Chapter 10
Introduction

David Swanson

Abstract The five case study chapters in this section are introduced and contextualised in relation to previous case study work in Interdisciplinary Mathematics Education. Alongside the benefits which come from interdisciplinary work, a common theme which emerges from the case studies is that of a potential for mathematics to disappear, or to become a mere tool, within such activities. Appealing to Vygotsky's theory of scientific concepts, it is argued that there is a crucial role for generalisation within interdisciplinary mathematics, and that the connections within mathematics require attention alongside the connections between mathematics and real world experience if mathematics is to more fully benefit from, and bring benefit to, interdisciplinary work.

Keywords Mathematics · Interdisciplinarity · Vygotsky · Scientific concepts · Generalisation

10.1 Case Studies in Inter-disciplinarity

In a previous survey of the literature on interdisciplinary mathematics (Williams et al., 2016), I noted that the activities researched within existing case studies divided fairly evenly into 3 categories: (i) those where mathematics appeared in another subject, or vice versa; (ii) thematic integration, where the subjects including mathematics were directed toward a common theme or project whilst retaining their particular disciplinarity; and (iii) project or problem based activity, where the problem or project had become the central question and individual subjects had begun to dissolve. It was argued that these categorizations mapped loosely (and with exceptions) onto the common categorizations of mono-disciplinary, multi-disciplinary, and inter-disciplinary. The case studies in this section arguably follow this pattern, with examples where mathematics appears in another subject; where thematic integration occurs, with dis-

D. Swanson (✉)
Manchester Institute of Education, The University of Manchester, B4.10 Ellen Wilkinson Building, Oxford Road Manchester M13 9PL, UK
e-mail: david.swanson@manchester.ac.uk

B. Doig et al. (eds.), *Interdisciplinary Mathematics Education*,
ICME-13 Monographs, https://doi.org/10.1007/978-3-030-11066-6_10

tinct subjects operating around a shared problem or project; and project work where individual subjects begin to lose their distinctions to various extents.

The previous survey also stressed how little research had been done in the field of inter-disciplinary mathematics, and that what did exist was dominated by small-scale case studies, often with a lack of conceptual clarity, whether terminologically or theoretically. The chapters in this section, therefore, bring very welcome new light and further clarity in a range of areas. This is not only in relation to previously more typical small-scale examples of interdisciplinarity found in practice, but also in relation to the developing existence of longer-term practices, and those that go beyond local to regional, or even (national) system-wide activity or innovation. The scale and form of the interdisciplinary activities involved include: an established national qualification, a long-running regional project at the edges of schooling, a multi-school project integrated into mainstream curricula of different subjects, and a single lesson sequence in one classroom.

In this introductory section I will first outline some of the key features of the forthcoming chapters. Then, drawing on some of the emerging themes, I will focus on an important distinction between mathematics as a tool and mathematics as a (conscious) generalisation. This aims to contribute an essential understanding that can begin to help overcome some of the limitations that can arise when mathematics joins with other subjects (both limitations for the learning of mathematics, but also for learning in areas of life and schooling outside of mathematics, whether in projects, problems or other subjects).

10.1.1 The Case Studies

In the first chapter of this section, den Braber, Krüger, Mazereeuw and Kuiper discuss a successful interdisciplinary STEM course in upper secondary school in the Netherlands, taking place in 40% of the schools there. The course on *Nature, life, and technology* is built around real world problems, and although separate from other subjects, it is staffed by a team of disciplinary teachers drawn from core subject teachers in mathematics, chemistry, biology, physics, and geography. Mathematics appears to have an exceptional role within the course, given one of only four key characteristics expected to be made visible through the curriculum, is ‘the role of mathematics in science’. The authors explore whether the stated objectives of the course in relation to mathematics are reflected in practice and the experience of teachers.

Findings indicate diversity in how mathematics operates within the course. In the curriculum the emphasis is on the use of mathematics rather than the learning of specific mathematical concepts, and seems to prioritise procedural over conceptual or strategic mathematical thinking. How this plays out in practice depends on the role of mathematicians within the team. However, although there are some positive stories, in practice only 50% of the teams have a mathematics teacher within them, despite this being named as a priority from the beginnings of the course, and generally students see mathematics as having a lower level or lesser role. The authors discuss the obstacles to mathematics teacher participation, the challenges that those teachers face when they take part, and ways they might overcome these difficulties.

There is a clear contrast between teachers' stated views on the importance of mathematics to the course, whatever their own subject, and that of students and what is often seen in practice. This may be partly due to mathematics importance being seen in its role as a tool for other subjects rather than its full richness. The authors touch on this question, and the contrasting perspectives of mathematics as a servant or queen of science. I will return to this question in the latter parts of this introduction and suggest a more helpful perspective that is of more use in interdisciplinary work—that of mathematics as a particularly helpful (conscious) generalisation of science.

Our next chapter, by Gorriz and Vilches, explores some delightful interdisciplinary work in secondary schools in Catalonia. The authors work with schools across the region on a variety of cross-curricular projects including the making of films using special effects, the design of packages of fixed volume but with shapes designed by students, and the creation of musical instruments.

What is different in their approach, to many others, is less in the bringing together of teachers in different subjects to design, collectively, the central activity, but more in the fact that teachers in each individual subject address central or related aspects of the task within their own classes, with close attention and alignment to the prescribed curriculum in each area. Common co-operative working guidelines are drawn up and then working guidelines within each subject. Student activities are then assessed through the writing of reports within each subject, and overall. So, for example, in the design of packaging the mathematics class focuses on the variations of complexity in particular shapes, and the calculation of volume, sometimes with computer assistance. The authors show how the activities lead to the development of interesting mathematics in the classroom. Meanwhile in science, for example, the nature of the food content is explored; in art, the aesthetics of the packaging is designed; in linguistics, the content of an advertisement is written; and in music, a jingle for the advertisement is created.

The authors argue that such activities are successful for students because of the consistency between subjects, the relation of education to problems close to the heart of students, the emphasis on collaborative work, and because students can approach the tasks at their own level. Teachers also gain much from the activity, although the time required for joint activity is seen to put, potentially, limits on its spreading without sympathetic teachers, or perhaps structural changes. Apart from this burden of additional time for teachers, this model of interdisciplinarity offers much potential due to its accommodation rather than challenge to existing curricula. How much that accommodation puts limits on the genuine integration of subjects, and whether that matters, is a topic for future debate.

In our third chapter, Hobbs, Doig and Plant discuss a regional project based in Victoria, Australia which aims to address the relatively low take-up of STEM qualifications in the area. The project, which engages teachers from 10 schools in a 2-year program, is unusual in its flexibility and range of outcomes. The schools work collaboratively with the local University to develop new knowledge, language, pedagogy, and curriculum, to support the development of the school's own 'STEM vision'. Cycles of professional development including intensive sessions, and support in school are developed on the basis of ongoing assessment and negotiation between the university and schools.

This process has led to a broad range of approaches to STEM and interdisciplinarity which are outlined in the chapter, alongside case studies which provide some rich illustrative detail of the diversity of results. Despite the differences, what unites them is their non-traditional pedagogic approaches, particularly through problem solving but also through strategies such as group work, peer-teaching and open-ended investigations. However, the important emphasis on outcomes such as engagement and creativity is seen to sometimes sideline conceptual development, and the dominance of particular subjects sometimes means that mathematics (and even science) are reduced to being mere tools.

Our fourth chapter, by Kastberg, Long, Lynch-Davis and D'Ambrosio, explores an example of inter-disciplinarity based around an art lesson for 10 and 11 year olds, but one where mathematics inspires and informs the activity, and students begin to develop a new, more positive, relationship with mathematics through the artistic task. The lesson, where groups of students are given the problem of enlarging artwork collectively, is argued to act as a counterpoint to the more common negative experiences of failure and alienation from the subject, partly through its disguising of mathematics in a fun and creative activity, but also through the way the activity allows the re-configuring of relationships, including peer-to-peer, student-to-teacher, and student-to-mathematics.

The space opened up by a focus on collaborative art and creativity encourages students to begin to shift authority and decision making from the teacher to their own collective discussions and activity. As part of this shift students begin to determine both their own working relationships and criteria for success. Students develop important aspects of proportional understanding in, and through, working this way. These are primarily perceptual aspects though, and the authors discuss the complex decisions to be made in such situations, where looking for opportunities to develop mathematics more explicitly may lose the gains that artistic and collaborative work offer.

This chapter therefore helps us to understand better the potentials within mainly mono-disciplinary contexts for teaching that can escape its boundaries and assist in the development of understanding in other disciplines, while still maintaining the positive features of the original discipline. It also raises questions about the limitations of such contexts to go further without losing those positive features. But perhaps, above all, it encourages us to imagine a less alienating school mathematics, where creativity, collaboration, and autonomy feature as much as they can in art.

In the final chapter in this section on case studies, Doig and Jobling provide us with a more general discussion on interdisciplinarity. Their chapter integrates a range of case studies to place the trend for STEM integration in its recent and more long-term historical context. In doing so, they provide a strong argument for the importance of project-based learning as an effective form of interdisciplinarity.

Early examples of interdisciplinarity include the post-war integration in schooling of the mathematics specifically required for manual trades. More modern examples given, such as the long-running Model Solar Vehicle Challenge in Victoria, arguably continue this lineage with an emphasis, for example, on technological design and electrical and mechanical engineering, providing experiences similar to those found within employment in those careers. Such recent examples are argued to be more commonly found on the edge of schooling rather than fully integral and the chapter explores a variety of existing forms to ground its analysis of the potential for making project work, and interdisciplinarity generally, more central to curricula.

The role of mathematics within the integration attempts discussed is not without its problems. In an example looking at mathematisations of acceleration, the expected mathematical model was found at times to complicate rather than assist the development of understanding. One survey of STEM integration projects discussed suggests that such projects are less effective in terms of student attainment gains when mathematics is included alongside other subjects. And finally, it is shown that there may be a tendency for mathematics to only be included in project-based learning in its limited role as a tool, for example, through measurement.

This last chapter therefore highlights some key points that arise in most of our other case studies. We find that engaging with real world problems (in project work, interdisciplinary problems, or art) brings great benefits for the learning of mathematics. It provides substance and meaning, but also engagement and many other things, including even joy, to mathematics. At the same time it also seems to take something away, partly through the potential for losing any conscious focus on mathematics. Through that we may be left with mathematics as a tool, and ultimately lose other benefits, even for the problems at hand, that would come from a richer and more conscious understanding of mathematics, and its own system and connections. One solution, seen within one of our case studies, (by Gorriz and Vilches) may be that alongside bringing subjects together, we may also need a separation, with space to develop mathematics for itself, that can then feed back into more general problem solving. But is this necessarily so? In what remains of this introduction I will focus on these issues.

10.1.2 Mathematics as Tool and Mathematics as (Conscious) Generalisation

> I do not trust teachers of other disciplines to be able to tie the bonds of mathematics with reality which have been cut by the mathematics teacher. (Freudenthal, 1971, p. 420)

In this slightly surly quote, from the irrepressible Han Freudenthal, his main concern is the damage done to mathematics (and perhaps learning in other disciplines) by the subject's separation from the real world contexts and practices that other subjects might provide. One can disagree with aspects of this quote, for example it seems to be blaming individual teachers whereas the forces that have led to the structural separation of the school curriculum into subjects seem beyond any individual. The agreement one can find with Freudenthal though, through studies of interdisciplinary

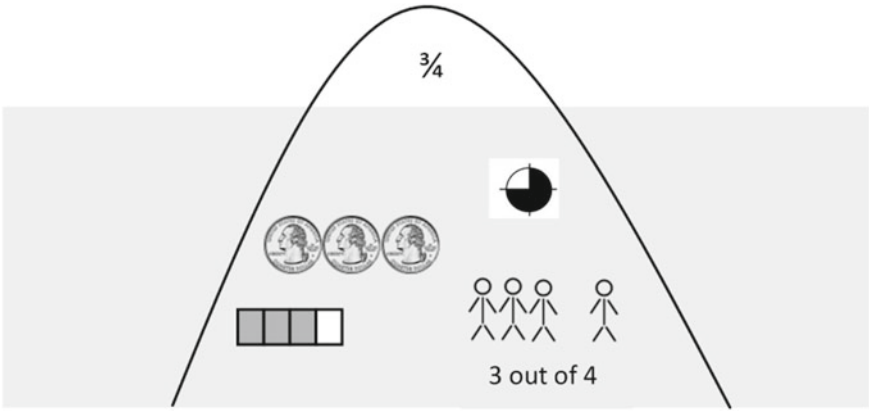

Fig. 10.1 A simplified version of the RME iceberg metaphor

work, including the case studies featured here, is on the essential problem that flows from the artificial separation of the subjects. Mathematics loses the rich concrete material that is an essential element within its abstractions.

In Realistic Mathematics Education (RME) (see Van den Heuvel-Panhuizen & Drijvers, 2014) which builds on Freudenthal's work, there is a popular metaphor of an iceberg, where a mathematical concept such as ¾, resting above the water, requires a range of contextual experiences and examples beneath the surface to support it (see Fig. 10.1).

This image on its own however could fit equally well with an idea of mathematics as a tool. Although ¾ here provides a generalisation across a range of experiences and contexts, it is not necessarily a conscious mathematical generalisation. There is little need for consciousness of what ¾ means as a mathematical thing, tool or object in order to use it. For it to become conscious requires something else to come in. Here, I borrow from Vygotsky's (1987) understanding of scientific concepts, which involves an understanding that abstractions and generalisations do not represent a shedding of concrete experience (typically, abstraction is viewed as removing such concrete details to leave formal mathematical objects), but instead the bringing in of the systemic relationships between concepts, including generalisations of generalisations. This is a complex idea, so let's simplify it (further) by adding something to our original iceberg metaphor.

Above the new line (see Fig. 10.2) lie the systematic connections with other mathematical concepts, such as the relationship between vulgar fractions and decimal fractions, equivalence of fractions, and ratio and proportion in general *et cetera.* This shift in focus is connected to that described as involving a shift from being an activity at one level to being an object of reflection at a higher level (Freudenthal, 1991, p. 96,—and it is therefore worth adding that this general philosophical point is not absent from either Freudenthal or RME, despite being absent from this visual metaphor to help illustrate RME).

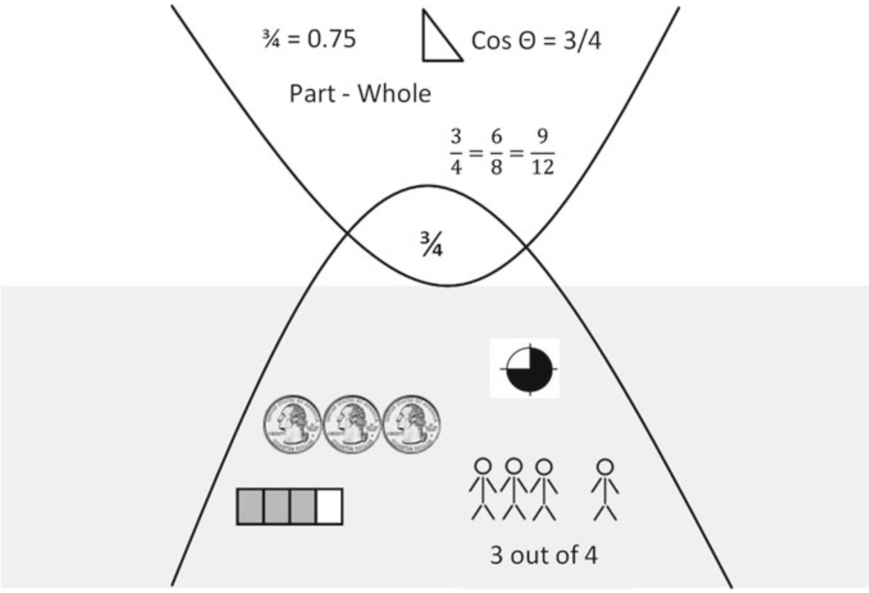

Fig. 10.2 A simplified extension of the RME iceberg metaphor to include mathematical connections

What the introduction of systematic relations between concepts does, in the image, as well as for any individual, is to carve out ¾ as a conscious mental object. This is one that can be picked up and used more readily, and one that becomes a point of conscious connection between the systematic relations of mathematics and any particular example, or experience. (I should add here the caveat that there are of course many ways to challenge this simplistic picture, e.g. can we really distinguish between things above one line and below the other, i.e. can they play similar roles, or, even, are there no objects at all but only processes and relationships? Still, I would suggest it is a useful initial metaphor).

In mathematics education, such generalising, and making of connections, plays a vital role in developing understanding of any particular mathematical activity (e.g. generalised understandings of proportion can mediate understanding of a whole range of topics usually separated in schooling). Mathematics, and systematic mathematical knowledge, can also play a role for the sciences, not just as a tool, but as a particularly useful conscious generalisation of scientific knowledge. It can provide a form of generalisation that unites a variety of physical relationships (e.g. the mathematics of proportion can be seen in Ohm's law, the wave equation, and distance, speed, time relationships) and this generalisation can then mediate and enrich understanding of any of the particular relationships.

This understanding helps us see how mathematics can be of use in interdisciplinary work beyond its role as a tool, but for this to happen this requires some space for specifically mathematical connections to be made (even if in parallel with the introduction of new contexts and experiences). One solution may therefore be that hinted at in the chapter authored by Gorriz et al., where cognate subjects remain distinct even while a joint project is developed. However there is a potential problem with this. To build on the original quote by Freudenthal above, school mathematics, in general has not only cut the bonds between mathematics and reality, but also between mathematics and itself. Curricula are often divided, and divided again, into small bite-size pieces, which perhaps aids memorisation, but certainly undermines understanding. So we find that the aspects of mathematics which are missing from, for example, project work, are also often missing in the mathematics classroom, where mathematics can almost become just a tool for doing mathematics (i.e. in memorised and applied processes). Therefore, to bring the full benefits to bear, of mathematics on inter-disciplinarity, yes, we can ensure there are spaces where mathematics itself is the focus, whether in a specifically mathematics classroom or not. But we also need to tie the bonds between mathematics and extra-mathematical reality, *and* between mathematics and itself, within the mono-disciplinary subject as much as outside, so that a more meaningful form of mathematics can come to dominate. Then, mathematics could share its full richness within interdisciplinary work, and shed both its role as merely a tool, and its reputation as a subject that brings too many negatives along with it.

References

Freudenthal, H. (1971). Geometry between the devil and the deep sea. *Educational Studies in Mathematics, 3*(3), 413–435.

Freudenthal, H. (1991). *Revisiting mathematics education: China lectures*. Dordrecht: Kluwer.

Swanson, D. (2016). Expressions of the commodity form: Alienation and mathematics education. In H. Straehler-Pohl, N. Bohlman, & A. Pais (Eds.), *The disorder of mathematics education*. Springer

Swanson, D. (2018). Vernacular: A montage of mathematical graffiti, quotations and commentary. *The Mathematics Enthusiast, 15*(1), 133–147.

Swanson, D., & Black, L. (2017). Integrating critical theory and practice in mathematics education. In *Proceedings of CERME 10* (pp.1569–1576).

Van den Heuvel-Panhuizen, M., & Drijvers, P. (2014). Realistic mathematics education. In S. Lerman (Ed.), *Encyclopedia of mathematics education* (pp. 521–525). Netherlands: Springer.

Vygotsky, L. S. (1987). Thinking and speech. In *The collected works of L.S.Vygotsky*, Vol. 1. New York: Plenum.

Williams, J., Roth, W.-M., Swanson, D., Doig, B., Groves, S., Omuvwie, M., Borromeo Ferri, R., & Mousoulides, N. (2016). *Interdisciplinary mathematics education: A state of the art (ICME-13 topical surveys)*. Dordrecht, The Netherlands: Springer.