ERRATUM

Errata and Comments

The following ‘red’ letters are errata that should be corrected (or inserted).

Chapter 1: Mathematical Descriptions and Models

1. Page 5: Eq. (1.10),

$$\begin{aligned} & y(k+n)+\cdots+a_{n-1}y(k+1)+a_n y(k) \\ & \quad = b_0 u(k+n)+\cdots+b_{n-1}u(k+1)+b_n u(k) \end{aligned}$$

2. Page 6: Eq. (1.18),

$$y(k)=\begin{bmatrix} b_n-a_n b_0 & b_{n-1}-a_{n-1}b_0 & \ldots & b_1-a_1 b_0 \end{bmatrix}\begin{bmatrix} x_1(k) \\ x_2(k) \\ \vdots \\ x_n(k) \end{bmatrix}+b_0 u(k).$$

3. Fig. 1.5: Time sequences of the solution for Example 1.6
4. Below Fig. 1.5: Note that the response is delayed by one step as shown in Fig. 1.5 if $y(k+1)=x_1(k)$ (i.e., $y(k)=x_1(k-1)$) is applied to the computer program for (1.55). This response corresponds to the result of Exercise (7).
5. Page 7: Eq. (1.21),

$$\boldsymbol{x}(k)=\boldsymbol{A}\boldsymbol{x}(k-1)+\boldsymbol{B}u(k-1),$$

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6. Page 13: Eq. (1.35),

$$
\begin{aligned}
&y(k+n)+\cdots+a_{n-1}y(k+1)+a_n y(k)\\
&\quad = b_0 u(k+n)+\cdots+b_{n-1}u(k+1)+b_n u(k).
\end{aligned}
$$

7. Page 13:

$$
\begin{cases}
\mathcal{Z}\{y(k+n)\} = z^n \hat{y}(z) - (y(0)z^n + \cdots + y(n-1)z)\\
\qquad \cdots\\
\mathcal{Z}\{a_{n-1}y(k+1)\} = a_{n-1}(z\hat{y}(z) - y(0)z)\\
\mathcal{Z}\{a_n y(k)\} = a_n \hat{y}(z).
\end{cases}
$$

8. Page 13: For simplicity, the initial conditions are assumed to be zero (i.e., $y(0) = y(1) = \cdots = y(n-1) = 0$ and also $u(0) = u(1) = \cdots = u(n-1) = 0$).

   Comments:
   There would be no contradiction, because $y(\kappa)$ and $u(\kappa)$ in (1.35) defined for $\kappa = k+n$ $(\kappa \geq n)$. However, in a computer simulation, backward expressions (1.19), (1.21), and (1.40) should be used.

9. Page 13: Eq. (1.36),

$$
(z^n + a_1 z^{n-1} + \cdots + a_{n-1}z + a_n)\hat{y}(z) = (b_0 z^n + b_1 z^{n-1} + \cdots + b_{n-1}z + b_n)\hat{u}(z).
$$

10. Page 14: Eq. (1.38), The z-transform with respect to $\kappa = k+2$ is given as

$$
(z^2 - z + 0.5)\hat{y}(z) - y(0)z^2 - y(1)z + y(0)z = (z+1)\hat{u}(z) - u(0)z.
$$

and

$$
(z^2 - z + 0.5)\hat{y}(z) - y(0)z^2 - y(1)z + y(0)z = \frac{z(z+1)}{z-1} - u(0)z
$$

11. Page 18:

$$
\frac{z^2+z)}{z^3 - 2z^2 + 1.5z - 0.5} \quad \Rightarrow \quad \frac{z^2+z}{z^3 - 2z^2 + 1.5z - 0.5}
$$

12. Page 18:

$$
\begin{cases}
\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} -a_1 & 1 \\ -a_2 & 0 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} u(k),\\
y(k) = x_1(k), \quad \text{where } a_1 = -1,\ a_2 = 0.5,\ \text{and } b_1 = b_2 = 1
\end{cases}
$$

   Tips:
   The z-transforms of the above equations are given as:

$$
\begin{bmatrix} z-1 & -1 \\ 0.5 & z \end{bmatrix} \begin{bmatrix} \hat{x}_1(z) \\ \hat{x}_2(z) \end{bmatrix} = \begin{bmatrix} \hat{u}(z) \\ \hat{u}(z) \end{bmatrix}
$$

Then,

$$\begin{bmatrix} \hat{x}_1(z) \\ \hat{x}_2(z) \end{bmatrix} = \frac{1}{z^2 - z + 0.5} \begin{bmatrix} z & 1 \\ 0.5 & z-1 \end{bmatrix} \begin{bmatrix} z/(z-1) \\ z/(z-1) \end{bmatrix}.$$

Thus,

$$\hat{y}(z) = \hat{x}_1(z) = \frac{z^2 + z}{(z^2) - z + 0.5)(z-1)} = \frac{z^2 + z}{z^3 - 2z^2 + 1.5z - 0.5}.$$

13. Page 19: The caption in Fig. 1.6,
Fig. 1.6 Block diagram for Example 1.6, where $a_1 = 1$, $a_2 = -0.5$, and $b_1 = b_2 = 1$
14. Page 23: Eq. (1.68),

$$G_1(z) := \tilde{\mathcal{Z}}\{G_1(s)\} = \frac{K_0}{1 - z^{-1}} + \frac{K_1}{1 - \mathrm{e}^{p_1 h} z^{-1}} + \cdots + \frac{K_n}{1 - \mathrm{e}^{p_n h} z^{-1}}.$$

15. Page 24: In Table 1.2, The fourth line in 'Discrete time',

$$\mathrm{e}^{p_k h} \quad \Rightarrow \quad kh\mathrm{e}^{p_k h}$$

16. Page 25: Eq. (1.76),

$$\boldsymbol{\Phi}(\tau) := \mathrm{e}^{\boldsymbol{A}\tau} = \boldsymbol{I} + \boldsymbol{A}\tau + \frac{\boldsymbol{A}^2\tau^2}{2!} + \cdots,$$

where

$$\boldsymbol{I} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

17. Page 26: Eqs. (1.78), (1.79), and (1.80),
My first manuscript was written as follows:

$$\begin{cases} \boldsymbol{x}(k+1) = \boldsymbol{\Phi}(h)\boldsymbol{x}(k) + \boldsymbol{\Gamma}(h)u(k), \quad \boldsymbol{\Gamma}(h) = \int_0^h \boldsymbol{\Phi}(\tau)\boldsymbol{B}\mathrm{d}\tau \\ y(k) = \boldsymbol{C}\boldsymbol{x}(k). \end{cases}$$

$$\begin{cases} z\hat{\boldsymbol{x}}(z) = \boldsymbol{\Phi}\hat{\boldsymbol{x}}(z) + \boldsymbol{\Gamma}\hat{u}(z) \\ \hat{y}(z) = \boldsymbol{C}\hat{\boldsymbol{x}}(z). \end{cases}$$

$$\hat{y}(z) = \boldsymbol{C}[\boldsymbol{I} - \boldsymbol{\Phi}z^{-1}]^{-1}\boldsymbol{\Gamma}z^{-1}\hat{u}(z).$$

These expressions might be preferable to (1.78), (1.79), and (1.80).

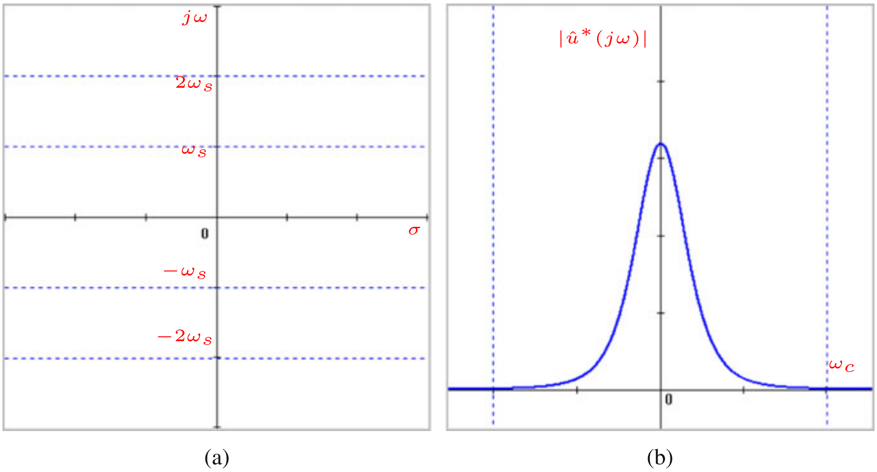

Fig. 1.23 Frequency shifting and spectrum

Chapter 2: Discretized Feedback Systems

1. Page 50: Eq. (2.7),

$$-\beta e^2 \le g(e)e \le \beta e^2.$$

2. Page 56: In Eq. (2.25),

$$e^{*\dagger}(z) \quad \Rightarrow \quad \hat{e}^{*\dagger}(z)$$

3. Page 69: Eq. (2.62),

$$\sum_{k,l=1,k\neq l}^{N} |x(k)|^2|y(l)|^2 - 2\sum_{k,l=1,k\neq l}^{N} |x(k)y(k)|\cdot|x(l)y(l)|$$

$$= \sum_{k,l=1,k\neq l}^{N} |x(k)y(l) - x(l)y(k)|^2,$$

4. Page 71: In the last line
$\cdots$ and the inequality problem is proved.

Chapter 3: Robust Stability Analysis

1. Page 75: Eq. (3.8),

$$\|e(k)\|_2 \le \|r(k)\|_2 + \sup_{|z|=1} |G(z)|(\rho\cdot\|e(k)\|_2 + \|d(k)\|_2).$$

2. Page 94: Eq. (3.46) in Theorem 3.3,

$$\eta(q_0, \omega_0) - \max_{q} \min_{\omega} \eta(q, \omega)$$

3. Page 94: The verification of robust stability using the above modified Hall diagram (off-axis M-circles) is based on the following theorem.

Chapter 4: Model Reference Feedback and PID Control

1. Page 114: In Eqs. (4.16), (4.17), (4.20), and (4.21),

$$G_I(s) \quad \Rightarrow \quad G_1(s)$$
$$G_I(z) \quad \Rightarrow \quad G_1(z).$$

2. Page 117: In Eq. (4.22),

$$\hat{f}(z) \quad \Rightarrow \quad \mathcal{F}(z).$$

3. Page 117: In the first line under Fig. 12,
$\cdots$, characteristic equation of the nominal feedback system is given as
4. Page 132:

$$\tilde{y}_3^{(3)} = \tilde{y}_3^{(2)} - \frac{a_{32}^{(2)}}{a_{22}^{(2)}} \tilde{y}_2^{(2)}, \quad \Rightarrow \quad \tilde{y}_3^{(3)} = \tilde{y}_3^{(1)} - \frac{a_{31}^{(1)}}{a_{11}^{(1)}} \tilde{y}_1^{(1)} - \frac{a_{32}^{(2)}}{a_{22}^{(2)}} \tilde{y}_2^{(2)},$$

5. Page 140: In Exercise (3), determine the characteristic equation of the nominal system, $\mathcal{F}(z) = 0$, for Example 4.1 (A) $\cdots$.
6. Page 140: (4) Show that the approximate PID control system in Fig. 4.18 is obtained from the model-reference feedback system in Fig. 4.17, when $\mathcal{D}_m(\cdot)$ and $\mathcal{D}_f(\cdot)$, are in high resolution.

Chapter 5: Multi-Loop Feedback Systems

1. Page 153:

$$\tilde{y}_n^{(n)} = \tilde{y}_n^{(n-1)} - \frac{a_{n,n-1}^{(n-1)}}{a_{n-1,n-1}^{(n-1)}} y_{n-1}^{(n-1)}$$

$$\Rightarrow \quad \tilde{y}_n^{(n)} = \tilde{y}_n^{(1)} - \frac{a_{n1}^{(1)}}{a_{11}^{(1)}} \tilde{y}_1^{(1)} - \frac{a_{n2}^{(2)}}{a_{22}^{(2)}} \tilde{y}_2^{(2)} - \cdots - \frac{a_{n,n-1}^{(n-1)}}{a_{n-1,n-1}^{(n-1)}} \tilde{y}_{n-1}^{(n-1)}$$

2. Page 153: $\cdots$, where $y_j^{(j)} \quad \Rightarrow \quad \cdots$, where $0 < \tilde{y}_j^{(j)} \leq y_j^{(j)}$

3. Page 153: $\cdots$ all principal minors of matrix $\mathcal{A}$ $\Rightarrow$ $\cdots$ all principal minors of matrix (5.15)
4. Page 163:

$$\tilde{y}_n^{(n)} = \tilde{y}_n^{(n-1)} - \frac{a_{n,n-1}^{(n-1)}}{a_{n-1,n-1}^{(n-1)}} y_{n-1}^{(n-1)}$$

$$\Rightarrow \quad \tilde{y}_n^{(n)} = \tilde{y}_n^{(1)} - \frac{a_{n1}^{(1)}}{a_{11}^{(1)}}\tilde{y}_1^{(1)} - \frac{a_{n2}^{(2)}}{a_{22}^{(2)}}\tilde{y}_2^{(2)} - \cdots - \frac{a_{n,n-1}^{(n-1)}}{a_{n-1,n-1}^{(n-1)}}\tilde{y}_{n-1}^{(n-1)}$$

5. Page 163: $\cdots$, where $y_j^{(j)}$ $\Rightarrow$ $\cdots$, where $0 < \tilde{y}_j^{(j)} \le y_j^{(j)}$
6. Page 163: $\cdots$ all principal minors of matrix $\mathcal{A}$ $\Rightarrow$ $\cdots$ all principal minors of matrix (5.48)
7. Page 177:

$$y_j \ge 0,\ J = 1, 2, \cdots, n \text{ and } a_{ij} \le 0,\ i \ne j$$

$$\Rightarrow \quad \tilde{y}_j \ge 0,\ j = 1, 2, \cdots, n \text{ and } a_{ij} \le 0,\ i \ne j$$

Chapter 6: Interval Polynomials and Robust Performance

1. Page 186: Equation (6.17) should be written as follows:

$$\begin{aligned}
&\tilde{F}(z) = D_{c1}(z)D_{c2}(z)D_{11}(z)D_{22}(z)D_{12}(z)D_{21}(z)\\
&+[K_1^-, K_1^+]N_{c1}(z)N_{11}(z)D_{c2}(z)D_{22}(z)D_{12}(z)D_{21}(z)\\
&+[K_2^-, K_2^+]N_{c2}(z)N_{22}(z)D_{c1}(z)D_{11}(z)D_{12}(z)D_{21}(z)\\
&+[K^-, K^+]N_{c1}(z)N_{c2}(z)N_{11}(z)N_{22}(z)D_{12}(z)D_{21}(z)\\
&-[K^-, K^+]N_{c1}(z)N_{c2}(z)N_{12}(z)N_{21}(z)D_{11}(z)D_{22}(z) = 0.
\end{aligned}$$

2. Page 191: Fig. 6.3 $\cdots$ for discrete control system $\Rightarrow$ Fig. 6.3 $\cdots$ for discrete control systems
3. Page 192:

$$\phi = \tan^{-1}\left(\frac{-\gamma + 2(1+\gamma^2)\theta - \gamma(1+\gamma^2)\theta^2}{1-(1+\gamma^2)\theta^2}\right)$$

The proof of Lemma 6.1 is given as follows:

$$x^2 + (y-\gamma)^2 = \frac{[1-(1+\gamma^2)\theta^2]^2 + [-\gamma + 2\theta(1+\gamma^2) - \gamma(1+\gamma^2)\theta^2]^2}{[1-2\gamma\theta + (1+\gamma^2)\theta^2]^2}$$

$$= \frac{(1+\gamma^2)[1+(1+\gamma^2)\theta^4 - 2\theta^2 + 4(1+\gamma^2)\theta^2 + \gamma^2(1+\gamma^2)\theta^4 - 4\gamma\theta - 4\gamma(1+\gamma^2)\theta^3 + 2\gamma^2\theta^2]}{[1-2\gamma\theta + (1+\gamma^2)\theta^2]^2}$$

$$= \frac{(1+\gamma^2)[1+4\gamma^2\theta^2+(1+\gamma^2)^2\theta^4-4\gamma\theta-4\gamma(1+\gamma^2)\theta^3+2(1+\gamma^2)\theta^2]}{[1-2\gamma\theta+(1+\gamma^2)\theta^2]^2} = 1+\gamma^2.$$

Thus, Lemma 6.1 has been proved. □

4. Page 204:

$$\tilde{F}(s) = [a_0^-, a_0^+]s^3 + [a_1^-, a_1^+]s^2 + [a_2^-, a_2^+]s + [a_3^-, a_3^+]$$

Chapter 7: Relation to Discrete Event Systems

1. Page 228: **Fig. 7.6** Petri net systems ⇒ **Fig. 7.6** Petri net systems
   (In the following figures, transitions τ_i are written in t_i)
2. Page 232: In (7.21),

$$\forall x0 \in S(\mathcal{X}_m; r) \quad \Rightarrow \quad \forall x_0 \in S(\mathcal{X}_m; r)$$

3. Page 234: Their notation is ⋯ ⇒ The notations are ⋯
4. Page 234:

$$\|x_i(t_k)\|_{\ell_1} = \sum_{k=0}^{\infty} |x_i(t_k)|$$

and

$$\|\boldsymbol{x}(t_k)\|_{\ell_1} = \begin{bmatrix} \|x_1(t_k)\|_{\ell_1} \\ \|x_2(t_k)\|_{\ell_1} \\ \vdots \\ \|x_n(t_k)\|_{\ell_1} \end{bmatrix}.$$

5. Page 235: In (7.28) and (7.33),

$$\boldsymbol{\Psi}(t_k)) \quad \Rightarrow \quad \boldsymbol{\Psi}(\boldsymbol{t_k})$$

6. Page 236:

$$\boldsymbol{I} - \left(\sum_{l=1}^{k} |\boldsymbol{\Phi}(t_k, t_l)|\right)\bar{\boldsymbol{\psi}} \quad \Rightarrow \quad \boldsymbol{I} - \left(\sum_{l=1}^{k} |\boldsymbol{\Phi}(t_k, t_l)|\right)\bar{\boldsymbol{\Psi}}$$

7. Page 237: ⋯ and three events. ⇒ ⋯ and two events.
8. Page 239:

$$\|\boldsymbol{x}(t_k)\|_{\ell_p} = \begin{bmatrix} \|x_1(t_k)\|_{\ell_p} \\ \|x_2(t_k)\|_{\ell_p} \\ \vdots \\ \|x_n(t_k)\|_{\ell_p} \end{bmatrix}$$

9. Page 239: The equations for continuous-time systems should be corrected as follows:

$$\|x_i(\tau)\|_{L_p} = \left(\int_0^\infty |x_i(\tau)|^p \mathrm{d}\tau\right)^{1/p}.$$

and

$$\|\boldsymbol{x}(\tau)\|_{L_p} = \begin{bmatrix} \|x_1(\tau)\|_{L_p} \\ \|x_2(\tau)\|_{L_p} \\ \vdots \\ \|x_n(\tau)\|_{L_p} \end{bmatrix},$$

furthermore,

$$\|\boldsymbol{\Psi}(\tau)\|_{L_1} = \begin{bmatrix} \int_0^\infty |\psi_{11}(\tau)|\mathrm{d}\tau & \int_0^\infty |\psi_{12}(\tau)|\mathrm{d}\tau & \dots & \int_0^\infty |\psi_{1n}(\tau)|\mathrm{d}\tau \\ \int_0^\infty |\psi_{21}(\tau)|\mathrm{d}\tau & \int_0^\infty |\psi_{22}(\tau)|\mathrm{d}\tau & \dots & \int_0^\infty |\psi_{2n}(\tau)|\mathrm{d}\tau \\ \vdots & \vdots & \ddots & \vdots \\ \int_0^\infty |\psi_{n1}(\tau)|\mathrm{d}\tau & \int_0^\infty |\psi_{n2}(\tau)|\mathrm{d}\tau & \dots & \int_0^\infty |\psi_{nn}(\tau)|\mathrm{d}\tau \end{bmatrix}.$$

Index

1. Page 242: Four discrete-type equation $\Rightarrow$ Forward discrete-time equation