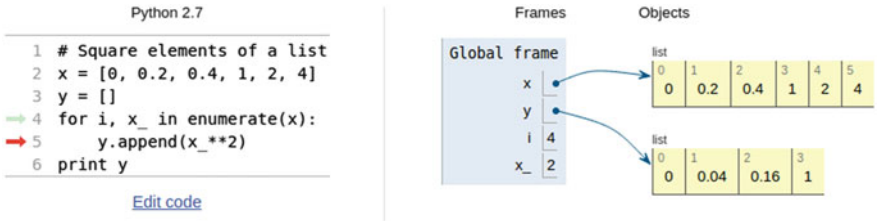
Loops and Branching

3



3.1 The for Loop

Many computations are repetitive by nature and programming languages have certain *loop structures* to deal with this. One such loop structure is the *for loop*.

3.1.1 Example: Printing the 5 Times Table

Assume the task is to print out the 5 times table. Before having learned about loop structures in programming, most of us would first think of coding this like:

```
# Naively printing the 5 times table
print('{:d}*5 = {:d}'.format(1, 1*5))
print('{:d}*5 = {:d}'.format(2, 2*5))
print('{:d}*5 = {:d}'.format(3, 3*5))
print('{:d}*5 = {:d}'.format(4, 4*5))
print('{:d}*5 = {:d}'.format(5, 5*5))
print('{:d}*5 = {:d}'.format(6, 6*5))
print('{:d}*5 = {:d}'.format(7, 7*5))
print('{:d}*5 = {:d}'.format(8, 8*5))
print('{:d}*5 = {:d}'.format(9, 9*5))
print('{:d}*5 = {:d}'.format(10, 10*5))
```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When executed, the 10 results are printed quite nicely as

```
1*5  = 5
2*5  = 10
...
...
```

With a `for` loop, however, the very same printout may be produced by just two (!) lines of code:

```
for i in [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]:     # Note... for, in and colon
    print('{:d}*5 = {:d}'.format(i, i*5))         # Note indent
```

With this construction, the *loop variable* `i` takes on each of the values 1 to 10, and for each value, the print function is called.

Since the numbers 1 to 10 appear in square brackets, they constitute a special structure called a *list*. The loop here would work equally well if the brackets had been dropped, but then the numbers would be a *tuple*:

```
for i in 1, 2, 3, 4, 5, 6, 7, 8, 9, 10:      # no brackets...
    print('{:d}*5 = {:d}'.format(i, i*5))
```

Both lists and tuples have certain properties, which we will come back to in Sect. 5.1.

3.1.2 Characteristics of a Typical for Loop

Loop Structure There are different ways to write `for` loops, but herein, they are typically structured as

```
for loop_variable in some_numbers:   # Loop header
    <code line 1>                        # 1st line in loop body
    <code line 2>                        # 2nd line in loop body
    ...
    ...                                # last line in loop body
# First line after the loop
```

where `loop_variable` runs through the numbers[1] given by `some_numbers`. In the very first line, called the *for loop header*, there are two reserved words, `for` and `in`. They are compulsory, as is the *colon* at the end. Also, the *block* of code lines inside a loop must be *indented*. These indented lines are referred to as the *loop body*. Once the indent is reversed, we are outside (and after) the loop (as commented with `# First line after the loop`, see code). One run-through of the loop body is called an *iteration*, i.e., in our example above with the 5 times table, the loop will do 10 iterations.

Loop Variable The name picked for the `loop_variable` is up to the programmer. In our times table example, the loop variable `i` appeared explicitly in the `print` command within the loop. Generally, however, the loop variable is not required to enter in any of the code lines within the loop, it is just *available* if you need it.

[1] In Python, the loop variable does not have to be a number. For example, a header like `for name in ['John', 'Paul', 'George', 'Ringo']:` is fine, causing the loop variable to be a name. If you place `print(name)` inside the loop and run it, you get each of the names printed. In this book, however, our focus will be loops with numbers.

This means that if we had, e.g., switched the print command inside our loop with `print('Hello!')` (i.e., so that `i` does not appear explicitly within the loop), `i` would still run through the numbers 1 to 10 as before, but `Hello!` would be printed 10 times instead.

The loop variable `i` takes on the values 1 to 10 *in the order listed*, and any order would be acceptable to Python. Thus, if we (for some reason) would like to reverse the order of the printouts, we could simply reverse the list of numbers, writing `[10, 9, 8, 7, 6, 5, 4, 3, 2, 1]` instead.

It should be noted that the loop variable is not restricted to run over integers. Our next example includes looping also over floating point values.

Indentation and Nested Loops In our simple times table example above, the print command inside the loop was indented 4 spaces, which is in accordance with the official style guide of Python.[2]

Strictly speaking, the style guide recommends an indent of *4 spaces per indentation level*. What this means, should become clear if we demonstrate how a `for` loop may appear within another `for` loop, i.e., if we show an arrangement with *nested loops*.

```
for i in [1, 2, 3]:
    # First indentation level (4 spaces)
    print('i = {:d}'.format(i))
    for j in [4.0, 5.0, 6.0]:
        # Second indentation level (4+4 spaces)
        print('      j = {:.1f}'.format(j))
    # First line AFTER loop over j
# First line AFTER loop over i
```

The meaning of indentation levels should be clear from the comments (see code), and it is straight forward to use more nested loops than shown here (see, e.g., Exercise 5.7). Note that, together with the colon, indenting is part of the syntax also for other basic programming constructions in Python (e.g., in `if-elif-else` constructions and functions).

When executing the nested loop construction, we get this printout:

```
i = 1
      j = 4.0
      j = 5.0
      j = 6.0
i = 2
      j = 4.0
      j = 5.0
      j = 6.0
i = 3
      j = 4.0
      j = 5.0
      j = 6.0
```

From the printout, we may infer how the execution proceeds. For *each* value of `i`, the loop over `j` runs through *all* its values before `i` is updated (or the loop is terminated). To test your understanding of nested loops, you are recommended to do Exercise 5.1.

[2] https://www.python.org/dev/peps/pep-0008/.

Other `for` Loop Structures In this book, you will occasionally meet `for` loops with a different structure, and as an example, take a look at this:

```
for i, j, k in (1, 2, 3), (4, 5, 6), (6, 7, 8):
    print(i, j, k)
```

Here, in the first iteration, the loop variables `i`, `j` and `k` will become 1, 2 and 3, respectively. In the second iteration, they will become 4, 5 and 6, respectively, and so on. Thus, the printout reads

```
1 2 3
4 5 6
6 7 8
```

As usual, each of `i`, `j` and `k` can be used in any desirable way within the loop.

3.1.3 Combining for Loop and Array

Often, loops are used in combination with arrays, so we should understand how that works. To reach this understanding, it is beneficial to do an example with just a small array.

Assume the case is to compute the average height of family members in a family of 5. We may choose to store all the heights in an array, which we then run through by use of a `for` loop to compute the average. The code (`average_height.py`) may look like this:

```
import numpy as np

N = 5
h = np.zeros(N)     # heights of family members (in meter)
h[0] = 1.60; h[1] = 1.85; h[2] = 1.75; h[3] = 1.80; h[4] = 0.50

sum = 0
for i in [0, 1, 2, 3, 4]:
    sum = sum + h[i]
average = sum/N

print('Average height: {:g} meter'.format(average))
```

When executed, the code gives 1.5 m as the average height, which compares favorably to a simple hand calculation. What happens here, is that we first sum up[3] all the heights being stored in the array `h`, before we divide by the number of family members `N` (i.e., just like we would do by hand). Observe how `sum` is initialized to 0 before entering the loop, and that with each iteration, a new height is added to `sum`. Note that the loop variable `i` takes on the integer index values of the array, which start with 0 and end with $N - 1$.

[3] Note this way of using a loop to compute a sum, it is a standard technique in programming.

Running Through Code "by Hand"

It is appropriate to stress that much understanding is often developed by first going through code "by hand", i.e. just read the code while doing calculations by hand, before comparing these hand calculations to what the code produces when run (often some print commands must be inserted in the code to enable detailed comparison).

Thus, to make sure you understand the important details in `average_height.py`, you are encouraged to go through that code by hand right now. Also, in a copy, insert a print command in the loop, so that you can compare the output from that program to your own calculations, iteration by iteration.

3.1.4 Using the `range` Function

At this point, the observant reader might argue: "Well, this `for` loop seems handy, but what if the loop must do a *really* large number of iterations? If the loop variable is to run through hundreds of numbers, we must spend all day typing those numbers into the loop header"!

An Example This is where the built-in `range` function enters the picture. When called, the `range` function will provide integers according to the arguments given in the function call. For example, we could have used `range` in `average_height.py` by just changing the header from

```
for i in [0, 1, 2, 3, 4]:       # original code line
```

to

```
for i in range(0, 5, 1):        # new code line
```

Here, `range(0, 5, 1)` is a function call, where the function `range` is told to provide the integers from 0 (inclusive) to 5 (exclusive!) in steps of 1. In this case, `range(0, 5, 1)` will provide exactly those numbers that we had in the original code, i.e., the loop variable `i` will run through the same values (0, 1, 2, 3 and 4) as before, and program computations stay the same.

With a little interactive test, we may confirm that the `range` function provides the promised numbers. However, since what is returned from the `range` function is an object of type `range`, the number sequence is not explicitly available.[4] Converting the range object to a list, however, does the trick.

```
In [1]: x = range(0, 5, 1)

In [2]: type(x)
Out[2]: range

In [3]: x
Out[3]: range(0, 5)
```

[4] In Python 2, the `range` function returned the requested numbers as a list.

```
In [4]: list(x)              # convert to list
Out[4]: [0, 1, 2, 3, 4]
```

A General Call to range With a header like

```
for loop_variable in range(start, stop, step):
```

and a $step > 0$, `loop_variable` will run through the numbers `start, start + 1*step, start + 2*step, ...., start + n*step`, where $start + n*step < stop <= start + (n + 1) * step$. So, the final number is as close as we can get to the specified `stop` without equalling, or passing, it. For a negative step ($step < 0$, example given below), the same thing applies, meaning that the final number can not equal, or be more negative, than the argument `stop`. Note that an integer `step` different from 1 and -1 is perfectly legal.

Different Ways of Calling range The function `range` is most often used in `for` loops to produce a required sequence of integers, but the function is not restricted to `for` loops only, of course. It may be called in different ways, e.g., utilizing default values. Some examples are

```
In [1]: list(range(1, 11, 1))
Out[1]: [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]

In [2]: list(range(1, 11))            # step not given, default 1
Out[2]: [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]

In [3]: list(range(11))               # start not given either, default 0
Out[3]: [0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10]

In [4]: list(range(1, -11, -1))
Out[4]: [1, 0, -1, -2, -3, -4, -5, -6, -7, -8, -9, -10]
```

When calling `range`, there is no argument specifying how many numbers should be produced, like we saw with the `linspace` function of `numpy`. With `range`, this is implied by the function arguments `start`, `stop` and `step`.

Computer Memory Considerations Note that `range` does *not* return all the requested numbers at once. Rather, the call to `range` will cause the numbers to be provided *one by one* during loop execution, giving one number per iteration. This way, simultaneous storing of *all* (loop variable) numbers in computer memory is avoided, which may be very important when there is a large number of loop iterations to make.

3.1.5 Using `break` and `continue`

It is possible to break out of a loop, i.e., to jump directly to the first code line after the loop, by use of a `break` statement. This might be desirable, for example, if a certain condition is met during loop execution, which makes the remaining iterations redundant.

With loops, it may also become relevant to skip any remaining statements of an ongoing iteration, and rather proceed directly with the next iteration. That is, contrary to the "loop stop" caused by a `break` statement, the loop continues to run after a `continue` statement, unless the end of the loop has been reached.

An example of how to use `break` and `continue` can be found in Sect. 5.2 (`times_tables_4.py`).

The `break` and `continue` statements may also be used in `while` loops, to be treated next.

3.2 The while Loop

The other basic loop construction in Python is the *while loop*, which runs as long as a condition is `True`. Let us move directly to an example, and explain what happens there, before we consider the loop more generally.

3.2.1 Example: Finding the Time of Flight

To demonstrate a `while` loop in action, we will make a minor modification of the case handled with `ball_plot.py` in Sect. 1.5. Now, we choose to find the time of flight for the ball.

The Case Assume the ball is thrown with a slightly lower initial velocity, say 4.5 ms^{-1}, while everything else is kept unchanged. Since we still look at the first second of the flight, the heights at the end of the flight will then become negative. However, this only means that the ball has fallen below its initial starting position, i.e., the height where it left the hand, so there is nothing wrong with that. In an array `y`, we will then have a series of heights which towards the end of `y` become negative. As before, we will also have an array `t` with all the times for corresponding heights in `y`.

The Program In a program named `ball_time.py`, we may find the time of flight as the time when heights switch from positive to negative. The program could look like this

```
import numpy as np

v0 = 4.5                      # Initial velocity
g = 9.81                      # Acceleration of gravity
t = np.linspace(0, 1, 1000)  # 1000 points in time interval
y = v0*t - 0.5*g*t**2         # Generate all heights

# Find index where ball approximately has reached y=0
i = 0
while y[i] >= 0:
    i = i + 1

# Since y[i] is the height at time t[i], we do know the
# time as well when we have the index i...
print('Time of flight (in seconds): {:g}'.format(t[i]))

# We plot the path again just for comparison
import matplotlib.pyplot as plt
plt.plot(t, y)
plt.plot(t, 0*t, 'g--')
plt.xlabel('Time (s)')
plt.ylabel('Height (m)')
plt.show()
```

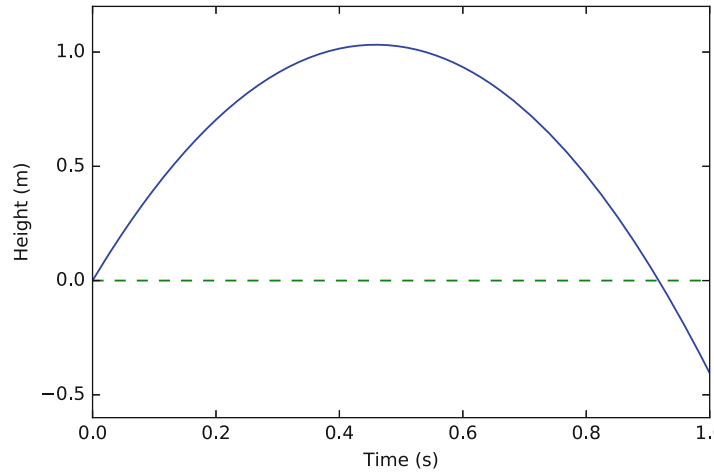

Fig. 3.1 Vertical position of ball

The loop will run as long as the condition `y[i] >= 0` evaluates to `True`. Note that the programmer introduced a variable by the name `i`, initialized it (`i = 0`) before the loop, and updated it (`i = i + 1`) in the loop. So, each time the condition `y[i] >= 0` evaluates to `True`, `i` is *explicitly* increased by 1, allowing a check of successive elements in the array `y`.

When the condition `y[i] >= 0` evaluates to `False`, program execution proceeds with the code lines after the loop. This means that, after skipping the comments, the time of flight is printed, followed by a plotting of the heights (to allow an easy check of the time of flight).

Reporting the Answer Remember that the height is computed at chosen points in time only, so, most likely, we do not have the time for when the height is *exactly* zero. Thus, reporting `t[i]` as the time of flight is an approximation. Another alternative, could be to report `0.5*(t[i-1] + t[i])` as the answer, reasoning that since `y[i]` is negative (which is why the loop terminated), `t[i]` must be too large.

Running the Program If you run this program, you get the printout

```
Time of flight (in seconds): 0.917918
```

and the plot seen in Fig. 3.1. The printed time of flight seems consistent with what we can read off from the plot.

3.2.2 Characteristics of a Typical while Loop

Loop Structure and Interpretation The structure of a typical `while` loop may be put up as

```
while some_condition:   # Loop header
    <code line 1>           # 1st line in loop body
```

```
    <code line 2>           # 2nd line in loop body
    ...
    ...
# This is the first line after the loop
```

The first line here is the *while loop header*. It contains the reserved word `while` and ends with a colon, both are compulsory. The *indented* lines that follow the header (i.e., `<code line 1>`, `<code line 2>`, etc.) constitute a *block* of statements, the *loop body*. Indentation is done as with `for` loops, i.e., 4 spaces by convention. In our example above with the ball, there was only a single line in the loop body (i.e., `i = i + 1`). As with `for` loops, one run-through of the loop body is referred to as an *iteration*. Once the indentation is reversed, the loop body has ended. Here, the first line after the loop is `# This is the first line after the loop`.

Between `while` and the colon, there is `some_condition`, which is a boolean expression that evaluates to either `True` or `False`. The boolean expression may be a compound expression with `and`, `or`, etc.

When a `while` loop is encountered by the Python interpreter, it evaluates `some_condition` the first time. If `True`, one iteration of the *loop body* is carried out. After this first iteration, `some_condition` is evaluated once again (meaning that program execution goes back up to the top of the loop). If `True` again, there is another iteration, and so on, just like we saw above with `ball_time.py`. Once `some_condition` evaluates to `False`, the loop is finished and execution continues with the first line after the loop. Note that if `some_condition` evaluates to `False` the very first time, the statements inside the loop will *not* be executed at all, and execution simply continues immediately with the first line after the loop.

Compared to a `for` loop, the programmer does not have to specify the number of iterations when coding a `while` loop. It simply runs until the boolean expression becomes `False`. Remember that if you want to use a variable analogously to the loop variable of a `for` loop, you have to explicitly update that variable inside the `while` loop (as we did with `i` in `ball_time.py` above). This differs from the automatic update of a loop variable in `for` loops.

Just as in `for` loops, there might be (arbitrarily) many code lines in a `while` loop. Also, nested loops work just like nested `for` loops. Having `for` loops inside `while` loops, and vice versa, is straight forward. Any `for` loop may also be implemented as a `while` loop, but `while` loops are more flexible, so not all of them can be expressed as a `for` loop.

Infinite Loops It is possible to have a `while` loop in which the condition never evaluates to `False`, meaning that program execution can not escape the loop! This is referred to as an *infinite loop*. Sometimes, infinite loops are just what you need, for example, in surveillance camera systems. More often, however, they are unintentional, and when learning to code, it is quite common to unintentionally end up with an infinite loop (just wait and see!). If you accidentally enter an infinite loop and the program just hangs "forever", press `Ctrl+c` to stop the program.

To check that you have gained a basic understanding of the `while` loop construction, you are recommended to do Exercise 3.4.

3.3 Branching (`if`, `elif` and `else`)

Very often in life,[5] and in computer programs, the next action depends on the outcome of a question starting with "if". This gives the possibility of branching into different types of action depending on some criterion.

As an introduction to branching, let us "build up" a little program that evaluates a water temperature provided by the program user.

3.3.1 Example: Judging the Water Temperature

Assume we want to write a program that helps us decide, based on water temperature alone (in degrees Celcius), whether we should go swimming or not.

One if-test As a start, we code our program simply as

```
T = float(input('What is the water temperature? '))
if T > 24:
    print('Great, jump in!')
# First line after if part
```

Even if you have never seen an `if` test before, you are probably able to guess what will happen with this code. Python will first ask the user for the water temperature. Let us assume that 25 is entered, so that `T` becomes 25 through the assignment (note that, since the `input` returns a string, we convert it to a float before assigning to `T`). Next, the condition `T > 24` will evaluate to `True`, which implies that the print command gets executed and "Great, jump in!" appears on the screen.

To the contrary, if 24 (or lower) had been entered, the condition would have evaluated to `False` and the print command would *not* have been executed. Rather, execution would have proceeded directly to the line after the `if` part, i.e., to the line `# First line after if part`, and continued from there. This would mean, however, that our program would give us *no response* if we entered a water temperature of 24, or lower.

Two if-tests Immediately, we realize that this is not satisfactory, so (as a "first fix") we extend our code with a second `if` test, as

```
T = float(input('What is the water temperature? '))
if T > 24:                                  # testing condition 1
    print('Great, jump in!')
if T <= 24:                                 # testing condition 2
    print('Do not swim. Too cold!')
# First line after if-if construction
```

This will work, at least in the way that we get a planned printout ("Do not swim. Too cold!") also when the temperature is 24, or lower. However, something is not quite right here. If `T` is 24 (or lower), the first condition will evaluate to `False`, and

[5] Some readers may perhaps be puzzled by this sentence, bringing in such a huge thing as life itself. The truth is, that this sentence *is* as deep as it appears. My dear co-author Hans Petter Langtangen wrote this sentence well into his cancer treatment. Hans Petter passed away on October 10th, 2016, a few months after the 1st edition of this book was published.

Python will proceed immediately by *also* testing the second condition. However, this test is superfluous, since we *know beforehand* that it will evaluate to `True`! So, in this particular case, using two separate `if` tests is not suitable (generally, however, separate `if` tests may be just what you need. It all depends on the problem at hand).

An if-else Construction For our case, it is much better to use an `else` part, like this

```
T = float(input('What is the water temperature? '))
if T > 24:                              # testing condition 1
    print('Great, jump in!')
else:
    print('Do not swim. Too cold!')
# First line after if-else construction
```

When the first condition evaluates to `False` in this code, execution proceeds directly with the print command in the `else` part, with no extra testing!

To students with little programming experience, this may seem like a very small thing to shout about. However, in addition to avoiding an unnecessary test with the `if-else` alternative, it also corresponds better to the actual logic: If the first condition is false, then the other condition has to be true, and vice versa. No further checking is needed.

An if-elif-else Construction Considering our "advisor program", we have to admit it is a bit crude, having only two categories. If the temperature is larger than 24 degrees, we are advised to swim, otherwise not. Some refinement seems to be the thing.

Let us say we allow some intermediate case, in which our program is less categoric for temperatures between 20 and 24 degrees, for example. There is a nice `elif` (short for `else if`) construction which then applies. Introducing that in our program (and saving it as `swim_advisor.py`), it reads

```
T = float(input('What is the water temperature? '))
if T > 24:                                  # testing condition 1
    print('Great, jump in!')
elif 20 <= T <= 24:                         # testing condition 2
    print('Not bad. Put your toe in first!')
else:
    print('Do not swim. Too cold!')
# First line after if-elif-else construction
```

You probably realize what will happen now. For temperatures above 24 and below 20, our "advisor" will respond just like in the previous version (i.e., the `if-else` version). However, for intermediate temperatures, the first condition will evaluate to `False`, which implies that the Python interpreter will continue with the `elif` line. Here, condition 2 will evaluate to `True`, which means that "Not bad. Put your toe in first!" will be printed. The `else` part is then skipped. As you might expect, more refinement would be straight forward to include by use of more `elif` parts.

Programming as a Step-Wise Process The reader should note that, besides demonstrating branching, the development of the previous program gave a (very) simple example of how code may be written by a step-wise approach. Starting

out with the simplest version of the code, complexity is added step by step, before arriving at the final version. At each step, you make sure the code works as planned. Such a procedure is generally a good idea, and we will address it explicitly again, when we program a (slightly) more comprehensive case in Sect. 4.2.

3.3.2 The Characteristics of Branching

A more general form of an `if-elif-else` construction reads

```
if condition_1:           # testing condition 1
    <code line 1>
    <code line 2>
    ...
elif condition_2:         # testing condition 2
    <code line 1>
    <code line 2>
    ...
elif condition_3:         # testing condition 3
    <code line 1>
    <code line 2>
    ...
else:
    <code line 1>
    <code line 2>
    ...
# First line after if-elif-else construction
```

Here we see an `if` part, two `elif` parts and an `else` part. Note the compulsory *colon* and *indented* code lines (a *block* of statements) in each case. As with loops, indents are conventionally 4 spaces. In such an arrangement, there may be "any" number of `elif` parts (also none) and the `else` part may, or may not, be present.

When interpreting an arrangement like this, Python starts checking the conditions, one after the other, from the top. If a condition (here, either `condition_1`, `condition_2` or `condition_3`) evaluates to `True`, the corresponding code lines are executed, before proceeding *directly* to the first line after the whole arrangement (here, to the line `# First line after if-elif-else construction`). This means that any remaining tests, and the `else` part, are simply skipped! If none of the conditions evaluate to `True`, the `else` part (when present) is executed.

3.3.3 Example: Finding the Maximum Height

We have previously modified `ball_plot.py` from Sect. 1.5 to find the time of flight instead (see `ball_time.py`). Let us now change `ball_plot.py` in a slightly different way, so that the new program instead finds the maximum height achieved by the ball.

The solution illustrates a simple, and very common, search procedure, looping through an array by use of a `for` loop to find the maximum value. Our program `ball_max_height.py` reads

```
import numpy as np
import matplotlib.pyplot as plt

v0 = 5                        # Initial velocity
g = 9.81                      # Acceleration of gravity
t = np.linspace(0, 1, 1000)   # 1000 points in time interval
y = v0*t - 0.5*g*t**2         # Generate all heights

# At this point, the array y with all the heights is ready,
# and we need to find the largest value within y.

largest_height = y[0]         # Starting value for search
for i in range(1, len(y), 1):
    if y[i] > largest_height:
        largest_height = y[i]

print('The largest height achieved was {:g} m'.format(largest_height))

# We might also like to plot the path again just to compare
plt.plot(t,y)
plt.xlabel('Time (s)')
plt.ylabel('Height (m)')
plt.show()
```

We focus our attention on the new thing here, the search performed by the `for` loop. The value in `y[0]` is used as a starting value for `largest_height`. The very first check then, tests whether `y[1]` is larger than this height. If so, `y[1]` is stored as the largest height. The `for` loop then updates `i` to 2, and continues to check `y[2]`, and so on. Each time we find a larger number, we store it. When finished, `largest_height` will contain the largest number from the array `y`.

When you run the program, you get

```
The largest height achieved was 1.27421 m
```

which compares favorably to the plot that pops up (see Fig. 1.1).

The observant reader has already seen the similarity of finding the maximum height and finding the time of flight, as we addressed previously in Sect. 3.2.1. In fact, we could alternatively have solved the maximum height problem here by utilizing that `y[i+1] > y[i]` as the ball moves towards the top. Doing this, our search loop could have been written

```
i = 0
while y[i+1] > y[i]:
    i = i + 1
```

When the condition `y[i+1] > y[i]` becomes `False`, we could report `y[i+1]` as our approximation of the maximum height, for example.

Getting indices right

To implement the traversing of arrays with loops and indices, is often challenging to get right. You need to understand the start, stop and step length values for the loop variable, and also how the loop variable (possibly) enters expressions inside the loop. At the same time, however, it is something that programmers do often, so it is important to develop the right skills on these matters.

You are encouraged to test your understanding of the search procedure in `ball_max_height.py` by doing Exercise 3.9. That exercise will ask you to compare what you get "by hand" to printouts from the code. It is of *fundamental importance* to get this procedure as an established habit of yours, so do the exercise right now!

3.3.4 Example: Random Walk in Two Dimensions

We will now turn to an example which represents the core of so-called *random walk* algorithms. These are used in many branches of science and engineering, including such different fields as materials manufacturing and brain research.

The procedure we will consider, is to walk a series of equally sized steps, and for each of those steps, there should be the same probability of going to the north (N), east (E), south (S), or west (W). No other directions are legal. How can we implement such an action in a computer program?

To prepare our minds for the coding, it might be useful to first reflect upon how this could be done for real. One way, is to use a deck of cards, letting the four suits correspond to the four directions: clubs to N, diamonds to E, hearts to S, and spades to W, for instance. We draw a card, perform the corresponding move, and repeat the process a large number of times. The resulting path mimics, e.g., a typical path followed by a diffusing molecule.

In a computer program, we can not draw cards, but we can draw random numbers. So, we may use a loop to repeatedly draw a random number, and depending on the number, we update the coordinates of our location. There are many ways to draw random numbers and "translate" them into our four directions, and the technical details will typically depend on the programming language. However, our technique here is universal: we draw a random number from the interval $[0, 1)$ and let $[0, 0.25)$ correspond to N, $[0.25, 0.5)$ to E, $[0.5, 0.75)$ to S, and $[0.75, 1)$ to W. We decide to simulate 1000 steps, each of length 1 (e.g., meter), starting from Origo in our coordinate system. To enable plotting our path, we use two arrays for storing the coordinate history, one for the x-coordinates and one for the corresponding y-coordinates.

The suggested code `random_walk_2D.py` then reads

```
import random
import numpy as np
import matplotlib.pyplot as plt

N = 1000                    # number of steps
```

```
d = 1                       # step length (e.g., in meter)
x = np.zeros(N+1)           # x coordinates
y = np.zeros(N+1)           # y coordinates
x[0] = 0;  y[0] = 0         # set initial position

for i in range(0, N, 1):
    r = random.random()          # random number in [0,1)
    if 0 <= r < 0.25:            # move north
        y[i+1] = y[i] + d
        x[i+1] = x[i]
    elif 0.25 <= r < 0.5:        # move east
        x[i+1] = x[i] + d
        y[i+1] = y[i]
    elif 0.5 <= r < 0.75:        # move south
        y[i+1] = y[i] - d
        x[i+1] = x[i]
    else:                        # move west
        x[i+1] = x[i] - d
        y[i+1] = y[i]

# plot path (mark start and stop with blue o and *, respectively)
plt.plot(x, y, 'r--', x[0], y[0], 'bo', x[-1], y[-1], 'b*')
plt.xlabel('x');  plt.ylabel('y')
plt.show()
```

Here, the initial position is explicitly set, even if `x[0]` and `y[0]` are known to be zero already. We do this, since the initial position is important, and by setting it explicitly, it is clearly not accidental what the starting position is. Note that if a step is taken in the x-direction, the y-coordinate is unchanged, and vice versa.

Executing the program produces the plot seen in Fig. 3.2, where the initial and final positions are marked in blue with a circle and a star, respectively. Remember that pseudo-random numbers are involved here, meaning that two consecutive runs will generally produce totally different paths.

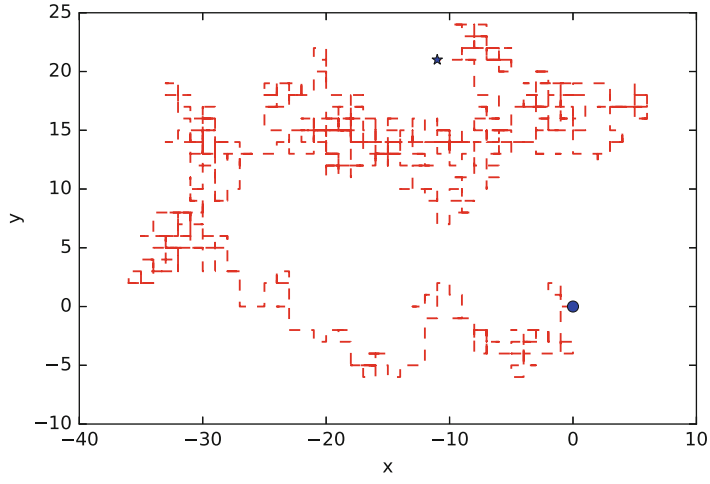

Fig. 3.2 One realization of a random walk (N-E-S-W) with a 1000 steps. Initial and final positions are marked in blue with a circle and a star, respectively

3.4 Exercises

Exercise 3.1: A `for` Loop with Errors
Assume some program has been written for the task of adding all integers $i = 1, 2, \ldots, 10$ and printing the final result:

```
for i in [1, 2, 3, 4, 5, 6, 7, 8, 9, 10)
sum = Sum + x
print 'sum: ', sum
```

a) Identify the errors in the program by just reading the code.
b) Write a new version of the program with errors corrected. Run this program and confirm that it gives the correct output.

Filename: `for_loop_errors.py`.

Exercise 3.2: The `range` Function
Write a slightly different version of the program in Exercise 3.1. Now, the `range` function should be used in the `for` loop header, and only the even numbers from [2, 10] should be added. Also, the (only) statement within the loop should read `sum = sum + i`.
Filename: `range_function.py`.

Exercise 3.3: A `while` Loop with Errors
Assume some program has been written for the task of adding all integers $i = 1, 2, \ldots, 10$:

```
some_number = 0
i = 1
while i < 11
    some_number += 1
print some_number
```

a) Identify the errors in the program by just reading the code.
b) Write a new version of the program with errors corrected. Run this program and confirm that it gives the correct output.

Filename: `while_loop_errors.py`.

Exercise 3.4: `while` Loop Instead of `for` Loop
Rewrite `average_height.py` from Sect. 3.1.3, using a `while` loop instead.
Filename: `while_instead_of_for.py`.

Exercise 3.5: Compare Integers a and b
Explain briefly, in your own words, what the following program does.

```
a = int(input('Give an integer a: '))
b = int(input('Give an integer b: '))
```

```
if a < b:
    print('\na is the smallest of the two numbers')
elif a == b:
    print('\na and b are equal')
else:
    print('\na is the largest of the two numbers')
```

Proceed by writing the program, and then run it a few times with different values for `a` and `b` to confirm that it works as intended. In particular, choose combinations for `a` and `b` so that all three branches of the `if` construction get tested.
Filename: `compare_a_and_b.py`.

Remarks The program is not too robust, since it assumes the user types an integer as input (a real number gives trouble).

Exercise 3.6: Area of Rectangle Versus Circle
Consider one circle and one rectangle. The circle has a radius $r = 10.6$. The rectangle has sides a and b, but only a is known from the outset. Let $a = 1.3$ and write a program that uses a `while` loop to find the largest possible integer b that gives a rectangle area smaller than, but as close as possible to, the area of the circle. Run the program and confirm that it gives the right answer (which is $b = 271$).
Filename: `area_rectangle_vs_circle.py`.

Exercise 3.7: Frequency of Random Numbers
Write a program that takes a positive integer N as input and then draws N random integers from the interval $[1, 6]$. In the program, count how many of the numbers, M, that equal 6 and print out the fraction M/N. Also, print all the random numbers to the screen so that you can check for yourself that the counting is correct. Run the program with a small value for `N` (e.g., `N = 10`) to confirm that it works as intended.

Hint Use `random.randint(1,6)` to draw a random integer between 1 and 6.

Filename: `count_random_numbers.py`.

Remarks For large N, this program computes the probability M/N of getting six eyes when throwing a dice.

Exercise 3.8: Game 21
Consider some game where each participant draws a series of random integers evenly distributed between 0 and 10, with the aim of getting the sum as close as possible to 21, but *not larger* than 21. You are out of the game if the sum passes 21.

After each draw, you are told the number and your total sum, and are asked whether you want another draw or not. The one coming closest to 21 is the winner.

Implement this game in a program.

Hint Use `random.randint(0,10)` to draw random integers in [0, 10].
Filename: `game_21.py`.

Exercise 3.9: Simple Search: Verification
Check your understanding of the search procedure in `ball_max_height.py` from Sect. 3.3.3 by comparing what you get "by hand" to printouts from the code. Work on a copy of `ball_max_height.py`. Comment out what you do not need, and use an array `y` of just 4 elements (or so). Fill that array with integers, so that you place a maximum value in a certain location. Then, run through that code *by hand* for every iteration of the loop, writing down the numbers in `largest_height`. Finally, place a print command in the loop, so that `largest_height` gets printed with every iteration. Run the program and compare to what you found by hand.
Filename: `simple_search_verify.py`.

Exercise 3.10: Sort Array with Numbers
Write a script that uses the `uniform` function from the `random` module to generate an array of 6 random numbers between 0 and 10.

The program should then sort the array so that numbers appear in increasing order. Let the program make a formatted print of the array to screen both before and after sorting. Confirm that the array has been sorted correctly.
Filename: `sort_numbers.py`.

Exercise 3.11: Compute π
Up through history, great minds have developed different computational schemes for the number π. We will here consider two such schemes, one by Leibniz (1646–1716), and one by Euler (1707–1783).

The scheme by Leibniz may be written

$$\pi = 8 \sum_{k=0}^{\infty} \frac{1}{(4k+1)(4k+3)},$$

while one form of the Euler scheme may appear as

$$\pi = \sqrt{6 \sum_{k=1}^{\infty} \frac{1}{k^2}}.$$

If only the first N terms of each sum are used as an approximation to π, each modified scheme will have computed π with some error.

Write a program that takes N as input from the user, and plots the error development with both schemes as the number of iterations approaches N. Your program should also print out the final error achieved with both schemes, i.e. when the number of terms is `N`. Run the program with $N = 100$ and explain briefly what the graphs show.
Filename: `compute_pi.py`.