Chapter 8
Quantitative Reasoning and Its Rôle in Interdisciplinarity

Robert Mayes

Abstract The Real Science, Technology, Engineering Mathematics (STEM) Project was conducted in middle schools and high schools in Georgia, USA. The project supported the development of interdisciplinary STEM modules and courses in over 20 schools. A project focus was development of five 21st century STEM reasoning abilities. In this chapter, I provide classroom activities from the Real STEM project that exemplify each form of reasoning: complex systems; model-based; computational; engineering design-based; and quantitative reasoning. Quantitative reasoning plays a critical rôle in authentic real-world interdisciplinary STEM problems, providing the tools to construct data informed arguments specific to the problem context, which can be debated, verified or refuted, modelled mathematically and tested against reality. Yet quantitative reasoning is often misrepresented, underdeveloped, and ignored in STEM classrooms. The chapter finishes with a discussion of the impact of Real STEM.

Keywords Quantitative reasoning · STEM reasoning · Authentic teaching · Learning progression

8.1 Introduction

Interdisciplinary Science, Technology, Engineering, and Mathematics (STEM) teaching and learning is a national obsession in the United States. There are calls to have STEM integrated into all schools from elementary level through to university. Why? First, there is the economic driver of increasing the number of students pursuing STEM areas to address growing STEM workforce needs. Second, there is the desire for STEM literate citizens, who can make informed decisions about grand challenges facing the next generation, challenges such as global climate change, clean water and the future of energy. Third, there is the proposed positive benefit of

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increasing student engagement, and persistence, in STEM areas through authentic teaching approaches, that provide hands-on, collaborative opportunities for students. The National Science Teachers Association, and the National Council of Teachers of Mathematics, promote standards that support STEM, including the Next Generation Science Standards (NGSS Lead States, 2013) and the Common Core State Standards for Mathematics (National Governors Association, 2010).

Mathematics is fundamental to interdisciplinary STEM, providing the processes to quantify a science problem, analyse engineering designs, and model large data sets. Applying mathematics to real-world interdisciplinary STEM problems requires more than knowledge of isolated mathematical algorithms. It requires the student to quantify the STEM context, and to select the appropriate mathematical tool for a given problem. The ability to apply mathematics within a real-world context is at the core of quantitative reasoning. Unfortunately, many students observed in our Real STEM Project, did not have good quantitative reasoning skills, not even those who were skilled at manipulation and calculation.

From Spring 2013 through to Spring 2017 the Real STEM Project supported over 39 teachers in 20 partner schools in creating and offering interdisciplinary STEM research, and design, experiences for students from age 12 to 18. The project advocated that quantitative reasoning is essential in integrating interdisciplinary STEM into school curricula. The Real STEM project went further by identifying five 21st century STEM reasoning modalities, that are of high demand in the work force, and support being a STEM literate citizen. These five STEM reasoning modalities are: complex systems reasoning, scientific model-based reasoning, technologic computational reasoning, engineering design-based reasoning, and mathematical quantitative reasoning. In this chapter, I begin by discussing what interdisciplinary STEM teaching and learning means, present these five STEM reasoning modalities and provide authentic problem-solving situations in which these reasoning modalities were explored by students in partner schools. This chapter structure enables me to illustrate problem-based learning and place-based education in authentic settings as experienced by Real STEM project teachers and students. Teacher (n = 39) and student responses (n = 898) to increased engagement through interdisciplinary STEM problems was very positive and the chapter concludes with a discussion of how to take such work forwards.

8.2 Interdisciplinary STEM: Authentic Teaching and Reasoning Modalities

STEM is the collective study of science, technology, engineering, and mathematics, with the goal of equipping students with the knowledge and skills to solve tough problems, gather and evaluate evidence, and make sense of information (U.S. Department of Education, 2015). STEM is, first and foremost, interdisciplinary. The term STEM is not needed if you have a great science programme, just call it a great science programme. STEM occurs when two, or more, of the areas of science, technology,

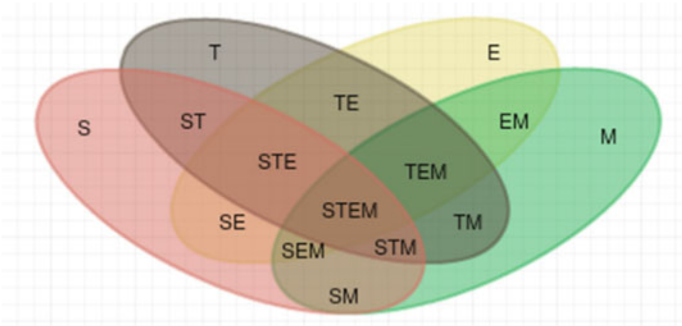

Fig. 8.1 STEM occurs in the intersections of the Venn diagram, not the single set spaces

engineering, and mathematics are brought to bear on a problem (Fig. 8.1). STEM engages students in authentic learning, by engaging them in real-world problems that are student centred and, when possible, tied to the student's context.

Real STEM project was based in what is known as Problem-Based Learning (PBL), a learner-centred approach that empowers learners to conduct research, integrate theory and practice, and apply knowledge and skills to develop a viable solution to a defined problem (Savery, 2006). Long-term retention, skill development, and student and teacher satisfaction, have been found to be benefits of problem-based learning when compared with traditional forms of instruction (Strobel & van Barneveld, 2009). Statistically significant gains in achievement, have been observed for middle school science students, experiencing science in a problem-based learning format (Williams, Pedersen, & Liu, 1998). Place-Based Education (PBE) uses the environment as a context for learning, and allows student input on the selection of the problem to be researched. Studies have found that PBE resulted in students who scored higher on standardised tests in reading, writing, mathematics, science, and social studies (Lieberman & Hoody, 1998; Bartosh, 2003; NEETF, 2000). Other results, from these PBE studies, indicated that students improve overall Grade Point Average (GPA), stay in school longer, and receive higher than average scholarship awards. Authentic learning has a number of qualities, including use of: real-world relevance, ill-defined problems, sustained investigation, collaboration, interdisciplinary perspective, and time for reflection.

The more student centred and problem-driven the STEM task is, the harder it is to target specific science, or mathematics, concepts. So, why should science and mathematics teachers implement authentic STEM tasks in their classrooms? Potential affective outcomes of authentic learning are increased student engagement and persistence, but what are the learning outcomes? Important learning outcomes include critical thinking, problem solving, and the ability to reason. Literally the student should gain a better understanding of how a scientist, computer scientist, engineer, and mathematician, solve a problem. Five STEM reasoning modalities are of high demand in the work force: complex systems reasoning, scientific model-based reasoning, technologic computational reasoning, engineering design-based reasoning,

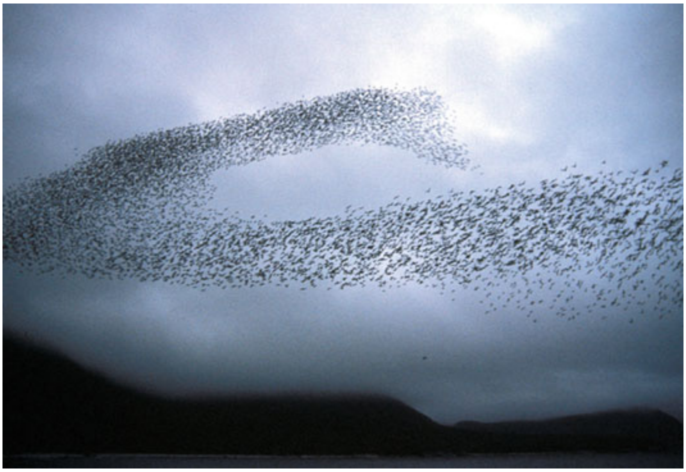

Fig. 8.2 Starlings flocking in Rome https://youtu.be/V-mCuFYfJdI

and mathematical quantitative reasoning. An exemplar for each reasoning modality is provided below, together with a focus on the rôle that quantitative reasoning plays in each.

8.2.1 Complex Systems Reasoning

Birds flocking is an exemplar of a complex system. Starlings flocking in Rome (Fig. 8.2) organise themselves based on simple interactions between birds (the agents), the result of which, are beautiful emerging patterns from random bird interactions. Flocking is an adaptation that, among other things, confuses predators such as the Peregrine Falcon.

Complex systems reasoning is the ability to analyse problems, like flocking behaviour, by recognising complexity, patterns, and interrelationships within a system featuring a large number of interacting components (agents, processes, etc.) whose aggregate activity is non-linear (not determined from the summations of the activity of individual components) and typically exhibits hierarchical self-organisation under selective pressures (Holland, 1992). Complex systems are characterised by a number of elements including: agent-based reasoning, where individual system elements and their interaction are considered; complexity, with a multi-scale hierarchical organisation of smaller systems within larger ones; emergence of patterns, from random interaction of agents; and self-organisation, to adapt to the environment. If the teacher's goal is real-world problem-driven experiences for students, then it is likely the problem lies within a complex system, such as a biological ecosystem.

Simulations of flocking allow students to explore and discover the three simple rules of flocking: Cohesion (steering to move toward the average position of local flockmates); Alignment (steering toward the average heading of local flockmates); and Separation (steering to avoid crowding local flockmates). A flocking simulation is provided in NetLogo, an open software programmable modelling environment, for simulating natural and social phenomena (Wilensky & Resnick, 1999). You can download NetLogo for free and it comes with an extensive library of simulations and lessons: https://ccl.northwestern.edu/netlogo/download.shtml.

The flocking simulation can be found in NetLogo by selecting File—Models Library—Biology—Flocking—Open. The simulation allows the student to vary population, vision, and the three rules of flocking. Running the simulation shows students the emerging behaviour of flocking, without the benefit of a lead bird. The concept of a system organising, without a leader, is surprising to students, and represents a central concept of complex system reasoning: self-organisation due to agent interaction.

So, where is the interdisciplinary STEM in the flocking example? Where is the mathematics? Science is evident, with biology, and environmental science, serving as the driver for the problem, while physics can be invoked through a study of flight. Science is one of the most common drivers in school interdisciplinary STEM tasks. If students are left to their own devices in exploring the simulation, the result is often a qualitative science account of flocking. But there is much more to the problem, if students are directed to view the problem through multiple STEM lenses, and to provide data-based arguments supporting their analysis. Technology is present in use of the simulation, the programming behind the simulation, and in engaging students in extending the model through programming. Engineering design, can be incorporated by having students engineer flying machines and compare them to the birds natural design. Mathematics underlies the development of the simulation. Quantitative reasoning, about the rules governing the interaction of the birds, provides for rich mathematical discussion.

Questions to ask the students to explore include:

- What is the most efficient minimum separation for flocking? This evokes distance, measurement, and inequality (distance to nearest bird < minimum separation).
- What is the effect of changing the maximum separation turn degrees? This evokes geometry.
- How do I calculate alignment of birds? This evokes trigonometry, specifically use of the arctangent.
- How do I calculate coherence of birds? This evokes use of sine and cosine.

8.2.2 Model-Based Reasoning

This occurs when students construct scientific models in order to explain observed phenomena (MUSE, 2015). Common problems in teaching the scientific method are that it becomes an algorithm that is followed linearly, and the result of the research is not related back to the real-world context. For example, a study of a local pond includes collection of dissolved oxygen data. The students collect the data, create a plot, and perhaps even determine trends, but they fail to discuss what the level of dissolved oxygen means for the pond ecosystem. The question of what type of fish could live in the pond, if dissolved oxygen is varying between 2 and 6 mg/litre is never addressed. Model-based reasoning is characterised by: development of a scientific model; revision of the model; determination of the acceptability of the model, in explaining all observations; predicting behaviour of the system; being consistent with other science; empirical assessment (the model explains the data and predicts future results); and conceptual assessment (the model fits with other accepted models).

Model-based reasoning can be elicited by having students engage in the seven step, model-based, reasoning process (Schwarz et al., 2009).

(1) Students observe an anchoring phenomenon of which they do not have a complete understanding.
(2) Students construct a model that expresses an idea or hypothesis about what is happening. This conceptual model includes a picture of the phenomenon with components identified, connections between components represented, and variables quantified when possible, which engages the student in quantitative reasoning.
(3) The students empirically test the model by determining if it reflects the reality or the phenomenon they observed.
(4) They evaluate the model against any data they have collected.
(5) The students test their model against other scientific ideas, laws, or theories, to see if it is consistent with known science.
(6) Once the original model is complete, the students are asked to revise the model to fit new evidence and known laws of science.
(7) Finally, they apply the model to make predictions and explain the phenomenon.

The phenomenon of electrolysis of water, provides a good exemplar for model-based reasoning. Set up the experiment as in Fig. 8.3, without discussing the concept prior to student observation of the phenomenon. Have students go through the seven step model-based reasoning process (Schwarz et al., 2009) described above. Pure water is an insulator, and the electrolysis may proceed too slowly, so add salt to speed up the reaction. Using NaCl as an electrolyte results in some impurity in the form of chlorine gas at the anode, but that should not be important for the purpose of this demonstration. Bubbles will form at the tips of pencils immediately. Oxygen gas bubbles (O_2) will form at + electrode (anode). Hydrogen gas bubbles H_2 will form at − electrode (cathode). The amount of H_2 will be twice the amount of O_2.

In this experiment science is again the driver, with chemistry and physics being evident. Technology, as computational reasoning, does not play a rôle. Engineering design, could be integrated into the experiment, by asking students to design an apparatus that allows them to collect and measure the gases being generated. Expect that student's initial models will be qualitative accounts, which include the

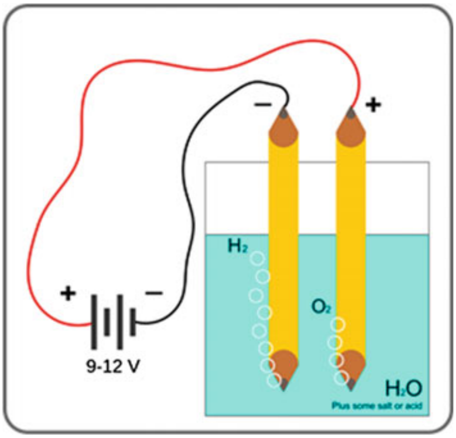

Fig. 8.3 Electrolysis of water experiment

key components of the experiment, provide an explanation of what is generating the bubbles, and may label the gases being generated. Most likely, it will not include a quantitative account, unless specifically prompted. Quantitative analysis of the electrolysis of water phenomenon includes rate of gas production, amount of gas produced, and balancing chemical equations that represent the reaction. The authentic, real-world, aspect of the experiment is the production of hydrogen as a renewable energy resource. This elicits questions of the feasibility of large scale production, and the cost of transforming electric energy into hydrogen, as a fuel source. Both of these provide further opportunities for integrating quantitative reasoning into the task.

8.2.3 Computational Reasoning

Computational reasoning is an analytical approach grounded in the computer sciences, that includes a range of concepts, applications, tools, and skills, that allow us to solve problems strategically, design systems, and understand human behaviour, by following a precise process, that engages computers to assist in automating a wide range of intellectual processes (Wing, 2006). Computational reasoning and large database analysis, are considered new paradigms of science, expanding beyond the traditional experimental and theoretical science paradigms. Computational reasoning is often viewed by teachers as programming, but, there is more to how computer scientists reason, than programming. Elements of computational reasoning include: abstraction by stripping a problem down to its bare essentials, then transferring the problem solving process to a wide variety of problems; algorithm design; automation of repetitive tasks to perform them quickly and efficiently; decomposition of a problem into steps that are implementable by machines; parallel processing; programming, simulation, and modelling; visualisation of large data sets; and capture and curation of data.

Computer programming is the most common activity implemented in classrooms to engage students in computational reasoning. Object oriented programming languages, such as Scratch (https://scratch.mit.edu/), make programming more approachable for students even at the 9–11 years of age level. However, teaching computational reasoning does not require expensive technology, or programming.

Computer scientists have their own ways of solving real-world problems. As much as scientists use model-based reasoning, and engineers use design-based reasoning, computer scientists employ algorithmic reasoning. An algorithm is a set of instructions designed to perform a specific task. For example, computer scientists are often asked to sort a list given some criteria, for example putting names into alphabetic order. Sorting allows the user to find a name more efficiently. There are many methods for sorting a list, some more efficient than others. The following sorting task engages students in creating their own sorting algorithm, and testing it against others, to see which sorts most efficiently. Provide the students with 8 film canisters of the same size but with different weights (e.g. filled with different amounts of sand) and a simple balance scale. Determine the best method of sorting the 8 containers provided so they are in order from lightest to heaviest. Make a comparison of efficiency with another group by swapping sorting algorithms and seeing who can sort the containers in the fewest moves. Clarity of the algorithm will be essential for others to complete the sort, so students need to determine if the algorithm was detailed, and clear enough, to follow easily.

This is a great computational reasoning task, but is it interdisciplinary STEM? The task does not include science, but engineering design could be incorporated if students are asked to engineer a measuring device for sorting the containers. Technology is emphasised by requiring algorithmic reasoning, and exploring different sorting algorithms. There are a number of sorting algorithms students might discover, such as the selection sort (lightest weight is found and removed, then repeat with remaining canisters) or the insertion sort (with each sort the new canister is placed in the appropriate position in the previously sorted canisters). Other sorts include the bubble sort, quick sort, and the mergesort. But where is the mathematics in the sorting task? The quantitative reasoning arises in the efficiency count. Ask students to provide a quantitative account of the efficiency of their algorithm. This engages the students in the discrete mathematics area of combinatorics (counting without counting). For example, for the insertion sort the students can begin by calculating the efficiency for an increasing number of containers. For 3 containers it is a maximum of 2 measures, for 4 containers 5 measures, for 5 containers 9 measures. A pattern emerges, where for n containers ($n > 2$) the number of measures is the sum of the integers from 2 to $n - 1$. Adjusting the formula for the sum of the first n integers, to account for starting at 2 (subtract 1) and ending at $n - 1$, the number of measures for n canisters would be given by the formula:

$$\frac{(n-1)n}{2} \tag{1}$$

In addition, a comparison of efficiency for different sorting algorithms provides the opportunity for integration of algebraic inequalities.

8.2.4 Engineering Design-Based Reasoning

Engineering design-based reasoning is the ability to engage in the engineering design process, thereby using a series of process steps to come up with a solution to a problem (PLTW, 2017). Many times the solution involves designing a product that meets certain criteria and, or, accomplishes a certain task. There are a variety of engineering design process models, but most have variations of the following steps:

(1) Define the problem including determining criteria the design must meet and the constraints on the design;
(2) Research the problem;
(3) Brainstorm solutions;
(4) Choose the best design;
(5) Build a prototype;
(6) Test the prototype; and
(7) Redesign.

As with the scientific method, it is important to stress that the design process is not a linear process, but a circular one which often requires jumping back to previous steps in the process. Engineering design tasks and scientific experiments were the two most common problem drivers in our observations of Real STEM project classes.

Alternative energy design problems provide a good engineering context. For example, designing an efficient wind turbine blade, provides an excellent interdisciplinary STEM task. A base for the wind turbine can be provided, including a stand, motor, and wiring (Fig. 8.4). The U.S. Department of Energy has detailed plans for a wind turbine base in Building the Basic PVC Wind Turbine (http://www1.eere.energy.gov/education/pdfs/wind_basicpvcwindturbine.pdf).

A student is only responsible for the design of the wind turbine blade. A variety of materials, for blade construction, can be provided for students to choose from, or the teacher can allow the students to forage for materials. Students work in teams to design blades from selected materials, and mount them on a hub that can be connected to the wind turbine base. The engineering design process guides all aspects of the blade development, including the critical component of identifying criteria, and constraints, for blade construction. Constraints of materials used to create blades provide an opportunity to integrate materials science into the task. Criteria can be set by students, and include outcomes, such as continuous blade rotation for at least a minute, blade rotation speed, or a minimum energy production requirement. A multimeter can be used to measure volts produced. Students keep a design notebook, where they log each step in the engineering design process.

Redesign is an essential part of engineering design. Students can consider a number of variables that may have an impact on blade performance: blade length and

Fig. 8.4 Wind turbine base and set up for testing blades

$$P = \frac{1}{2} \rho \pi r^2 V^3$$

P = Power in the Wind (watts)

ρ = Density of the Air (kg/m^3)

r = Radius of your swept area (m)

V = Wind Velocity (m/s)

π = 3.14

Fig. 8.5 Power in the wind formula

pitch, number of blades, material used in blades, including smoothness of surface, and blade shape. Engineering design tasks have great potential for interdisciplinary STEM learning, but observations of Real STEM classes indicated that the potential was often not realized. If students are not held to rigorous engineering design standards, then the tasks may devolve into a trial and error mode, where there is no observable science, technology, or mathematics. The wind turbine blade task can engage students in the earth systems science topic of wind and weather, as well as physics of power. The task supports the use of technology including circuits, motors, and a multimeter. It also has plentiful mathematical applications. A simple equation for power can be used to provide a quantitative reasoning aspect to the problem (Fig. 8.5). With this equation students can determine the power generated by a typical house fan with wind velocity $V = 5$ m/s (metres per second), density of air $\rho = 1.0$ kg/m^3 (kilograms per cubic metre), and radius of swept area $r = 0.2$ m.

8.2.5 Quantitative Reasoning

Having discussed the integration of mathematics into the other four STEM reasoning modalities, now I focus on quantitative reasoning itself. The Culturally Relevant Ecology, Learning Progressions, and Environmental Literacy project (Mayes, Peterson, & Bonilla, 2012, 2013) developed a definition of quantitative reasoning, as well as a learning progression proposing a trajectory of QR development across ages 12–18.

Quantitative Reasoning (QR), in context, is mathematics and statistics, applied in real-life, authentic, situations, that have an impact on individual's life as a constructive, concerned, and reflective citizen. QR problems are context-dependent, interdisciplinary, open-ended, tasks that require critical thinking and the capacity to communicate a course of action (Mayes et al., 2013).

Once QR was defined, the research team began to construct, based on the literature and professional experience, a framework for QR that would be evaluated through development of a learning progression. The four key components of QR in the framework and key researchers' work upon which they were determined are:

1. Quantification Act (QA): The mathematical process of conceptualizing an object, and an attribute of it, so that the attribute has a unit measure (Thompson, 2011; Dingman & Madison, 2010).
2. Quantitative Literacy (QL): The use of fundamental mathematical concepts in sophisticated ways for the purpose of describing, comparing, manipulating, and drawing conclusions, from variables developed in the quantification act (Steen, 2001; Madison, 2003; Briggs, 2004).
3. Quantitative Interpretation (QI): Ability to use models to discover trends, and make predictions (Madison & Steen, 2003; Thompson & Saldanha, 2000).
4. Quantitative Modelling (QM): The ability to create representations that explain a phenomenon, and to revise them based on fit to reality (Duschl, Schweingruber, & Shouse, 2007; Schwarz et al., 2009; Lehrer, Schauble, Carpenter, & Penner, 2000).

A learning progression is a set of empirically grounded, and testable, hypotheses about how, with appropriate instruction, students' understanding of, and ability to use, core scientific concepts, explanations, and related scientific practices, grow and become more sophisticated over time (Corcoran, Mosher, & Rogat, 2009). Learning progressions provide levels of understanding through which students develop mastery of a concept over an extended period of time, such as over six years from ages 12–18. The QR learning progression is conceptualised as having four levels: the lower anchor, upper anchor and two intermediate levels of understanding. The lower anchor is grounded in data collected on 12 year olds understanding of QR (Mayes et al., 2014b). The upper anchor is based on expert views of what a scientifically literate citizen, who is well versed in QR, should know, and be able to apply by age 18. A learning progression defines progress variables, which are essential categories for the overall concept across which the levels are established. The progress variables for the QR learning progression were drawn from the four components in the QR framework (Mayes, Forrester, Christus, Peterson, & Walker, 2014a). The Quantification Act, and Quantitative Literacy, components were combined under Quantification Act (QA), with the expectation that once a variable is conceptualized, then fundamental mathematical concepts allow one to compare, contrast, manipulate, and combine the variables to form mathematical expressions. Quantitative literacy is essential to moving, from quantification, to building and interpreting models. This reduction of the framework left three progress variables in the QR learning progression: Quantification Act (QA), Quantitative Interpretation (QI), and Quantitative Modelling (QM).

Finally, each of the progress variables were elucidated by identifying a collection of elements, that indicate essential capabilities within the categories that were determined through student interviews, and tested, throughout development of the learning progression:

- Quantification Act Elements: Variation, Quantitative Literacy, Context, Variable
- Quantitative Interpretation Elements: Trends, Predictions, Translation, Revision
- Quantitative Modelling Elements: Create model, Refine model, Reason with model, Statistical analysis.

For a detailed presentation of the learning progression see Mayes et al. (2014a).

8.2.5.1 Quantitative Reasoning Examples

Mathematics is not typically the driver for STEM in schools. In STEM, S, and E, are the most common drivers, with T, and M, playing support rôles. Can mathematics be the driver in STEM? Certainly, verification of a mathematical statement, or a mathematical argument supporting a conjecture, can be a driver in STEM, if the conjecture is connected to a real-world STEM application. The difficulty is engaging a broad range of students in such mathematical discourse. Quantitative reasoning provides the opportunity for mathematics to play a more central rôle in STEM.

For authentic real-world interdisciplinary STEM problems, the quantification act is the ability to mathematise the problem, moving from a qualitative account to a quantitative description, by establishing quantitative variables, connecting the variables through exploring covariation, and building algebraic expressions. Quantitative modelling is the creation and refinement of a model, reasoning with mathematical models, and the use of statistical inference, to test hypotheses springing from analysis of data gathered on the problem. Quantitative interpretation is using a model to determine trends and make predictions, revision of models to fit reality, and the translation between multiple models of the same problem. Here I use the topic of sustainable energy, and environmental impacts, as a context for exploring the rôle of quantitative reasoning in STEM (Mayes & Myers, 2014). The amount of data, and variety of representations, in the area of energy challenge quantitative interpretation abilities.

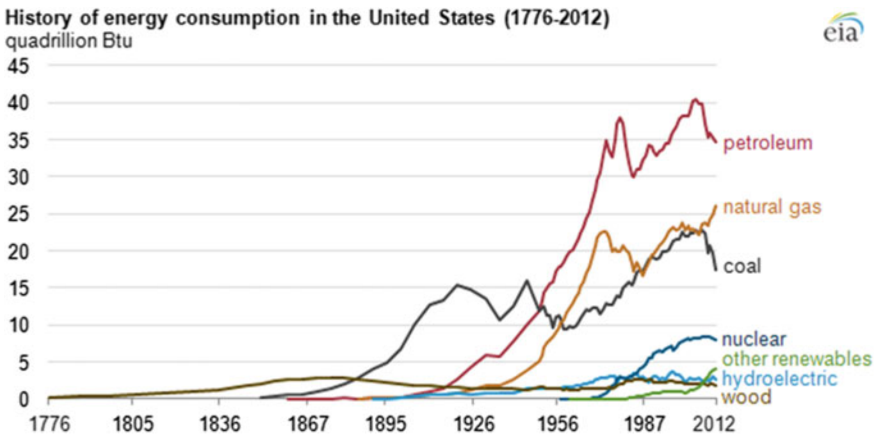

Fig. 8.6 Energy models (Mayes & Myers, 2014). U.S. Energy History. *Source* U.S. Energy Information Administration (2012)

Quantitative reasoning is recognised as an essential component for making informed decisions (Mayes et al., 2012). Quantitative reasoning, moves past quantities and values, to conceptualise and interpret, the relationships and contexts defining them (Thompson, 1993; Ramful & Ho, 2015). Within an interdisciplinary framework, quantitative reasoning, is envisioned as the application of mathematical concepts and models across domains to discover trends, and make inferences and predictions (Mayes & Koballa, 2012; Elrod, 2014). Further, quantitative reasoning distances itself from traditional mathematics, through its emphasis on ill-defined, open ended, real world problems (Mayes et al., 2012; Elrod, 2014). Unlike traditional mathematics, which places emphasis on calculations, and manipulations of abstract representations, quantitative reasoning is distinct in its emphasis on the underlying meaning of mathematical functions, and its application to authentic real-world problems (Elrod, 2014). Quantitative reasoning problems are context dependent, interdisciplinary, and open-ended, tasks that require critical thinking and the capacity to communicate a course of action. The energy exemplar, below, elucidates major rôles for quantitative reasoning in interdisciplinary STEM. Consider the representations in Fig. 8.6. Students need to understand the variable attributes and measures (QA) before they can interpret a model (QI).

Modelling data, and testing statistical hypothesis (QM), are critical for many real-world STEM problems. Given the data on U.S. Oil Consumption and Production in Table 8.1, there are a number of quantitative analyses that can be performed in analysing these data.

Table 8.1 U.S. Oil Consumption and Production, 1990–2018 (Year 1 is 1990)

Year	Production	Consumption	Year	Production	Consumption
1	7.36	16.99	16	5.18	20.80
2	7.42	16.71	17	5.09	20.69
3	7.17	17.03	18	5.08	20.68
4	6.85	17.24	19	5.00	19.50
5	6.66	17.72	20	5.35	18.77
6	6.56	17.72	21	5.48	19.18
7	6.46	18.31	22	5.65	18.88
8	6.45	18.62	23	6.49	18.49
9	6.25	18.92	24	7.47	18.96
10	5.88	19.52	25	8.76	19.11
11	5.82	19.70	26	9.41	19.53
12	5.80	19.65	27	8.85	19.63
13	5.74	19.76	28	9.35	19.97
14	5.65	20.03	29	9.91	20.30
15	5.44	20.73			

Production and consumption unit is million barrels per day ($\times 10^6$). U.S. Energy Information Administration https://www.eia.gov/outlooks/steo/data/browser

8.2.5.2 Descriptive Statistics Analysis

Use Excel to find measures of centre (mean, median, mode) and spread (range, standard deviation). What do these descriptive statistics tell you about the data sets?

Should you use the mean, median, or mode for this data set? Should you use the range, or standard deviation? Why? Construct a histogram of the data, to explore issues of data distribution type, related to which measure of centre and spread to use. Statistical display:

- QM Model—Data Display: Which is the best data display to use for these data (frequency table, bar chart, histogram, pie chart, scatter plot, dot plot, stem and leaf plot, box and whisker plot)? Why?
- QM Trends and Predictions: Use the data display you selected to discuss trends in the production and consumption data. Make a prediction of production and consumption in 2020.

Modelling:

- QM Mathematical Model: Create a mathematical model for production by year, using a line, or curve, of best fit. Use the model to extend the discussion of trend, and verify your prediction.
- Now find the line of best fit for the consumption by year. Predict consumption in 2020. Extension: Attempt a curve of best fit, for example a parabola. Does it fit

Table 8.2 Selecting a statistical test

Test	Hypothesis	Indep't Var'ble	Dep't Var'ble	Co-variates	Contin. or Cat. Ind'p't Var	Contin' or Cat. Dep't Var	Dist'n
t test	Grp Comp	1	1	0	Cat.	Cont.	Normal
ANOVA	Grp Comp	1or more	1	0	Cat.	Cont.	Normal
Chi-sq.	Grp Comp	1	1	0	Cat.	Cat.	Non-normal
Pearson R	Relate Var's	1	1	0	Cont.	Cont.	Normal

better than the line of best fit? Is the additional complication of a quadratic model worth it?

Hypothesis testing:

- QM Hypothesis Testing: Is the difference between production and consumption significant? We can test that question by comparing the means of the production and consumption data sets. First, examine a visual display of the two data sets, to see if they appear to be significantly different. Construct box and whisker plots for both production and consumption.

  An easy online boxplot tool is the Boxplot Grapher (http://www.imathas.com/stattools/boxplot.html).

  Do the plots support the hypothesis that there is a significant difference? Can we say the difference is significant using only a visual display?
- QM Statistical Hypothesis Testing: While comparing box and whiskers plots provides some intuition about differences in data sets, determining if there is a statistically significant difference requires conducting a formal statistical analysis. First, determine the best statistical test to use to assess the null hypothesis that the two data sets are not statistically different. Table 8.2 identifies four basic statistical tests, and the conditions under which they should be used. Which works best for this problem?
- The descriptive statistics, called for above, provide some information on which to conjecture about the type of distribution criteria. Use histograms for the two data sets for a visual representation of distribution type (normal or nonnormal). Which statistical test is best to use considering what you know about the production and consumption data sets?
- Use Excel to run the best statistical test for the null hypothesis. Use the Data—Data Analysis—t-Test: Two-Sample assuming Unequal Variances (NOTE t-test has several versions, this is the best for this data). Run with an alpha level of 0.05 (5% risk that we accept a difference that does not exist, that is the null hypothesis true

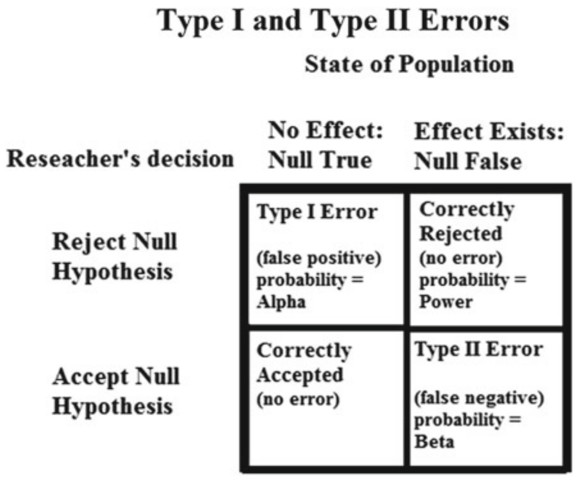

Fig. 8.7 Type I and Type II errors

but we say it is false). Interpret the resulting table. Is there a statistically significant difference between the data sets?

8.2.5.3 Inferential Analysis—Hypothesis Testing

It is important that students understand the concept behind hypothesis testing. We begin with a null hypothesis, that there is no difference between the data sets. We set an alpha level, typically $\alpha = 0.05$, which indicates the maximum risk we are willing to take that any observed differences are due to chance. So for $\alpha = 0.05$ we are willing to risk 5% of the time that we say there is a significant difference when there is not (Fig. 8.7). This is called a Type I error, where we have a false positive which is the worst possible outcome. We can also be in error when there is an effect but the test does not pick it up. This is called a Type 2 error, but we have a false negative which is at least more conservative, and therefore, less of a concern.

- State the Type 1 and Type 2 errors in terms of the production-consumption comparison.

Unless we have a good reason to believe that prior to the experiment the relationship will occur in one direction, such as that consumption will always exceed production, then we use a two-tailed test. If we do have a sense of direction for the outcome we use a one-tailed test. Let's use a two-tailed test for our comparison. Hypothesis testing is a probability game that indicates if we should accept or reject the null hypothesis (Fig. 8.8). If the probability value p is high then the null hypothesis is likely true and we do not reject it. If the probability value is less than $\alpha = 0.05$ then it is highly unlikely any difference is due only to chance (fewer than 5% of studies would result in the difference due to random sampling error only) so we reject the null hypothesis.

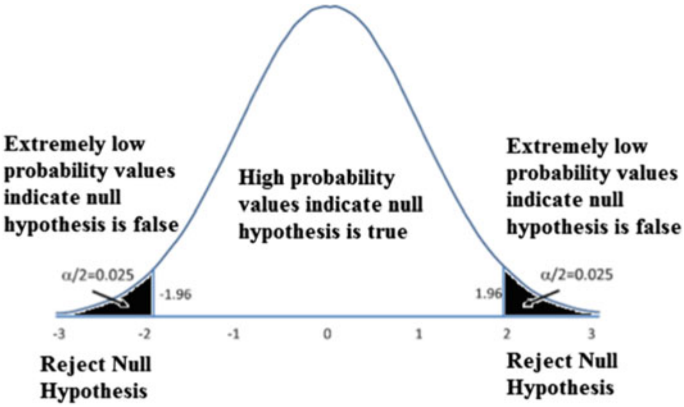

Fig. 8.8 A two-tailed test of a null hypothesis at an alpha level of 0.05

- State the above, with reference to the production-consumption comparison.

This concludes the presentation of examples of quantitative reasoning. The Real STEM Project classroom observations revealed that students struggled with QA and QI, and that teachers rarely engaged them in QM. Too often, interdisciplinary STEM tasks assigned by teachers, did not move the students beyond qualitative science descriptions or engineering designs lacking in quantitative analysis.

8.2.6 Evaluation

Teacher (n = 39) and student responses (n = 898) to increased engagement through interdisciplinary STEM problems was very positive. Teacher focus groups indicated positive interest and activity in developing STEM partnerships with businesses and research institutes. The teachers had areas of concern with implementing the STEM reasoning modalities, and sustaining collaboration with teachers in other STEM areas. The middle schools (ages 12–14) found it easier to have teachers collaborating on STEM research and design courses, due to the cross disciplinary team structures that exist in many middle schools, and the availability of flexible courses as a natural place to implement STEM courses. For example, Connections Courses, which provide opportunities for middle grade students to explore high school career pathways, were the common vehicle for establishing an interdisciplinary STEM course at the middle school level. The subject area silo structure of high schools, and the barrier of developing and staffing new courses in STEM, made it more of a challenge for high schools (ages 15–18) to sustain interdisciplinary STEM courses. In order to overcome the teacher collaboration and structure issues, of implementing interdisciplinary STEM programmes, it is essential to have administrative support and participation. The most successful high school STEM courses thrived when given administrative support.

Student surveys were conducted to determine student reaction to the STEM courses. Overall, the results indicated that students expressed statistically significant increases in Intrinsic Motivation, Self-Management and Self-Regulation, and Intent to Persist in STEM. The largest student gains observed were in the Intrinsic Motivation construct. For example, before taking the STEM courses 54% of students said that they enjoyed challenging classwork; after completing the courses 75% of students agreed that they preferred classwork that was challenging. Likewise, before taking the courses 62% of students agreed that the content they were learning could be used in other classes. After the courses, 81% of the students felt that they would be able to use what they learned in their other classes. A second survey examined student (1) interest in STEM fields, (2) confidence in their ability to perform academically in STEM fields, (3) feelings about the importance of understanding STEM, (4) interest in taking classes and pursuing post-secondary education in STEM fields, and (5) interest in STEM careers. There were significant differences in all five categories, supporting improved student attitudes and beliefs, upon completion of a STEM course.

8.3 Conclusion

The push to incorporate interdisciplinary STEM into existing science and mathematics classes, as well as for development of new STEM research and design courses, provides an excellent opportunity for interdisciplinarity for mathematics. STEM problems are real-world, complex, and require cross-disciplinary applications. Quantitative reasoning is a natural fit for such problems, consisting of the tools and concepts supporting quantification, interpretation, and modelling of STEM problems. The challenge for STEM in general, and mathematics specifically, is that quantitative reasoning abilities are not well developed in most students. We need to develop mathematical reasoning across STEM in an interdisciplinary manner.

The Real STEM project provides a model for developing and integrating interdisciplinary STEM courses into traditional middle schools and high schools. In the USA, intensive interdisciplinary STEM programmes are often the province of specialized STEM magnet schools or academies. The demand for STEM understanding far exceeds these specialized schools, both for workforce needs, and for STEM literate citizens. But, there are extensive barriers to integrating STEM into traditional middle schools and high schools, including curricula guided by excessive high stakes testing, extensive curriculum implementation guidelines, that limit flexibility in both topics taught and order in which they are presented, teachers' fear of going beyond their disciplinary boundaries, inflexible school schedules, that inhibit cross-disciplinary planning time, and lack of administrative support for interdisciplinary STEM and authentic teaching. So what can stakeholders take from the Real STEM project?

Practitioner stakeholders can use the Real STEM tenets as a guide to implementing interdisciplinary STEM in their classrooms. ICARE is an acronym for the key tenets:

- Interdisciplinary STEM that integrates the four STEM subjects across mathematics, science, engineering, and technology courses.
- Collaboration both within schools, through interdisciplinary STEM Professional Learning Communities (PLC), and external to the school, though partnerships that bring STEM experts into the classroom from the community, business, and industry, and research institutes such as universities and government research entities.
- Authentic teaching strategies, that engage students in real-world problems, and provide opportunities for student-centred research and design performance tasks.
- Reasoning in STEM that moves beyond entertaining activities, to performance tasks that reflect understanding and reasoning.
- Education for Understanding, that identifies enduring understandings, and essential questions, that motivate students to engage in developing and demonstrating deeper conceptual understanding.

Practitioners should focus on providing opportunities for model-based reasoning, design-based reasoning, and quantitative reasoning. All of these permeate the four subject areas of STEM, and when applied in real-world contexts, provides the opportunity to incorporate social sciences. Implementing the ICARE tenets requires teachers and administrators to work together to overcome barriers, such as, common PLC planning time, development of community partnerships, and flexibility in shuffling curriculum to allow for collaborative lessons across subject areas.

Policy-maker stakeholders should take notice of our research, that indicated the Real STEM project's impact on improving positive student engagement and the students' struggle with interdisciplinary STEM reasoning. Reduce excessive testing and curricular control, allowing teachers the flexibility to use authentic teaching strategies to improve student understanding of STEM.

For researchers, the project outcomes include a Quantitative Reasoning Learning Progression, a diagnostic assessment of interdisciplinary STEM for middle and high school grades (ages 12–18), student and teacher attitude surveys, a classroom observation protocol, and an exemplar for using Rasch Analysis to analyze these tools. Quantitative reasoning does not have a home within any of the STEM subject areas, not even mathematics. There is a need for research on the teaching and learning of quantitative reasoning in STEM. How do we meet the challenge of sustaining interdisciplinary quantitative reasoning across subject matter silos which constitute today's schools?

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