

Chapter 8

Concluding Remarks

We have endeavored in this book to give the reader a look at the ever-evolving Bergman theory. Some of the results here are 90 years old, and others were proved quite recently.

The Bergman's ideas have proved to be remarkably robust and fruitful. They continue to yield new techniques and new paths for research. They have played a key role in the development of complex geometry and of partial differential equations in one and several complex variables, in complex function theory, and in extremal problems. The biholomorphic invariance of the Bergman kernel and metric has proved to be particularly important.

We have with pleasure presented the ideas connected with the Bergman representative coordinates. This is a much underappreciated aspect of the Bergman theory, and one that deserves further development. The proof of the Lu Qi-Keng's theorem serves to illustrate what a powerful idea it is. It also played a decisive role in the original proof of the Greene–Krantz semicontinuity theorem.

Fefferman's work on biholomorphic mappings of strictly pseudoconvex domains gives yet another illustration of the centrality and power of the Stefan Bergman's ideas. The Fefferman's asymptotic expansion has proved to be one of the central ideas in the modern function theory of several complex variables.

The work of Greene and Krantz has also served to illustrate the geometric force of the Bergman kernel and Bergman metric. Krantz and Li, and Kim and Krantz, have developed these ideas even further.

We look forward to many years of future activity in the Bergman geometric theory and the Bergman function theory.