Chapter 7
The Effects of Network Relationships on Global Supply Chain Vulnerability

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Abstract In this chapter, we analyze the effects of levels of social relationship on the global supply chain networks vulnerability. Relationship levels in our framework are assumed to influence transaction costs as well as risk for the decision-makers. We propose a network performance measure for the evaluation of the global supply chain networks efficiency and vulnerability. The measure captures risk, transaction cost, price, transaction flow, revenue, and demand information in the context of the decision-makers behavior the network. The network consists of manufacturers, retailers, and consumers. Manufacturers and retailers are multicriteria decision-makers who decide about their production and transaction quantities as well as the level of social relationship they want to pursue in order to maximize net return and minimize risk. The model allows us to investigate the interplay of the heterogeneous decision-makers in the supply chain and to compute the resultant equilibrium pattern of product outputs, transactions, product prices, and levels of social relationship. The results show that high levels of relationship can lead to lower overall cost and therefore lower price and higher product transaction. Moreover, we use the performance measure to assess which nodes in the supply networks are the most vulnerable in the sense that their removal will impact the performance of the network in the most significant way.

7.1 Introduction

In recent years, growing competition and emphasis on efficiency and cost reduction, as well as the satisfaction of consumer demands, have brought new challenges for businesses in the global marketplace. As a result companies are outsourcing and offshoring large portions of manufacturing, sourcing in low-cost countries, reducing inventories, streamlining the supply base, and collaborating more intensively with other supply chain actors [12, 19]. However, the increase in interfirm dependence as well as longer and more complex supply chain setups with globe-

spanning operations have increased the vulnerability of supply chains to unexpected events [6, 17, 18, 36]. For example, recently, the threat of illness in the form of SARS (see [10]) has disrupted supply chains, as have terrorist threats (cf. [34]) and the natural disaster of Hurricane Katrina in 2005. Indeed, at the same time that supply chains have become increasingly globalized, their operation environment has become unpredictable and filled with uncertainty.

Supply chain disruptions can materialize from various areas internal and external to a supply chain. The main supply chain vulnerability drivers can be divided in three groups, demand-side, supply-side, and Catastrophic events. The demand-side drivers would include demand uncertainty, customer dependence, and disruptions in the physical distribution of products to the end-customers. The supply-side drivers include supplier business risks, production capacity constraints on the supply market, quality problems, technological changes, and product design changes [39]. Moreover these drivers would increase supply chain vulnerability even more if the firm uses single sourcing or fewer suppliers. Catastrophic events refer to natural hazards, socio-political instability, civil unrest, economic disruptions, and terrorist attacks [26, 24]. Therefore we believe that firms should proactively assess and manage the uncertainties in supply chain by creating a portfolio of relationships with their suppliers and demand markets in order to guard against costly supply chain disruptions.

The value of relationship is not only economical but also technical and social [14]. Strong supply chain relationships enable firms to react to changes in the market, create customer value and loyalty, which lead to improve profit margins [11]. The benefits are reduction of production, transportation and administrative costs. On the technical development the greatest benefit is the possibility of sharing the resources of suppliers and shortening the lead-times. Spekman and Davis [35] found that supply chain networks that exhibit collaborative behaviors tend to be more responsive and that supply chain-wide costs are, hence, reduced. These results are also supported by Dyer [9] who demonstrated empirically that a higher level of trust (relationship) lowers transaction costs (costs associated with negotiating, monitoring, and enforcing contracts). Baker and Faulkner [1] present an overview of papers by economic sociologists that show the important role of relationships due to their potential to reduce risk and uncertainty. Uzzi [37] and Gadde and Snehota [14] suggest that multiple relationships can help companies deal with the negative consequences related to dependence on supply chain partners. Krause et al. [25] found that buyer commitment and social capital accumulation with key suppliers can improve buying company performance. However, Christopher and Jüttner [5] indicate that the value of the relationship depends on the substitutability of the buyers or sellers, the indispensability of goods, savings resulting from partner's practices and the degree of common interest.

In this chapter, we analyze the effects of relationships on a multitiered global supply chain network efficiency and vulnerability. Wakolbinger and Nagurney [38] and Cruz et al. [8] developed a framework for the modeling and analysis of supply chains networks that included the role that relationships play. Their contribution was apparently the first to introduce relationship levels in terms of flows on networks,

along with logistical flows in terms of product transactions, combined with pricing. However, their models did not considerer the effects of relationship levels on supply chain efficiency and vulnerability.

This chapter models the multicriteria decision-making behavior of the various decision-makers in a multitiered global supply chain network, which includes the maximization of profit and the minimization of risk through the inclusion of the social relationship, in the presence of both business-to-business (B2B) and business-to-consumer (B2C) transactions. We describe the role of relationships in the global supply chain networks. Decision-makers in a given tier of the network can decide on the relationship levels that they want to achieve with decision-makers associated with the other tiers of the network. Establishing/maintaining a certain relationship level induces some costs, but may also lower the risk and the transaction costs. We explicitly describe the role of relationships in influencing transaction costs and risk. Both the risk functions and the relationship cost functions are allowed to depend on the relationship levels. In addition, we analyze the effects of the levels of social relationship on the supply chain efficiency and vulnerability. Hence, we truly capture the effects of networks of relationships in the global supply chain framework.

This chapter is organized as follows. In Sect. 7.2, we develop the multitiered, multiperiod supply chain network model. We describe decision-makers' optimizing behavior and establish the governing equilibrium conditions along with the corresponding variational inequality formulation. In Sect. 7.3, we present the supply chain network efficiency and vulnerability measures. In Sect. 7.4, we present numerical examples. Section 7.5 provides managerial implications. We conclude the chapter with Sect. 7.6 in which we summarize our results and suggest directions for future research.

7.2 The Global Supply Chain Networks Model

In this Section, we develop the network model with manufacturers, retailers, and demand markets in a global context. We assume that the manufacturers are involved in the production of a homogeneous product and we consider L countries, with I manufacturers in each country, and J retailers, which are not country-specific but, rather, can be either physical or virtual, as in the case of electronic commerce. There are K demand markets for the homogeneous product in each country and H currencies in the global economy. We denote a typical country by l or $\hat{l}$, a typical manufacturer by i, and a typical retailer by j. A typical demand market, on the other hand, is denoted by k and a typical currency by h. We assume that each manufacturer can conduct transactions with the retailers in different currencies. The demand for the product in a country can be associated with a particular currency.

The network in Fig. 7.1 represents the global supply chain network consisting of three tiers of decision-makers. The top tier of nodes consists of the manufacturers in the different countries, with manufacturer i in country l being referred to as
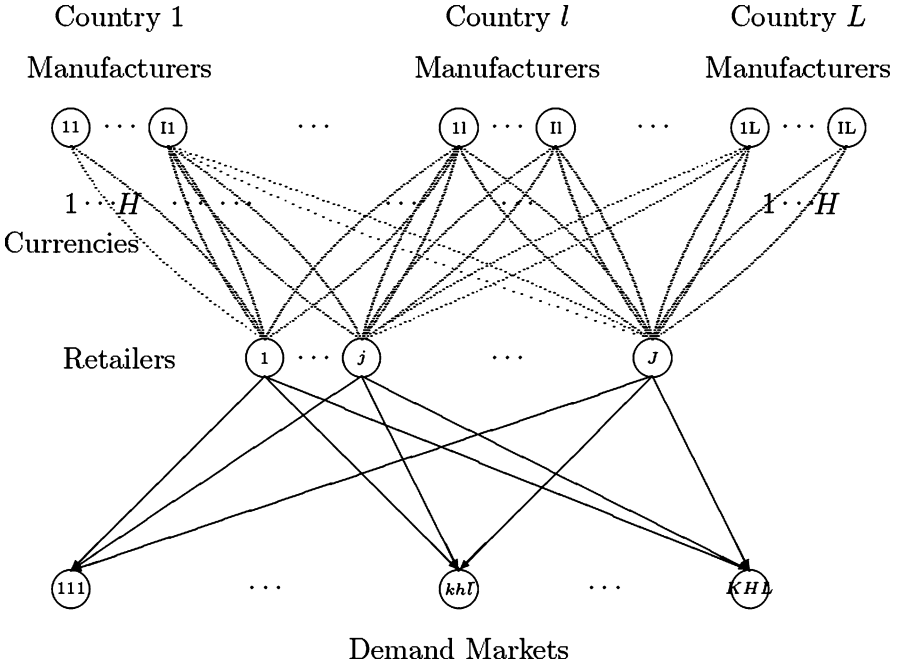
Country 1   Country l Country L

Manufacturers   Manufacturers   Manufacturers

Fig. 7.1 The structure of the global supply chain network

manufacturer il and associated with node il. There are, hence, IL top-tiered nodes in the network. The middle tier of nodes of the network consists of the retailers (which recall need not be country-specific) and who act as intermediaries between the manufacturers and the demand markets, with a typical retailer j associated with node j in this (second) tier of nodes. The bottom tier in the supply chain network consists of the demand markets, with a typical demand market k in currency h and country $\hat{l}$, being associated with node $kh\hat{l}$ in the bottom tier of nodes. There are, as depicted in Fig. 7.1, J middle (or second) tiered nodes corresponding to the retailers and KHL bottom (or third) tiered nodes in the global supply chain network.

We have identified the nodes in the network and now we turn to the identification of the links joining the nodes in a given tier with those in the next tier. We assume that each manufacturer i in country l involved in the production of the homogeneous product can transact with a given retailer in any of the H available currencies, as represented by the H links joining each top tier node with each middle tier node j; $j = 1, \ldots, J$. Furthermore, each retailer (intermediate) node j; $j = 1, \ldots, J$, can transact with each demand market denoted by node $kh\hat{l}$. The product transactions represent the flows on the links of the supply chain network in Fig. 7.1.

We also assume that each manufacturer i in country l can establish a portfolio of relationships with retailers. Furthermore, each retailer (intermediate) node j; $j = 1, \ldots, J$, can establish a relationship level with a demand market denoted by

node $kh\hat{l}$ and with manufacturers. We assume that the relationship levels are non-negative and that they may attain a value from 0 through 1. A relationship level of 0 indicates no relationship and a relationship of 1 indicates the highest possible relationship. These relationship levels are associated with each of nodes of the first two tiers of the network in Fig. 7.1.

Note that there will be prices associated with each of the tiers of nodes in the global supply chain network. The model also includes the rate of appreciation of currency h against the basic currency, which is denoted by e_h (see [28]). These "exchange" rates are grouped into the column vector $e \in R^H$. The variables for this model are given in Table 7.1. All vectors are assumed to be column vectors.

Table 7.1 Variables in global supply chain networks

Notation	Definition
q	IL-dimensional vector of the amounts of the product produced by the manufacturers in the countries with component il denoted by q^{il}
Q^1	$ILJH$-dimensional vector of the amounts of the product transacted between the manufacturers in the countries in the currencies with the retailers with component $^{il}_{jh}$ denoted by q^{il}_{jh}
Q^2	$JKHL$-dimensional vector of the amounts of the product transacted between the retailers and the demand markets in the countries and currencies with component $^{j}_{kh\hat{l}}$ denoted by $q^{j}_{kh\hat{l}}$
η^1	IL-dimensional vector of the relationships levels of manufacturers with component ^{il} denoted by η^{il}
η^2	J-dimensional vector of the relationship levels of retailers with component ^{j} denoted by η^{j}
ρ^{il}_{1jh}	Price associated with the product transacted between manufacturer il and retailer j in currency h
$\rho^{j}_{2kh\hat{l}}$	Price associated with the product transacted between retailer j and demand market k in currency h and country $\hat{l}$
$\rho3$	KHL-dimensional vector of the demand market prices of the product at the demand markets in the currencies and in the countries with component khl denoted by $\rho3khl$

We now turn to the description of the functions and assume that they are measured in the base currency (dollar). We first discuss the production cost, transaction cost, handling, and unit transaction cost functions given in Table 7.2. Each manufacturer is faced with a certain production cost function that may depend, in general, on the entire vector of production outputs. Furthermore, each manufacturer and each retailer are faced with transaction costs. The transaction costs are affected/influenced by the amount of the product transacted and the relationship levels. As indicated in the introduction, relationship levels affect transaction costs [9, 35]. This is especially

Table 7.2 Production, handling, transaction, and unit transaction cost functions

Notation	Definition
$f^{il}(q) = f^{il}(q^{il})$	The production cost function of manufacturer i in country l
$c_j(Q^1)$	The handling/conversion cost function of retailer j
$c_{jh}^{il}\left(q_{jh}^{il}, \eta^{il}\right)$	The transaction cost function of manufacturer il transacting with retailer j in currency h
$\hat{c}_{jh}^{il}\left(q_{jh}^{il}, \eta^{j}\right)$	The transaction cost function of retailer j transacting with manufacturer il in currency h
$c_{kh\hat{l}}^{j}\left(q_{kh\hat{l}}^{j}, \eta^{j}\right)$	The transaction cost function of retailer j transacting with demand market $kh\hat{l}$
$\hat{c}_{kh\hat{l}}^{j}(Q^2, \eta^2)$	The unit transaction cost function associated with consumers at demand market khl in obtaining the product from retailer j

important in international exchanges in which transaction costs may be significant. Hence, the transaction cost functions depend on flows and relationship levels.

Each retailer is also faced with what we term a handling/conversion cost (cf. Table 7.2), which may include, for example, the cost of handling and storing the product plus the cost associated with transacting in the different currencies. The handling/conversion cost of a retailer is a function of how much he has obtained of the product from the various manufacturers in the different countries and what currency the transactions took place. For the sake of generality, however, we allow the handling functions to depend also on the amounts of the product held and transacted by other retailers.

The consumers at each demand market are faced with a unit transaction cost. As in the case of the manufacturers and the retailers, higher relationship levels may potentially reduce transaction costs, which mean that they can lead to quantifiable cost reductions. The unit transaction costs depend on the amounts of the product that the retailers and the manufacturers transact with the demand markets as well as on the vectors of relationships established with the demand markets. The generality of the unit transaction cost function structure enables the modeling of competition on the demand side. Moreover, it allows for information exchange between the consumers at the demand markets who may inform one another as to their relationship levels which, in turn, can affect the transaction costs.

We now turn to the description of the relationship production cost functions and, finally, the risk functions and the demand functions. We assume that the relationship production cost functions as well as the risk functions are convex and continuously differentiable. The demand functions are assumed to be continuous.

We start by describing the relationship production cost functions that are given in Table 7.3. We assume that each manufacturer may actively try to achieve a certain relationship level with a retailer as proposed in Golicic et al. [15]. Furthermore, each retailer may actively try to achieve a certain relationship level with a manufacturer

Table 7.3 Relationship productions cost functions

Notation	Definition
$b^{il}\left(\eta^{il}\right)$	The relationship production cost function associated with manufacturer il
$b^{j}\left(\eta^{j}\right)$	The relationship production cost function associated with retailer j

and/or demand market. These relationship production cost functions may be distinct for each such combination. Their specific functional forms may be influenced by such factors as the willingness of retailers or demand markets to establish/maintain a relationship as well as the level of previous business relationships and private relationships that exist. Hence, we assume that these production cost functions are also affected and influenced by the relationship levels. Crosby and Stephens [7] indicate that the relationship strength changes with the amount of buyer-seller interaction and communication. In a global setting, cultural differences, difficulties with languages, and distances, may also play a role in making it more costly to establish (and to maintain) a specific relationship level (cf. [20]).

The concept of relationship levels was inspired by a paper by Golicic et al. [15] who introduced the concept of relationship magnitude. That research strongly suggested that different relationship magnitudes lead to different benefits and those different levels of relationship magnitudes can be achieved by putting more or less time and effort into the relationship. The idea of a continuum of relationship strength is also supported by several theories of relationship marketing that suggest that business relationships vary on a continuum from transactional to highly relational (cf. [13]). The model by Wakolbinger and Nagurney [38] operationalized the frequently mentioned need to create a portfolio of relationships (cf. [4, 15]). The optimal portfolio balanced out the various costs and the risk, against the profit and the relationship value and included the individual decision-makers preferences and risk aversions.

We now describe the risk functions as presented in Table 7.4. We note that the risk functions in our model are functions of both the product transactions and the relationship levels. Jüttner et al. [21] suggest that supply chain-relevant risk sources falls into three categories: environmental risk sources (e. g., fire, social-political actions, or "acts of God"), organizational risk sources (e. g., production uncertainties), and network-related risk sources. Johnson [22] and Norrman and Jansson [31] argue that network related risk arises from the interaction between organizations within the supply chain, e. g., due to insufficient interaction and cooperation. Here, we model

Table 7.4 Risk funktions

Notation	Definition
$r^{il}\left(Q^{1},\eta^{1}\right)$	The risk incurred by manufacturer il in his transactions
$r^{j}\left(Q^{1},Q^{2},\eta^{2}\right)$	The risk incurred by retailer j in his transactions

supply chain organizational risk and network-related risk by defining the risk as a function of product flows as well as relationship levels. We use relationship levels (levels of cooperation) as a way of possibly mitigating network-related risk. We also note that by including the currency appreciation rate (e_h) in our model, in order to convert the prices to the base currency (dollar), we are actually mitigating any exchange rate risk. Of course, in certain situations; see also Granovetter [16], the risk may be adversely affected by higher levels of relationships. Nevertheless, the functions in Table 7.4 explicitly include relationship levels and product transactions as inputs into the risk functions and reflect this dependence.

The demand functions as given in Table 7.5 are associated with the bottom-tiered nodes of the global supply chain network. The demand of consumers for the product at a demand market in a currency and country depends, in general, not only on the price of the product at that demand market (and currency and country) but also on the prices of the product at the other demand markets (and in other countries and currencies). Consequently, consumers at a demand market, in a sense, also compete with consumers at other demand markets.

Table 7.5 Demand functions

Notation	Definition
$d_{kh\hat{l}}(\rho 3)$	The demand for the product at demand market k transacted in currency h in country $\hat{l}$ as a function of the demand market price vector

We now turn to describing the behavior of the various economic decision-makers. The model is presented, for ease of exposition, for the case of a single homogeneous product. It can also handle multiple products through a replication of the links and added notation. We first focus on the manufacturers. We then turn to the retailers, and, subsequently, to the consumers at the demand markets.

7.2.1 The Behavior of the Manufacturers

The manufacturers are involved in the production of a homogeneous product and in transacting with the retailers. Furthermore, they are also involved in establishing the corresponding relationship levels. The quantity of the product produced by manufacturer il must satisfy the following conservation of flow equation:

$$q^{il} = \sum_{j=1}^{J} \sum_{h=1}^{H} q_{jh}^{il}, \qquad (7.1)$$

which states that the quantity of the product produced by manufacturer il is equal to the sum of the quantities transacted between the manufacturer and all retailers.

Hence, in view of (7.1), and as noted in Table 7.2, we have that for each manufacturer il the production cost $f^{il}(q) = f^{il}\left(q^{il}\right)$.

Each manufacturer il tries to maximize his profits. He faces total costs that equal the sum of his production cost plus the total transaction costs and the costs that he incurs in establishing and maintaining his relationship levels. His revenue, in turn, is equal to the sum of the price that he can obtain times the exchange rate multiplied by quantities of the product transacted. Furthermore, each manufacturer tries to minimize his risk generated by interacting with the retailers subject to his individual weight assignment to this criterion.

7.2.2 The Multicriteria Decision-Making Problem Faced by a Manufacturer

We can now construct the multicriteria decision-making problem facing a manufacturer which allows him to weight the criteria of profit maximization and risk minimization in an individual manner. Manufacturer il's multicriteria decision-making objective function is denoted by U^{il}. Assume that manufacturer il assigns a nonnegative weight α^{il} to the risk generated. The weight associated with profit maximization serves as the numeraire and is set equal to 1. The nonnegative weights measure the importance of the risk and, in addition, transform it value into monetary units. Let now ρ_{1jh}^{il*} denote the actual price charged by manufacturer il for the product in currency h to retailer j. We later discuss how such price is recovered. We can now construct a value function for each manufacturer (cf. [8, 23]) using a constant additive weight value function. Therefore, the multicriteria decision-making problem of manufacturer il can be expressed as:

$$\text{Maximize} \quad U^{il} = \sum_{j=1}^{J} \sum_{h=1}^{H} (\rho_{1jh}^{il*} \times e_h) q_{jh}^{il} - f^{il}(q^{il}) - \sum_{j=1}^{J} \sum_{h=1}^{J} c_{jh}^{il}(q_{jh}^{il}, \eta^{il})$$
$$- b^{il}(\eta^{il}) - \alpha^{il} r^{il}(Q^1, \eta^1) \tag{7.2}$$

subject to:

$$q_{jh}^{il} \geq 0, \forall j, h\,, \tag{7.3}$$
$$0 \leq \eta^{il} \leq 1\,. \tag{7.4}$$

The first four terms on the right-hand side of the equal sign in (7.2) represent the profit which is to be maximized, the next term represents the weighted total risk which is to be minimized. The relationship values lie in the range between 0 and 1 and, hence, we need constraint (7.4).

7.2.3 The Optimality Conditions of Manufacturers

Here we assume that the manufacturers compete in a noncooperative fashion follow-
ing Nash [29, 30]. Hence, each manufacturer seeks to determine his optimal strate-
gies, that is, product transactions, given those of the other manufacturers. The opti-
mality conditions of all manufacturers i; $i = 1, \ldots, I$; in all countries: l; $l = 1, \ldots,$
L simultaneously, under the above assumptions (cf. [2, 8, 27]), can be compactly ex-
pressed as: determine $(Q^{1*}, \eta^{1*}) \in k^1$, satisfying

$$
\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{j=1}^{J}\sum_{h=1}^{H}\left[\frac{\partial f^{il}(q^{il*})}{\partial q_{jh}^{il}} + \frac{\partial c_{jh}^{il}(q_{jh}^{il*}, \eta^{il*})}{\partial q_{jh}^{il}} + \alpha^{il}\frac{\partial r^{il}(Q^{1*}, \eta^{1*})}{\partial q_{jh}^{il}}\right.
$$
$$
\left. -\rho_{1jk}^{il*} \times e_h\right] \times \left[q_{jh}^{il} - q_{jh}^{il*}\right]
$$
$$
+ \sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{j=1}^{J}\sum_{h=1}^{H}\left[\frac{\partial c_{jh}^{il}(q_{jh}^{il*}, \eta^{il*})}{\partial \eta^{il}} + \frac{\partial b^{il}(\eta^{il*})}{\partial \eta^{il}}\right.
$$
$$
\left. +\alpha^{il}\frac{\partial r^{il}(Q^{1*}, \eta^{1*})}{\partial \eta^{il}}\right] \times \left[\eta^{il} - \eta^{il*}\right] \geq 0, \quad \forall (Q^1, \eta^1) \in k^1,
$$

(7.5)

where
$$
k^1 \equiv \left[(Q^1, \eta^1)|q_{jh}^{il} \geq 0, \ 0 \leq \eta^{il} \leq 1, \quad \forall i, l, j, h\right].
$$
(7.6)

The inequality (7.5), which is a variational inequality (cf. [27]) has a meaningful
economic interpretation. From the first term in (7.5) we can see that, if there is a pos-
itive volume of the product transacted from a manufacturer to a retailer, then the
marginal cost of production plus the marginal cost of transacting plus the weighted
marginal cost of risk must be equal to the price (times the exchange rate) that the
retailer is willing to pay for the product. If that sum, in turn, exceeds that price then
there will be no product transacted.

The second term in (7.5) show that if there is a positive relationship level (and
that level is less than one) then the marginal cost associated with the level is equal
to the marginal reduction in transaction costs plus the weighted marginal reduction
in risk.

7.2.4 The Behavior of the Retailers

The retailers (cf. Fig. 7.1), in turn, are involved in transactions both with the manu-
facturers in the different countries, as well as with the ultimate consumers associated
with the demand markets for the product in different countries and currencies and

represented by the bottom tier of nodes in both the global supply chain network and the social network.

As in the case of manufacturers, the retailers have to bear some costs to establish and maintain relationship levels with manufacturers and with the consumers, who are the ultimate purchasers/buyers of the product. Furthermore, the retailers have associated transaction costs in regards to transacting with the manufacturers, which we assume can be dependent on the type of currency as well as the manufacturer. Retailers also are faced with risk in their transactions. As in the case of the manufacturers, the transaction cost functions and the risk functions depend on the amounts of the product transacted as well as the relationship levels.

Each retailer j tries to maximize profits and to minimize his individual risk associated with his transactions with these criteria weighted in an individual fashion.

7.2.5 A Retailer's Multicriteria Decision-Making Problem

Retailer j assigns a nonnegative weight δ^j to his risk. The weight associated with profit maximization is set equal to 1 and serves as the numeraire (as in the case of the manufacturers). The actual price charged for the product by retailer j is denoted by $\rho_{2kh\hat{l}}^{j*}$, and is associated with transacting with consumers at demand market k in currency h and country $\hat{l}$. Later, we discuss how such prices are arrived at. We are now ready to construct the multicriteria decision-making problem faced by a retailer which combines the individual weights with the criteria of profit maximization and risk minimization. Let U^j denote the multicriteria objective function associated with retailer j with his multicriteria decision-making problem expressed as:

Maximize

$$
U^j = \sum_{k=1}^{K} \sum_{h=1}^{H} \sum_{\hat{l}=1}^{L} (\rho_{2kh\hat{l}}^{j*} \times e_h) q_{kh\hat{l}}^j - c_j(Q^1) - \sum_{i=1}^{I} \sum_{l=1}^{L} \sum_{h=1}^{H} \hat{c}_{jh}^{il}(q_{jh}^{il}, \eta^j)
$$
$$
- \sum_{k=1}^{K} \sum_{h=1}^{H} \sum_{\hat{l}=1}^{L} c_{kh\hat{l}}^j(q_{kh\hat{l}}^j, \eta^j) - b^j(\eta^j) - \sum_{i=1}^{I} \sum_{l=1}^{L} \sum_{h=1}^{H} (\rho_{1jh}^{il*} \times e_h) q_{jh}^{il}
$$
$$
- \delta^j r^j(Q^1, Q^2, \eta^2) \tag{7.7}
$$

subject to:

$$
\sum_{k=1}^{K} \sum_{h=1}^{H} \sum_{\hat{l}=1}^{L} q_{kh\hat{l}}^j \le \sum_{i=1}^{I} \sum_{l=1}^{L} \sum_{h=1}^{H} q_{jh}^{il}, \tag{7.8}
$$

$$
q_{jh}^{il} \ge 0, \ q_{kh\hat{l}}^j \ge 0, \quad \forall i, l, k, h, \hat{l}, \tag{7.9}
$$

$$
0 \le \eta^j \le 1, \ \forall j. \tag{7.10}
$$

The first six terms on the right-hand side of the equal sign in (7.7) represent the profit which is to be maximized, the next term represents the weighted risk which is to be minimized. Constraint (7.8) states that consumers cannot purchase more of the product from a retailer than is held "in stock".

7.2.6 The Optimality Conditions of Retailers

We now turn to the optimality conditions of the retailers. Each retailer faces the multicriteria decision-making problem (7.7), subject to (7.8), the nonnegativity assumption on the variables (7.9), and the assumptions for the relationship values (7.10). As in the case of the manufacturers, we assume that the retailers compete in a noncooperative manner, given the actions of the other retailers. Retailers seek to determine the optimal transactions associated with the demand markets and with the manufacturers. In equilibrium, all the transactions between the tiers of the decision-makers will have to coincide, as we will see later in this section.

If one assumes that the handling, transaction cost, and risk functions are continuously differentiable and convex, then the optimality conditions for all the retailers satisfy the variational inequality: determine $(Q^{1*}, Q^{2*}, \eta^{2*}, \lambda^*) \in k^2$, such that

$$
\sum_{j=1}^{J} \sum_{i=1}^{I} \sum_{l=1}^{L} \sum_{h=1}^{H} \left[\delta^j \frac{\partial r^{j*}}{\partial q_{jh}^{il}} + \frac{\partial c_j(Q^{1*})}{\partial q_{jh}^{il}} + \rho_{1jh}^{il*} \times e_h + \frac{\partial \hat{c}_{jh}^{il}(q_{jh}^{il*}, \eta^{j*})}{\partial q_{jh}^{il}} - \lambda_j^* \right]
$$
$$
\times \left[q_{jh}^{il} - q_{jh}^{il*} \right]
$$

$$
+ \sum_{j=1}^{J} \sum_{k=1}^{K} \sum_{h=1}^{H} \sum_{\hat{l}=1}^{L} \left[\delta^j \frac{\partial r^{j*}}{\partial q_{kh\hat{l}}^{j}} + \frac{\partial c_{kh\hat{l}}^{j}(q_{kh\hat{l}}^{j*}, \eta^{j*})}{\partial q_{kh\hat{l}}^{j}} + \rho_{2kh\hat{l}}^{j*} \times e_h + \lambda_j^* \right]
$$
$$
\times \left[q_{kh\hat{l}}^{j} - q_{kh\hat{l}}^{j*} \right]
$$

$$
+ \sum_{j=1}^{J} \sum_{k=1}^{K} \sum_{h=1}^{H} \sum_{\hat{l}=1}^{L} \left[\delta^j \frac{\partial r^{j*}}{\partial \eta^j} + \frac{\partial c_{kh\hat{l}}^{j}(q_{kh\hat{l}}^{j*}, \eta^{j*})}{\partial \eta^j} + \frac{\partial \hat{c}_{jh}^{il}(q_{jh}^{il*}, \eta^{j*})}{\partial \eta^j} \right.
$$
$$
\left. + \frac{\partial b^j(\eta^{j*})}{\partial \eta^j} \right] \times [\eta^j - \eta^{j*}]
$$

$$
+ \sum_{j=1}^{J} \left[\sum_{i=1}^{I} \sum_{l=1}^{L} \sum_{h=1}^{H} q_{jh}^{il*} - \sum_{k=1}^{K} \sum_{h=1}^{H} \sum_{l=1}^{L} q_{kh\hat{l}}^{j*} \right] \times [\lambda_j - \lambda_j^*] \geq 0,
$$

$$
\forall (Q^1, Q^2, \eta^2, \lambda) \in k^2, \tag{7.11}
$$

where $r^{j*} = r^j(Q^1, Q^2, \eta^{2*})$ and

$$k^2 \equiv \left[(Q^1, Q^2, \eta^2, \lambda) | q_{jh}^{il} \geq 0, q_{kh\hat{l}}^j \geq 0, 0 \leq \eta^j \leq 1, \lambda_j \geq 0, \quad \forall i, l, j, h, k, \hat{l} \right].$$
(7.12)

Here λ_j denotes the Lagrange multiplier associated with constraint (7.8) and λ is the column vector of all the retailers' Lagrange multipliers. These Lagrange multipliers can also be interpreted as shadow prices. Indeed, according to the fourth term in (7.11), λ_j^* serves as the price to "clear the market" at retailer j.

The economic interpretation of the retailers' optimality conditions is very interesting. The first term in (7.11) states that if there is a positive amount of product transacted between a manufacturer/retailer pair and currency h, that is, $q_{jh}^{il*} > 0$, then the shadow price at the retailer, λ_j^*, is equal to the price charged for the product plus the various marginal costs and the associated weighted marginal risk. In addition, the second term in (7.11) shows that, if consumers at demand market $kh\hat{l}$ purchase the product from a particular retailer j, which means that, if the $q_{kh\hat{l}m}^{j*}$ is positive, then the price charged by retailer j, $\rho_{2kh\hat{l}}^{j*}$, is equal to λ_j^* plus the marginal transaction costs in dealing with the demand market and the weighted marginal costs for the risk that he has to bear. One also obtains interpretations from (7.11) as to the economic conditions at which the relationship levels associated with retailers interacting with either the manufacturers or the demand markets will take on positive values.

7.2.7 The Consumers at the Demand Markets

We now describe the consumers located at the demand markets. The consumers can transact through with the retailers. The consumers at demand market k in country $\hat{l}$ take into account the price charged for the product transacted in currency h by retailer j, which is denoted by $\rho_{2kh\hat{l}}^{j*}$ and the exchange rate, plus the transaction costs, in making their consumption decisions. The equilibrium conditions for demand market $kh\hat{l}$, thus, take the form: for all retailers: $j = 1, \ldots, J$, demand markets k; $k = 1, \ldots, K$; and currencies: h; $h = 1, \ldots, H$

$$\rho_{2kh\hat{l}}^{j*} \times e_h + \hat{c}_{kh\hat{l}}^j(Q^{2*}, \eta^{2*}) \begin{cases} = \rho_{3kh\hat{l}}^*, & \text{if } q_{kh\hat{l}}^{j*} > 0, \\ \geq \rho_{3kh\hat{l}}^*, & \text{if } q_{kh\hat{l}}^{j*} = 0, \end{cases}$$
(7.13)

In addition, we must have that for all $k, h, \hat{l}$

$$d_{kh\hat{l}}(\rho_3^*) \begin{cases} = \sum_{j=1}^{J} q_{kh\hat{l}}^{j*}, & \text{if } \rho_{3kh\hat{l}}^* > 0, \\ \leq \sum_{j=1}^{J} q_{kh\hat{l}}^{j*}, & \text{if } \rho_{3kh\hat{l}}^* = 0, \end{cases}$$
(7.14)

Conditions (7.13) state that consumers at demand market $kh\hat{l}$ will purchase the product from retailer j, if the price charged by the retailer for the product times the exchange rate plus the transaction cost (from the perspective of the consumer) does not exceed the price that the consumers are willing to pay for the product in that currency and country, i.e., $\rho^*_{3kh\hat{l}}$. Note that, according to (7.13), if the transaction costs are identically equal to zero, then the price faced by the consumers for a given product is the price charged by the retailer for the particular product.

Condition (7.14), on the other hand, states that, if the price the consumers are willing to pay for the product at a demand market is positive, then the quantity of the product at the demand market is precisely equal to the demand.

In equilibrium, conditions (7.13) and (7.14) will have to hold for all demand markets and these, in turn, can be expressed also as an inequality analogous to those in (7.5) and (7.11) and given by:

determine $(Q^{2*}, \rho^*_3) \in R^{(J+1)KHL}_+$, such that

$$\sum_{j=1}^{J} \sum_{k=1}^{K} \sum_{h=1}^{H} \sum_{\hat{l}=1}^{L} \left[\rho^{j*}_{2kh\hat{l}} \times e_h + \hat{c}^j_{kh\hat{l}}(Q^{2*}, \eta^{2*}) - \rho^*_{3kh\hat{l}} \right] \times \left[q^j_{kh\hat{l}} - q^{j*}_{kh\hat{l}} \right]$$

$$+ \sum_{k=1}^{K} \sum_{h=1}^{H} \sum_{\hat{l}=1}^{L} \left[\sum_{j=1}^{J} q^{j*}_{kh\hat{l}} - d_{kh\hat{l}}(\rho^*_3) \right] \times \left[\rho_{3kh\hat{l}} - \rho^*_{3kh\hat{l}} \right] \geq 0 ,$$

$$\forall (Q^2, \rho_3) \in R^{(J+1)KHL}_+ . \tag{7.15}$$

In the context of the consumption decisions, we have utilized demand functions, whereas profit functions, which correspond to objective functions, were used in the case of the manufacturers and the retailers. Since we can expect the number of consumers to be much greater than that of the manufacturers and retailers we believe that such a formulation is more natural. Also, note that the relationship levels in (7.15) are assumed as given. They are endogenous to the integrated model as is soon revealed.

7.2.8 The Equilibrium Conditions of the Network

In equilibrium, the product flows that the manufacturers in different countries transact with the retailers must coincide with those that the retailers actually accept from them. In addition, the amounts of the product that are obtained by the consumers in the different countries and currencies must be equal to the amounts that the retailers actually provide. Hence, although there may be competition between decision-makers at the same level of tier of nodes of the network there must be cooperation between decision-makers associated with pairs of nodes. Thus, in equilibrium, the prices and product transactions must satisfy the sum of the optimality conditions

(7.5) and (7.11) and the equilibrium conditions (7.15). We make these statements rigorous through the subsequent definition and variational inequality derivation.

Definition 1 (Network Equilibrium). The equilibrium state of the global supply chain network is one where the product transactions and relationship levels between the tiers of the network coincide and the product transactions, relationship levels, and prices satisfy the sum of conditions (7.5), (7.11), and (7.15).

The equilibrium state is equivalent to the following:

Theorem 1 (Variational Inequality Formulation). The equilibrium conditions governing the global supply chain network model according to Definition 1 are equivalent to the solution of the variational inequality given by:
determine $(Q^{1*}, Q^{2*}, \eta^{1*}, \eta^{2*}, \lambda^*, \rho_3^*) \in k$, satisfying:

$$\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{j=1}^{J}\sum_{h=1}^{H}\left[\frac{\partial f^{il}(q^{il*})}{\partial q_{jh}^{il}} + \frac{\partial c_{jh}^{il}(q_{jh}^{il*},\eta^{il*})}{\partial q_{jh}^{il}} + \frac{\partial \hat{c}_{jh}^{il}(q_{jh}^{il*},\eta^{j*})}{\partial q_{jh}^{il}} + \frac{\partial c_j(Q^{1*})}{\partial q_{jh}^{il}}\right.$$

$$\left. + \alpha^{il}\frac{\partial r^{il}(Q^{1*},\eta^{1*})}{\partial q_{jh}^{il}} + \delta^j\frac{\partial r^j(Q^{1*},Q^{2*},\eta^{2*})}{\partial q_{jh}^{il}} - \lambda_j^*\right] \times \left[q_{jh}^{il} - q_{jh}^{il*}\right]$$

$$+ \sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\left[\delta^j\frac{\partial r^j(Q^{1*},Q^{2*},\eta^{2*})}{\partial q_{kh\hat{l}}^{j}} + \frac{\partial c_{kh\hat{l}}^{j}(q_{kh\hat{l}}^{j*},\eta^{j*})}{\partial q_{kh\hat{l}}^{j}}\right.$$

$$\left. + \hat{c}_{kh\hat{l}}^{j}(Q^{2*},\eta^{2*}) + \lambda_j^* - \rho_{3kh\hat{l}}^*\right] \times \left[q_{kh\hat{l}}^{j} - q_{kh\hat{l}}^{j*}\right]$$

$$+ \sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{j=1}^{J}\sum_{h=1}^{H}\left[\frac{\partial c_{jh}^{il}(q_{jh}^{il*},\eta^{il*})}{\partial \eta^{il}} + \frac{\partial b^{il}(\eta^{il*})}{\partial \eta^{il}} + \alpha^{il}\frac{\partial r^{il}(Q^{1*},\eta^{1*})}{\partial \eta^{il}}\right]$$

$$\times \left[\eta^{il} - \eta^{il*}\right] + \sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\left[\delta^j\frac{\partial r^j(Q^{1*},Q^{2*},\eta^{2*})}{\partial \eta^j} + \frac{\partial c_{kh\hat{l}}^{j}(q_{kh\hat{l}}^{j*},\eta^{j*})}{\partial \eta^j}\right.$$

$$\left. + \frac{\partial \hat{c}_{jh}^{il}(q_{jh}^{il*},\eta^{j*})}{\partial \eta^j} + \frac{\partial b^j(\eta^{j*})}{\partial \eta^j}\right] \times \left[\eta^j - \eta^{j*}\right] + \sum_{j=1}^{J}\left[\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{h=1}^{H}q_{jh}^{il*}\right.$$

$$\left. - \sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}q_{kh\hat{l}}^{j*}\right] \times \left[\lambda_j - \lambda_j^*\right] + \sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\left[\sum_{j=1}^{J}q_{kh\hat{l}}^{j*} - d_{kh\hat{l}}(\rho_3^*)\right]$$

$$\times \left[\rho_{3kh\hat{l}} - \rho_{3kh\hat{l}}^*\right] \geq 0, \quad \forall(Q^1,Q^2,\eta^1,\eta^2,\lambda,\rho_3) \in k, \tag{7.16}$$

where

$$k \equiv \Big[(Q^1, Q^2, \eta^1, \eta^2, \lambda, \rho_3) | q_{jh}^{il} \geq 0, q_{kh\hat{i}}^{j} \geq 0, 0 \leq \eta^{il} \leq 1 \ ,$$

$$0 \leq \eta^{j} \leq 1, \rho_{3kh\hat{i}} \geq 0, \lambda_j \geq 0, \ \forall i, l, j, h, k, \hat{l} \Big] \ . \tag{7.17}$$

Proof. Summation of inequalities (7.5), (7.11), and (7.14), yields, after algebraic simplification, the variational inequality (7.15).

We now put variational inequality (7.15) into standard form which will be utilized in the subsequent sections. For additional background on variational inequalities and their applications, see the book by Nagurney [27]. In particular, we have that variational inequality (7.15) can be expressed as:

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in k \ , \tag{7.18}$$

Where

$$X \equiv (Q^1, Q^2, \eta^1, \eta^2, \lambda, \rho_3) \text{ and } F(X) \equiv (F_{iljh}, F_{jkh\hat{i}}, \hat{F}_{iljh}, \hat{F}_{jkh\hat{i}}, F_j, F_{kh\hat{i}})$$

With indices: $i = 1, \ldots, I; l = 1, \ldots, L; j = 1, \ldots, J; h = 1, \ldots, H; \hat{l} = 1, \ldots, L;$, and the specific components of F given by the functional terms preceding the multiplication signs in (7.14), respectively. The term $\langle \cdot, \cdot \rangle$ denotes the inner product in N-dimensional Euclidean space.

We now describe how to recover the prices associated with the first two tiers of nodes in the global supply chain network. Such a pricing mechanism guarantees that the optimality conditions (7.5) and (7.11) as well as the equilibrium conditions (7.14) are satisfied individually through the solution of variational inequality (7.16).

Clearly, the components of the vector ρ_3^* are obtained directly from the solution of variational inequality ((7.16) as will be demonstrated explicitly through several numerical examples in Sect. 7.5). In order to recover the second tier prices associated with the retailers and the appreciation/exchange rates one can (after solving variational inequality (7.16) for the particular numerical problem) either (cf. (7.13) or (7.15)) set $\rho_{2kh\hat{i}}^{j*} \times e_h = \Big[\rho_{3kh\hat{i}}^{*} - \hat{c}_{kh\hat{i}}^{j}(Q^{2*}, \eta^{2*})\Big]$, for any $j, k, h, \hat{l}, m$ such that $q_{kh\hat{i}}^{j*} > 0$ or (cf. (7.11)) for any $q_{kh\hat{i}}^{j*} > 0$, set

$$\rho_{2kh\hat{i}}^{j*} \times e_h = \Big[\delta^j \frac{\partial r^j(Q^{1*}, Q^{2*}, \eta^{2*})}{\partial q_{kh\hat{i}}^{j}} + \frac{\partial c_{kh\hat{i}m}^{j}(q_{kh\hat{i}}^{j*}, \eta^{j*})}{\partial q_{kh\hat{i}}^{j}} + \lambda_j^* \Big].$$

Similarly, from (7.5) we can infer that the top tier prices can be recovered (once the variational inequality (7.16) is solved with particular data) thus:

for any i, l, j, h, m, such that $q_{jh}^{il*} > 0$, set $\rho_{1jh}^{il*} \times e_h = \left[\frac{\partial f^{il}(q^{il*})}{\partial q_{jh}^{il}} + \alpha^{il} \frac{\partial r^{il}(Q^{1*}, \eta^{1*})}{\partial q_{jh}^{il}} + \frac{\partial c_{jh}^{il}(q_{jh}^{il*}, \eta^{il*})}{\partial q_{jh}^{il}} \right]$, or

equivalently, (cf. (7.11)), to $\left[\lambda_j^* - \delta^j \frac{\partial r^j(Q^{1*},Q^{2*},\eta^{2*})}{\partial q_{jh}^{il}} - \frac{\partial c_j(Q^{1*})}{\partial q_{jh}^{il}} - \frac{\partial \hat{c}_{jh}^{il}(q_{jh}^{il*},\eta^{j*})}{\partial q_{jh}^{il}} \right]$.

Under the above pricing mechanism, the optimality conditions (7.5) and (7.11) as well as the equilibrium conditions (7.15) also hold separately (as well as for each individual decision-maker) (see also, e. g., [8, 28]).

Existence of a solution to variational inequality (7.16) follows from the standard theory of variational inequalities, under the assumption that the functions are continuous, since the feasible set k is compact (cf. [27]). Also, according to the theory of variational inequalities, uniqueness of solution, in turn, is then guaranteed, provided that the function $F(X)$ that enters variational inequality (7.16) is strictly monotone on k.

Note that, if the equilibrium values of the flows (be they product or relationship levels) on links are identically equal to zero, then those links can effectively be removed from the network (in equilibrium). Moreover, the size of the equilibrium flows represents the "strength" of the respective links. In addition, the solution of the model reveals the true network structure in terms of the optimal relationships (and their sizes) as well as the optimal product transactions, and the associated prices.

7.2.9 Remark

We note that manufacturers as well as retailers may be faced with capacity constraints. Capacity limitations can be handled in the above model since the production cost functions, as well as the transaction cost functions and the handling cost functions can assume nonlinear forms (as is standard in the case of modeling capacities on roads in congested urban transportation networks (cf. [33])). Of course, one can also impose explicit capacity constraints and this would then just change the underlying feasible set(s) so that k would need to be redefined accordingly. However, the function $F(X)$ in variational inequality (7.18) would remain the same (see, e. g., [27]). Finally, since we consider a single homogeneous product the exchange rates e_h are assumed fixed (and relative to a base currency). Once can, of course, investigate numerous exchange rate and demand scenarios by altering the demand functions and the fixed exchange rates and then recomputing the new equilibrium product transaction, price, and relationship level equilibrium patterns.

7.3 The Supply Chain Network Efficiency and Vulnerability Measures

In this section, we propose the global supply chain network efficiency measure and the associated network component importance definition.

Definition 2 (The global supply chain network efficiency measure). The global supply chain network efficiency measure, ε, for a given network topology G, and demand price $\rho_{3kh\hat{l}}$, and available product from manufacturer il and retailer j, is defined as follows:

$$\varepsilon(G) = \frac{\sum_{k=1}^{K} \sum_{h=1}^{H} \sum_{\hat{l}=1}^{L} \frac{d_{kh\hat{l}}(\rho_3^*)}{\rho_{3kh\hat{l}}^*}}{K * H * L} , \tag{7.19}$$

where $K * H * L$ is the number of demand markets in the network, and $d_{kh\hat{l}}(\rho_3^*)$ and $\rho_{3kh\hat{l}}^*$ denote the equilibrium demand and the equilibrium price for demand market $kh\hat{l}$, respectively.

The global supply chain network efficiency measure, ε defined in (7.19) is actually the average demand to price ratio (cf. [32]). It measures the overall (economic) functionality of the global supply chain network. When the network topology G, the demand price functions, and the available product are given, a global supply chain network is considered to be more efficient if it can satisfy higher demand at lower prices.

By referring to the equilibrium conditions (7.13), we assume that if there is a positive transaction between a retailer with a demand market at the equilibrium, the price charged by the retailer plus the respective unit transaction costs is always positive. Hence, the prices paid by the demand market will always be positive and the above network efficiency measure is well-defined.

The importance of the network components is analyzed, in turn, by studying their impact on the network efficiency through their removal. The network efficiency of a global supply chain network can be expected to deteriorate when a critical network component is eliminated from the network. Such a component can include a link or a node or a subset of nodes and links depending on the network problem under study. Furthermore, the removal of a critical network component will cause more severe damage than that of a trivial one. Hence, the importance of a network component is defined as follows (cf. [32]):

Definition 3 (Importance of a global supply chain network component). The importance of network component g of global supply chain network G, $I(g)$, is measured by the relative global supply chain network efficiency drop after g is removed from the network:

$$I(g) = \frac{\varepsilon(G) - \varepsilon(G - g)}{\varepsilon(G)} , \tag{7.20}$$

where $\varepsilon(G - g)$ is the resulting global supply chain network efficiency after component g is removed.

It is worth pointing out that the above importance of the network components is well-defined even in a supply chain network with disconnected manufacturer/demand market pairs. In our supply chain network efficiency measure, the elimination of a transaction link is treated by removing that link from the network

while the removal of a node is managed by removing the transaction links entering or exiting that node. The above procedure(s) to handle disconnected manufacturer/demand market pairs will be illustrated in the numerical examples in Sect. 7.4, when we compute the importance of the supply chain network components and their rankings.

Supply chain vulnerability arises from two sources, the risk within the supply chain and the external risk. The risk within the supply chain network is caused by sub-optimal interaction and co-operation between the entities along the chain. Such supply chain risks result from a lack of visibility, lack of 'ownership' and inaccurate forecasts. External risks are risks that arise from interactions between the supply chain and its environment. Such interactions include disruptions caused by strikes, terrorism and natural catastrophes. These risks impact the vulnerability of the supply chain. Thus, supply chain vulnerability can be defined as an exposure to serious disturbance, arising from risks within the supply chain as well as risks external to the supply chain.

Hence, vulnerability (ε_A) is a measure of the average decrease in efficiency of the network over the time horizon after attack and takes into account the entire history of the attack and the rapidity of the decline [3].

$$\varepsilon_A = \frac{\sum_{n=1}^{m} \varepsilon_n}{N} \tag{7.21}$$

Let ε_1, ε_2, ..., ε_N, be the efficiency the network after the elimination of 1, 2, up to N nodes. ε_A measures the average loss of efficiency during N attacks so that networks with a more rapid decline will have higher ε_A scores.

Consequently, supply chain risk management should aims at identifying the areas of potential risk and implementing appropriate actions to contain that risk. Therefore, we believe that the best way to mitigate supply chain risk and to reduce supply chain vulnerability as a whole is through coordinated and relationship approach amongst supply chain members.

7.4 Numerical Examples

In this section, we analyze the global supply chain networks efficiency and vulnerability. For each example, our network efficiency and vulnerability measures are computed and the importance and the rankings of nodes are also reported.

Example 1

The first set of numerical examples consisted of one country, two manufacturers, two currencies, two retailers, and two demand markets for the product. Hence, $L = 1$, $I = 2$, $H = 2$, $J = 2$, and $K = 2$, for this and the subsequent two numerical exam-
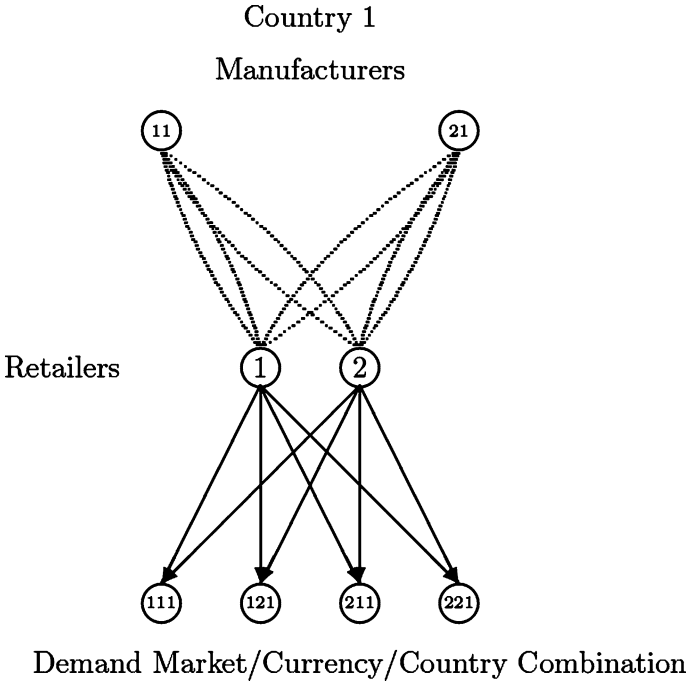
Fig. 7.2 Global supply chain network for Example 1

ples. The global supply chain network for the first example is depicted in Fig. 7.2. The examples below were solved using the Euler method (see [8, 28]).

The data for the first example were constructed for easy interpretation purposes (cf. Tables 7.2, 7.3, 7.5)

The transaction cost functions faced by the manufacturers associated with transacting with the retailers were given by:

$$c_{jh}^{il}(q_{jh}^{il}, \eta^{il}) = 0.5 \left(q_{jh}^{il}\right)^2 + 3.5q_{jh}^{il} - \eta^{il}, \text{ for } i = 1, 2; l = 1; j = 1, 2; h = 1, 2.$$

The production cost functions faced by the manufacturers were

$$f^{il}(q^{il}) = 0.5 \left(\sum_{j=1}^{2} \sum_{h=1}^{2} q_{jh}^{il}\right)^2, \quad \text{for } i = 1, 2; l = 1, 2.$$

The handling costs of the retailers were given by:

$$c_j(Q^1) = .5 \left(\sum_{j=1}^{2} \sum_{h=1}^{2} q_{jh}^{il}\right)^2, \quad \text{for } j = 1, 2.$$

The transaction costs of the retailers associated with transacting with the manufacturers in the two countries were given by:

$$\hat{c}_{jh}^{il}\left(q_{jh}^{il},\eta^{il}\right) = 1.5\left(q_{jh}^{il}\right)^2 + 3q_{jh}^{il}, \quad \text{for } i = 1,2; l = 1,2; j = 1,2; h = 1,2 \,.$$

The relationship cost functions were:

$$b^{il}(\eta^{il}) = 2\eta^{il}, \quad \text{for } i = 1,2,$$
$$b^{j}(\eta^{j}) = \eta^{j}, \quad \text{for } j = 1,2.$$

The demand functions at the demand markets were:

$$d_{111}(\rho_3) = -2\rho_{3111} - 1.5\rho_{3121} + 1000, \quad d_{121}(\rho_3) = -2\rho_{3121} - 1.5\rho_{3111} + 1000,$$
$$d_{211}(\rho_3) = -2\rho_{3211} - 1.5\rho_{3221} + 1000, \quad d_{221}(\rho_3) = -2\rho_{3221} - 1.5\rho_{3211} + 1000.$$

and the transaction costs between the retailers and the consumers at the demand markets (see (6.12)) were given by:

$$\hat{c}_{khl}^{j}(Q^2) = q_{khl}^{j} - \eta^{j} + 5, \quad \text{for } j = 1,2; k = 1,2; h = 1,2; l = 1\,.$$

We assumed for this and the subsequent examples that the transaction costs as perceived by the retailers and associated with transacting with the demand markets were all zero, that is, $c_{khl}^{j}(q_{khl}^{j}) = 0$, for all j,k,h,l.

The Euler method converged and yielded the following equilibrium product shipment pattern:

$$q_{jh}^{il*} = 15.605, \quad \forall i,l,j,h\,, \quad q_{khl}^{j*} = 15.605, \quad \forall j,k,h,l\,.$$

The vector γ^* had components: $\gamma_1^* = \gamma_2^* = 256.190$, and the computed demand prices at the demand markets were: $\rho_{3111}^* = \rho_{3121}^* = \rho_{3211}^* = \rho_{3221}^* = 276.797$.

Next, we analyze the importance of the network components by studying their impact on the network efficiency through their removal. Table 6 shows the results of this analysis.

Table 7.6 The importance of the supply chain decision-makers

Nodes	Importance Value	Ranking
Manufacturer (il)	0.3204	2
Retailer (j)	0.4854	1
Demand Market (khl)	−0.3086	3

Note that, given the cost structure and the demand price functions, since the retailer's nodes carry the largest amount of equilibrium product flow, they are ranked the most important. The negative importance values for demand markets are due to

the fact that the existence of each demand market brings extra flows on the transaction links and nodes and, therefore, increases the marginal transaction cost. The removal of one demand market has two effects: first, the contribution to the network performance of the removed demand market becomes zero; second, the marginal transaction cost on links/nodes decreases, which decreases the equilibrium prices and increases the demand at the other demand markets. If the performance drop caused by the removal of the demand markets is overcompensated by the improvement of the demand-price ratio of the other demand markets, the removed demand market will have a negative importance value. It simply implies that the negative externality caused by the demand market has a larger impact than the performance drop due to its removal.

The vulnerability score for this supply chain network is $\varepsilon_A = 0.1657$.

Example 2

The first numerical example consisted of one country, two source agents, two currencies, two intermediaries, and two financial products. Hence, $L = 1$, $I = 2$, $H = 2$, $J = 2$, and $K = 2$. The network for the first example is depicted in Fig. 7.3. The data for this example is the same as in Example 1 except that in this example we allow the manufacturers to also transact directly with demand markets.
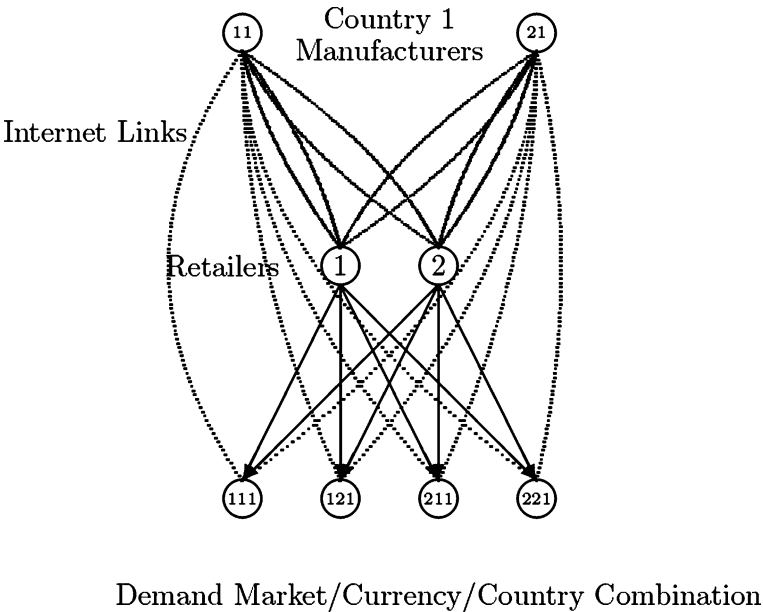

Fig. 7.3 Global supply chain network for Example 2

The transaction costs, in turn, associated with the transactions between manufacturers and the demand markets (from the perspective of the consumers) were given by:

$$\hat{c}_{kh\hat{\imath}}^{il}(q_{kh\hat{\imath}}^{il}) = 0.1 q_{kh\hat{\imath}}^{il} + 1, \quad \forall i, l, \hat{l}, k, h.$$

The Euler method converged and yielded the following equilibrium supply chain flow pattern:

$$q_{jh}^{il*} = 2.017, \quad \forall i, l, j, h,$$

$$q_{khl}^{il*} = 26.82, \quad \forall i, l, k, h, l,$$

$$q_{khl}^{j*} = 2.017, \quad \forall j, k, h, l.$$

The vector γ^* had components: $\gamma_1^* = \gamma_2^* = 268.58$, and the computed demand prices at the demand markets were: $\rho_{3111}^* = \rho_{3121}^* = \rho_{3211}^* = \rho_{3221}^* = 269.236$. The importance of the nodes of this network and their ranking are reported in Table 7.7.

For this network the most important nodes are the manufacturers nodes since they transact and have relationships with retailers and demand markets. The vulnerability score for this supply chain network is $\varepsilon_A = 0.019$.

Table 7.7 The importance of the supply chain decision-makers

Nodes	Importance Value	Ranking
Manufacturer (il)	0.486	1
Retailer (j)	−0.03	2
Demand Market (khl)	−0.19	3

Example 3

In this numerical example, the global supply chain network was as given in Fig. 7.4. This example consisted of two countries with two manufacturers in each country; two currencies, two retailers, and two demand markets. Hence, $L = 2$, $I = 2$, $H = 2$, $J = 2$, and $K = 2$.

The data for this example is the replication of the data for Example 1, from 1 country to two countries. Therefore, the demand functions at the demand markets were:

$$d_{111}(\rho_3) = -2\rho_{3111} - 1.5\rho_{3121} + 1000, \quad d_{121}(\rho_3) = -2\rho_{3121} - 1.5\rho_{3111} + 1000,$$

$$d_{211}(\rho_3) = -2\rho_{3211} - 1.5\rho_{3221} + 1000, \quad d_{221}(\rho_3) = -2\rho_{3221} - 1.5\rho_{3211} + 1000,$$

$$d_{112}(\rho_3) = -2\rho_{3112} - 1.5\rho_{3122} + 1000, \quad d_{122}(\rho_3) = -2\rho_{3122} - 1.5\rho_{3112} + 1000,$$

$$d_{212}(\rho_3) = -2\rho_{3212} - 1.5\rho_{3222} + 1000, \quad d_{222}(\rho_3) = -2\rho_{3222} - 1.5\rho_{3212} + 1000.$$

Country 1   Country 2

Manufacturers   Manufacturers

Demand Market/Currency/Country Combination

Fig. 7.4 Global supply chain network for Example 3

The Euler method converged (in 318 iterations) and yielded the following equilibrium product shipment pattern:

$$q_{jh}^{il*} = 16.305 , \quad \forall i, l, j, h , \quad q_{khl}^{j*} = 16.305 , \quad \forall j, k, h, l .$$

The vector γ^* had components: $\gamma_1^* = \gamma_2^* = 266.882$ and the computed demand prices at the demand markets were: $\rho_{3111}^* = \rho_{3121}^* = \rho_{3211}^* = \rho_{3221}^* = \rho_{3212}^* = \rho_{3222}^* = 277.379$.

The importance of the nodes of this network and their ranking are reported in Table 7.8. The vulnerability score for this supply chain network is $\varepsilon_A = 0.060$. This network is less vulnerable than the one from Example 1, since the retailers can get their products from many more manufacturers and sell them to many more demand markets as compare to the retailers in Example 1.

Table 7.8 The importance of the supply chain decision-makers

Nodes	Importance Value	Ranking
Manufacturer (il)	0.18	2
Retailer (j)	0.42	1
Demand Market (khl)	-0.307	3

We note that supply chain in Example 1 is the most vulnerable, follow by the network in Example 3. A vulnerable supply chain would have a high vulnerability score. Network in Example 2 is the least vulnerable since manufacturers have more options to sell their products, to retailers and demand market. The results in this chapter can be used to assess which nodes and links in supply chain networks are the most vulnerable in the sense that their removal will impact the performance of the network in the most significant way. Moreover, for each member of the supply chain network the highest is it relationship or connectivity the lowest is its vulnerability. Therefore, supply chain risk management should be based on clear performance requirements and lines of communication or relationship between all members of the chain. It is important to note that the vulnerability of the supply chain depends on the design and structure of the network and on the type of relationship between its members.

7.5 Managerial Implications

As managers determine the appropriate practices to manage the supply chain efficiency and vulnerability, the supply chain design and structure should take in consideration risk issues. When considering risk issues it is important to the firm to create a portfolio of relationships. Strong supply chain relationships enable firms to react to changes in the market, improve forecasting and create supply chain visibility, which lead to improve profit margins [11]. The benefits are reduction of production, transportation and administrative costs. Moreover, multiple relationships can help companies deal with the negative consequences related to dependence on supply chain partners. Thus, supply chain vulnerability can be minimized as an exposure to serious disturbance is reduced. Companies will be able to deal with disturbance arising from risks within the supply chain as well as risks external to the supply chain when there is an optimal interaction and co-operation between the entities along the chain.

The benefits of investing in social relationship are not only economical but also technical and social. On the technical development the greatest benefit is the possibility of sharing the resources of suppliers and shortening the lead-times. Spekman and Davis [35] found that supply chain networks that exhibit collaborative behaviors tend to be more responsive and that supply chain-wide costs are, hence, reduced. These results are also supported by our economic model where we demonstrated that a higher level of relationship lowers transaction costs and risk and uncertainty. As a result, supply chain prices are reduced and the overall transactions increases.

Risk management in supply chain should be based on clear performance requirements, optimal level of investment in supply chain relationships, and on process alignment and cooperation within and between the entities in the supply chain. Therefore, the structure and design of the supply chain will determine how vulnerable it will be.

7.6 Conclusions

In this chapter, we develop a framework for the analysis of the optimal levels of social relationship in a global supply chain network consisting of manufacturers, retailers, and consumers. We propose a network performance measure for the evaluation of supply chain networks efficiency and vulnerability. The measure captures risk, transaction cost, price, transaction flow, revenue, and demand information in the context of the decision-makers behavior. Manufacturers and retailers are multicriteria decision-makers who decide about their production and transaction quantities as well as the amount of social relationship they want to pursue in order to maximize net return and minimize risk.

We construct the finite-dimensional variational inequality governing the equilibrium of the competitive global supply chain network. The model allows us to investigate the interplay of the heterogeneous decision-makers in the global supply chain and to compute the resultant equilibrium pattern of product outputs, transactions, product prices, and levels of social relationship. A computational procedure that exploits the network structure of the problem is applied to several numerical examples.

Results of our numerical examples highlight the importance of considering the impact of relationship levels in a global supply chain context. Furthermore, they stress the importance of a network perspective, the importance of each decision makers in the network and the overall efficiency and vulnerability of the global supply chain network . These examples, although stylized, have been presented to show both the model and the computational procedure. Obviously, different input data and dimensions of the problems solved will affect the equilibrium product transaction, levels of social relationship, price patterns, and the supply chain vulnerability.

Future research will extend this framework to include other criteria and the introduction of dynamics. Future research may also focus on the study of type of relationship investments required in each stages of the supply chain network. It is important to know to what extent do the relationships between elements in supply chain differ depending on the stage of the alliance and on aspects of the costs and benefits of the relationship. How do the combinations of these elements play in each time period and stage of the supply chain? These questions await empirical study. We feel we have only scratched the surface and look forward to future studies that will help researchers better conceptualize and theorize specific aspect of supply chain relationship, vulnerability and risk management.

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