

# **DISTORTION OF BAR CODE SIGNALS IN LASER SCANNING**

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## **1 Introduction**

One of the most common methods of reading bar codes is to scan them with a laser beam and process the received optical signal. The received light is proportional to the integral of the beam profile multiplied by the region of the bar code covered by the beam. The finite spot size of the laser beam acts as a filter on the original bilevel bar code signal. This filtering can cause intersymbol interference between adjacent edges of the bar code signal. An edge detector attempts to reconstruct the original bilevel signal from this filtered signal. The edges detected in the filtered waveform are in different locations than those in the original bilevel bar code signal resulting in a distortion of the bar code signal. If this distortion is too large, it can prevent the bar code from being properly decoded. Thus, there is a limit on the spot size of the laser beam that can be used to scan a given density bar code.

The laser beam profile that is most often used in bar code scanners can be approximated by a Gaussian distribution. The width of the beam changes depending on the distance from the scanner to the bar code.<sup>1</sup> As the size of the laser beam becomes too large, relative to the bars and spaces of the bar code, it becomes impossible to read the bar code. Thus, the working range (i.e. the range of distances between the laser scanner and the bar code) over which the bar code is readable is limited by this growth in the spot size and the maximum spot size that can be tolerated.

In this paper a detailed analysis of the distortion of bar code signals caused by the finite spot size of the laser beam is performed. In Section 2 a general method of analysis is proposed for determining the edge shifts in the bar code signal due to the filtering of the optical beam. In Section 3 the technique is applied to a comprehensive study of one of the most popular bar code symbologies, Universal Product Code (UPC). A measure of distortion is proposed which relates directly to the readability of the bar code. A study of which UPC bar code characters are most sensitive to the edge shift is performed. The distortion of the most sensitive UPC characters is shown as a function of the optical spot size.

## 2 Edge Shifts of Gaussian Filtered Bilevel Signal

A bar code is a sequence of vertical bars and spaces where the information stored in the bar code is encoded in the relative widths of the bars and spaces.<sup>2</sup> The specific pattern of the bars and spaces depends on the coding method used, often referred to as the symbology, as well as the actual data that is encoded in the bar code. In this paper we will deal with the most common type of bar code the Universal Product Code (UPC).<sup>3</sup>

The bar code is scanned by a flying spot laser scanner. The scanner receives the laser signal reflecting off the bar code. The laser beam is assumed to scan at a fixed velocity and have a Gaussian beam profile. The received signal is a time domain function that results from scanning the bar code. Mathematically the bar code can be represented, in the time domain, as a sequence of step functions of alternating polarity,

$$x(t) = \sum_{i=1}^N (-1)^{(i-1)} u(t - T_i) \quad (1)$$

where  $u(t)$  is the unit step function and the  $T_i$  represents the time when the signal transitions from either black to white or white to black (i.e. the edges). Here we assume that the edges are ordered:  $T_1 < T_2 < \dots < T_N$ . The width of the bars and spaces are given by the difference between adjacent edges (e.g.  $T_{i+1} - T_i$ ). This bar code signal,  $x(t)$ , is filtered by the optical spot which has a Gaussian impulse response,

$$h(t) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-t^2/2\sigma^2} \quad (2)$$

where the standard deviation (in the time domain),  $\sigma$ , is just the standard deviation in the spatial domain divided by the velocity,  $\sigma = \sigma_x/v$ . It is often common to refer to the *spot size* of the optical beam which is given by  $4\sigma$ , in the time domain (or  $4\sigma_x$  is the spatial domain). The received signal is the convolution of the bar code signal and the optical beam,<sup>4</sup>

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau) h(t - \tau) d\tau. \quad (3)$$

Given the received signal,  $y(t)$ , there are numerous methods of detecting the edge locations and reconstructing an estimate of the original bar code signal. One of the more common methods is to detect the inflection-point of the signal which is given by the zero crossing of the second derivative. The edge locations are given by the solutions to the equation:  $\ddot{y}(t) = 0$ . Due to the linearity of the convolution equation we can write,

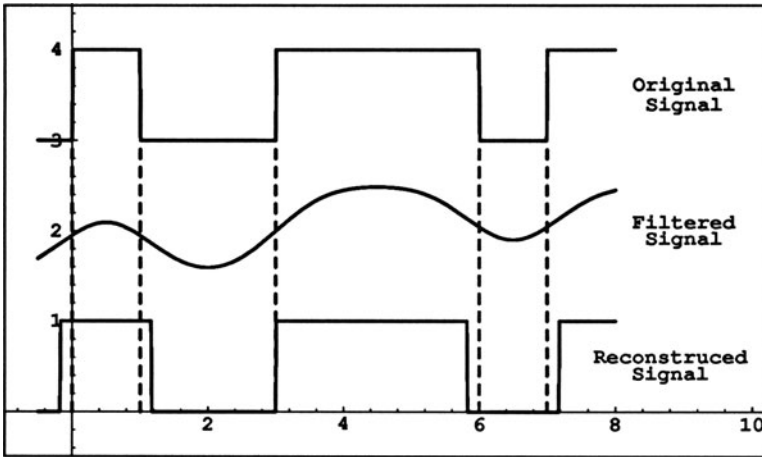
$$\ddot{y}(t) = \dot{x}(t) * \dot{h}(t). \quad (4)$$

Since  $x(t)$  is a sum of unit step functions of alternating polarity then  $\dot{x}(t)$  is a sum of impulses of alternating polarity,

$$\dot{x}(t) = \sum_{i=1}^N (-1)^{(i-1)} \delta(t - T_i). \quad (5)$$

Substituting this into Eq. (4), simplifying, and equating to zero gives,

$$\ddot{y}(t) = \sum_{i=1}^N (-1)^{(i-1)} \dot{h}(t - T_i) = 0. \quad (6)$$



**Figure 1.** Original Bar Code Signal, Filtered Signal, and the Reconstructed Signal, with  $\sigma = 0.6$

Substituting the derivative of  $h(t - T_i)$  into Eq. (6) gives the following equation for the edge locations,

$$\sum_{i=1}^N (-1)^{(i-1)} (t - T_i) e^{-((t-T_i)^2/2\sigma^2)} = 0. \quad (7)$$

Since this equation is nonlinear it is best solved using a numerical root finding routine like that available in Mathematica.<sup>5</sup> In finding the root of a nonlinear equation it is always necessary to have an initial estimate to initiate the search. Here, the search is initialized at the original edge locations,  $T_i$ . In solving Eq. (7) we obtain the inflection-points of  $y(t)$  which are estimates of the edges in  $x(t)$ . If we call these inflection-points  $\hat{T}_i$  then the reconstructed bar code signal is given by,

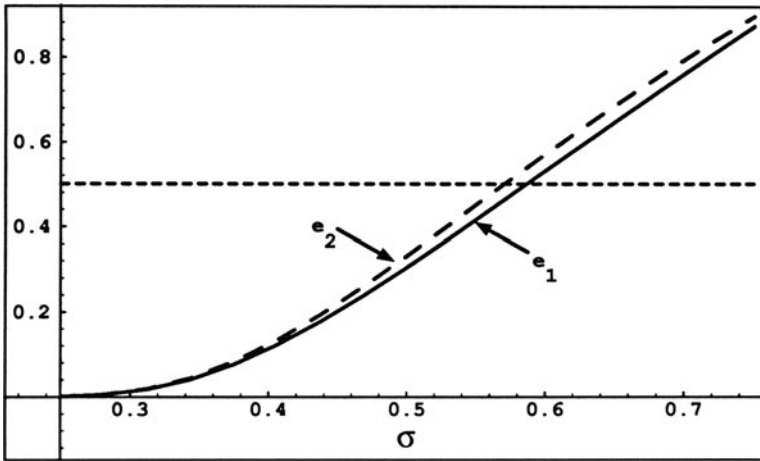
$$\hat{x}(t) = \sum_{i=1}^N (-1)^{(i-1)} u(t - \hat{T}_i) \quad (8)$$

Figure 1 shows a segment of a UPC bar code scanned by a Gaussian beam with  $\sigma = 0.6$ . Shown are the original bilevel bar code signal, the Gaussian filtered signal, and the reconstructed bilevel signal.

### 3 Distortion of UPC Characters

In this section the method of determining the shift in the edges due to Gaussian filtering is applied to the UPC bar code symbology. A UPC bar code consists of a sequence of characters surrounded by guard bars. Each UPC character is composed of two bars and two spaces. The width of the bars and spaces are multiples of a unit width called a *module*, and they sum to seven modules.<sup>2,3</sup> Each UPC character can encode a single digit from 0 to 9. The objective in scanning the bar code is to process the received signal and determine what character combination was originally printed.

Once there is a method of determining the edge locations of the Gaussian filtered signal, several questions can be posed which relate to bar code scanning. First, which character is most distorted by the optical beam? Second, for what value of  $\sigma$  is the



**Figure 2.** The Solid Line is  $e_1$  for Character “3”, and the Dashed Line is  $e_2$  of Character “5”, both as a Function of  $\sigma$  for a Unit Module Size

distortion sufficiently large so that the character is no longer decodable? In order to answer both of these questions, it is necessary to introduce some measure of distortion. The proposed distortion measure is directly related to the decodability of the UPC characters.

It is convenient to group the widths of the two bars and two spaces representing a UPC character into a vector. For example, the bar space pattern for the UPC character “7” can be written in vector notation as  $\mathbf{u} = (1\ 3\ 1\ 2)^T$ , where the components of  $\mathbf{u}$  are the widths of the bars and spaces. In the decoding procedure it is useful to deal with the vector represented by the sum of adjacent elements. The reason for this is that the sum of adjacent elements (one bar and one space) is not effected by uniform ink spread.<sup>3</sup> The vector representing the sums of adjacent pairs,  $\mathbf{v}$ , is given by  $v_i = u_i + u_{i+1}$  for  $i = 1, 2, 3$ . For example, the UPC character “7” represented this way is  $\mathbf{v} = (4\ 4\ 3)^T$ . In some of the literature the first vector,  $\mathbf{u}$ , consisting of the widths of the bars and spaces is referred to as the x-sequence and the second vector,  $\mathbf{v}$ , consisting of sums of adjacent pairs is referred to as the t-sequence.

The standard decode procedure for a UPC character begins by normalizing the measured widths of the bars and spaces so they sum to seven. The widths of the bars and spaces are obtained from the locations of the detected edges,  $\hat{T}_i$ . We define this as the normalized vector  $\hat{\mathbf{u}}$ . This is an estimate of the original vector  $\mathbf{u}$  representing the UPC character. An estimate of  $\mathbf{v}$  can easily be obtained adding adjacent pairs of elements of  $\hat{\mathbf{u}}$ , namely  $\hat{v}_i = \hat{u}_i + \hat{u}_{i+1}$  for  $i = 1, 2, 3$ . Next, each component of estimate vector, either  $\hat{\mathbf{u}}$  or  $\hat{\mathbf{v}}$ , is rounded off to the nearest integer and that vector is compared to the patterns for the different characters. For most characters the  $\mathbf{v}$  vector is used since it is insensitive to ink spread; but, for some characters it is necessary to use the  $\mathbf{u}$  vector.

Therefore, if any of the components of  $\hat{\mathbf{u}}$  or  $\hat{\mathbf{v}}$  differ from the component of actual character,  $\mathbf{u}$  or  $\mathbf{v}$ , by more than 0.5 then the character will not be decoded. This suggests the use of an  $\infty$ -norm<sup>6</sup> in defining a distortion measure for UPC characters,

$$e_1 \triangleq \|\hat{\mathbf{u}} - \mathbf{u}\|_{\infty} = \max_i |\hat{u}_i - u_i| \quad (9)$$

$$e_2 \triangleq \|\hat{\mathbf{v}} - \mathbf{v}\|_\infty = \max_i |\hat{v}_i - v_i|. \quad (10)$$

The first type of error applies to decoding when using the vector of bar and space widths; while the second applies when using the vector composed of the sum of adjacent element widths. So, in general, both are necessary.

An exhaustive study was performed to find the UPC characters that were most sensitive to the Gaussian filtering. In doing this study, it was necessary to not only consider the current UPC character but also the adjacent UPC characters since the Gaussian filtering causes the edges in those characters to interfere with the current character. So for each character it was necessary to consider all possible adjacent characters. Actually, only those edges that are within one to two modules of the current character have any effect. In comparing different UPC characters, it was determined that character "3" has the highest  $e_1$  error, and character "5" has the highest  $e_2$  error, for a given spot size. It turns out that some characters have large errors of one type but a relatively small error of the other type, so it is necessary to consider both types of errors. To summarize the results Figure 2 shows a plot of the errors (with the worst case adjacent characters) for these two worst case characters as a function of  $\sigma$ , where the module size is assumed to be unity (i.e.  $\sigma$  has been normalized by the module size). The dotted line at 0.5 shows the threshold at which the characters become undecodable. This occurs at about  $\sigma = 0.58$ .

## 4 Conclusions

A method of determining the edge location of a Gaussian filtered bilevel signal is proposed. The method is applied to a systematic study of distortion of UPC bar code characters. A measure of distortion of a UPC bar code is introduced which is derived from the standard decode procedure. The characters that are most sensitive to this type of systematic distortion were identified and the distortion of those characters was plotted as a function of the optical spot size.

This method could be used to study other bar code symbologies to determine which bar code symbologies are most sensitive to this type of distortion. Also, other distortion measures could be introduced to study the merits of different decoding procedures.

## References

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