems' or 'My special problems' (see Krauthausen & Scherer, 2014, p. 144ff.; Van den Heuvel-Panhuizen 1996, p. 144ff.;). Open problems can also be varied in the cognitive demand they ask; some problems are more straightforward and others require real problem solving (see Klavir & Hershkovitz, 2008; Krauthausen & Scherer, 2014, p. 100ff.).

8.4.2.2 Example 4: Open Problems Completely Posed by the Students Themselves

Open problems can also come completely from the students themselves. In a mini project, in which fifth-graders of a school for learning disabled students had figured out that they could walk 4 km an hour. They themselves came with the problem of how many kilometres they might cover in one full day, that is in 24 hours. Problems such as 24×4 had not yet been treated in their classroom previously (see also Scherer, 2003). Nevertheless, several students found solutions.

Sandrina wrote down the problems from 1×4 up to 24×4 without directly calculating them (Fig. 8.6a). She started with the easiest one and worked out the following results one after another, in which the results did not correspond with the multiplication in the same line. Finally, she stopped with this laborious way of working. Jan wanted to split up 24×4 into $10 \times 4 + 10 \times 4 + 4 \times 4$, which represents an effective way of working out this multiplication. He also started with calculating the results of the multiplication table up to 10 (Fig. 8.6b). Unfortunately,

$5 \times 8 = 40$ and many students write down 400 as the result. With calculations beyond 1000 and thus a higher quantity of zeroes, these uncertainties can increase.

In general, certain relations such as ‘exchange problems’ (e.g., using the commutative property $6 \times 8 = 8 \times 6$) or ‘derived problems’ (e.g., $6 \times 8 = 5 \times 8 + 1 \times 8$) have to be explicitly practised by offering students problems with numbers by which the making use of these relations is explicitly elicited. Or, as shown earlier, open problems with given numbers may help to reach this goal as the students will use the given numbers inevitably in a more or less systematic way and the patterns become obvious.

A more conscious selection of numbers and taking into account their relations should not be underestimated in instruction. For the students, the use of these relations demands for understanding of connections between problems (Ter Heege, 1999), between operations (multiplication and division, but also between multiplication and addition) as well as having insight in relations between different levels of representation (Scherer, 2003). Only in this way can knowledge about the result of 20×4 which is not immediately accessible be effectively reconstructed by using 2×4 .

This, of course, requires that in education much attention is paid to developing understanding of these number relations, and the operative principle is of major importance (see Sect. 8.3.4; Wittmann, 1985; see also Ter Heege, 1999; Van den Heuvel-Panhuizen, 2001, p. 76ff.).

8.4.3 Making Use of Picture Books for Learning Mathematics

The study carried out by my doctoral student Anna Vogtländer on pre-school children’s competencies when engaged in picture books reading (Vogtländer, 2015) was also inspired by RME research. This study is in line with the nowadays generally accepted recognition of the importance of early mathematics education (see Van den Heuvel-Panhuizen & Elia, 2014). In Germany, the research on young children’s mathematical development was provoked in particular by international comparative studies like TIMSS and PISA and several empirical studies, like the SCHOLASTIK study (e.g., Weinert & Helmke, 1997). The SCHOLASTIK study indicates that school beginners with a low level of achievement maintain their relative position until the end of their primary education, that is, low-achieving students hardly seem to catch up on their peers. Therefore, it is important to create and offer stimulating learning environments, to support young children in their individual learning processes.

One way of supporting children’s mathematical competences is making use of picture books. There are interesting research developments concerning the use of such books in relation to young children’s learning of mathematics in the Netherlands (Scherer, Van den Heuvel-Panhuizen, & Van den Boogaard, 2007; Van den Heuvel-Panhuizen & Van den Boogaard, 2008; Van den Heuvel-Panhuizen, Van den Boogaard, & Doig, 2009; Van den Heuvel-Panhuizen, Van den Boogaard, & Scherer,

2007). These studies showed that selected picture books, which are not written to teach mathematics explicitly, have the power to stimulate mathematical thinking. The studies illustrated how picture books can offer a meaningful context for learning mathematics and can give children an informal entry to mathematical ideas early on.

These studies were one of the starting points for Vogtländer's research project in the field of early childhood mathematics education (Vogtländer, 2015). The purpose of this study is to investigate the mathematical thinking and learning processes of young children during reading sessions with picture books. Although the authors of the selected picture books did not have the intention of teaching children mathematics, the picture books gave children opportunities to talk about mathematics anyway. Moreover, reading picture books can be an activity that can motivate children to discover and explore mathematical contents by themselves based on their existing competencies.

8.4.4 Primary Students' Preconceptions of Negative Numbers

The study done by my doctoral student Christian Rütten, who focused on primary school students' pre-conceptions of negative numbers (Rütten, 2016) is another example of RME influence on German research on mathematics education, and in particular the recognition within RME that students can already be familiar with mathematical procedures and possess conceptions of mathematical objects before official instruction at school. In Rütten's study it was revealed that whereas in Germany negative numbers are not dealt with in mathematics until the 5th grade, students in the lower primary grades do already know something about negative numbers and integer operations. In his study, the pre-instructional knowledge of integers of nearly 300 German third- and fourth-graders was assessed through a paper-and-pencil test followed by individual interviews. Characteristic of this assessment was that, contrary to the traditional written tests, students were offered room for informal notation and reasoning (Van den Heuvel-Panhuizen, 1990, 1996), which made it possible to recognise more detailed students' knowledge. Related to the RME approach, motivating and supporting task contexts "offer the opportunity to sound out the students' abilities while avoiding obstructions which are caused by formal notation" (Van den Heuvel-Panhuizen & Gravemeijer, 1991, p. 142). A characteristic of the Dutch type of assessment is the use of open problems that allow the students to answer with individual strategies and at different levels, and to show what they are capable of. By offering meaningful pictures and representations, these tasks are nearly self-explanatory, and do not need more information to handle. Furthermore, in all tasks, students are prompted to reason about their answers or write down notes on scrap paper.

Guided by this general characteristic, Rütten (2016) investigated primary students' pre-conceptual knowledge of negative numbers by enabling them to encounter negative numbers in realistic contexts (thermometer, elevator, games, number line, etc.) without using conventional symbols. By means of open problems, the test allows the

students to show their own ideas and in part idiosyncratic symbolisation relating to these new mathematical objects.

The results of the administered test showed a wide range of primary students' ideas of integers. Furthermore, the test allowed to specify some phenomena earlier described in other studies, e.g., the idea of a divided number line (Peled, Mukhopadhyay, & Resnick, 1989) or the idea that decimals with zero are less than zero (e.g., Stacey, Helme, & Steinle, 2001). In summary, also in this study the Dutch approach for paper-and-pencil tests provides a productive instrument for a qualitative inquiry.

8.5 Conclusions and Perspectives

As pointed out, the Dutch approach to mathematics education had—and still has—a high impact on German research. In the following I will recapitulate them.

8.5.1 *Competence-Oriented Diagnosis and Instruction*

With regard to low achievers the diagnosis plays an important role. Diagnosis should be seen in a competence-related view, which means the diagnosis of difficulties and of existing abilities at the same time. This can be realised through interviews, but also through written tests (Van den Heuvel-Panhuizen, 1990, 1991, 1996). Instead of underestimating low achievers' abilities, it is necessary to give them the opportunity to show what they are capable of. If the problems are open, the level is not fixed at the beginning, and that can boost students' confidence. In this chapter, I gave several examples in which specific difficulties with particular mathematical contents were pointed out and the power of students' own work was illustrated. These examples showed that even low achievers can choose alternative strategies, make use of structures and relations, find patterns and show creative and effective work. However, the orientation on competencies and the opportunity of productive work cannot completely solve all the diverse problems of low achievers in mathematics, and the new teaching approach will probably cause a lot of difficulties in the beginning, as the students have to become familiar with working in an active and responsible manner. Yet, by competence-oriented diagnosis and instruction, misconceptions and difficulties can be identified more easily and at an early time. If the students only have to work on problems as regularly presented in textbooks, some specific mistakes as discussed in this chapter will not become apparent.

8.5.2 Own Productions and Open Problems

Students often approach open problems with enthusiasm because the results are assessed not only as right or wrong, and therefore the fear of failure is reduced (Grossman, 1975). The open approach offers the opportunities for natural differentiation, as the students can work on several levels of difficulties and be successful at their own level (Krauthausen & Scherer, 2014). The danger of under- or overestimating the weaker students, as well as the better ones, can be thoroughly reduced (Scherer, 1999; Wittmann, 1990, p. 159). Learning-disabled students are often underestimated or misjudged, and we often limit their potential that would come to the fore with a more open approach. Supporting students means making certain demands on them and aiming at long-term learning processes and not only thinking of short-term success in learning.

To optimally take advantage of students' capabilities it is indispensable for the teacher to be familiar with the mathematics in order to be able to recognise and evaluate the possibly uncommon discoveries of the students.

8.5.3 Support of Own Strategies

For successfully solving problems, also initially unfamiliar problems, it seems especially essential to encourage learning-disabled students to follow their own methods. At the same time, unfamiliar problems should be explicitly made subject of discussion in class. Only in this way, these students learn that they themselves can solve problems with their own ideas (see also Ter Heege, 1985, p. 380). Especially for solving word problems, or context problems in general, own notations and independently developed strategies play a central role. The knowledge gained in this way can be easier remembered and applied and it also contributes to supporting self-confidence and independency (see also Isenbarger & Baroody, 2001, p. 468).

8.5.4 Role of Mistakes

Over and above there is a need for a change in attitude: difficulties and errors should be regarded as natural concomitants of the learning process and not just short-term solutions should be searched for. Difficulties and errors should be cleared for students—whenever possible—in a meaningful way. Then, students will in the end benefit from the ideas, strategies and attempts that initially were not successful. Teaching should not be only oriented on pure results. It is important that students should “feel no shame or embarrassment when they present erroneous solutions in front of the others” (Cobb, Wood, & Yackel, 1991, p. 165).

At this point, it has to be emphasised that specific difficulties are not necessarily to be understood as features of the students themselves, but that they can also be consequences of the kind of instruction they have experienced. With a small-step instruction conception, which can currently still be encountered in German schools for students with learning disabilities, many mathematical topics are usually introduced and worked on in isolation from each other. For example, students learn the task 6×8 in the 8-row and at another point in time they learn 8×6 in the 6-row. The fact that students do not then use the relation of the commutative law is not a surprise.

8.5.5 *Last but Not Least*

Overall, for students with learning disabilities the quality of teaching is of great importance:

Good teaching that emphasises the structure of a subject is probably even more valuable for the less able student than for the gifted one, for it is the former rather than the latter who is most easily thrown off the track by poor teaching. (Bruner, 1969, p. 9)

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