ERRATA

Chapter 3
Tensor Properties of Crystals

R.C. Powell, *Symmetry, Group Theory, and the Physical Properties of Crystals*,
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On page 72, Table 3.5 has the forms of 3rd rank tensors for all the crystallographic point groups. Five of these have factors of 2 in some of the components. These factors of 2 can be found in other published tables of 3rd rank tensors but they are only valid for a different type of notation that is not used in the book. To be consistent with the tensor notation used throughout this book, no factors of 2 should appear in these tensors. The Erratum provides a corrected Table 3.5 with the factors of 2 eliminated from all the tensor elements.

In addition, on page 73 an example is given in the second paragraph using one of the tensors from Table 3.5 that contains erroneous factors of 2 in some of its components. The Erratum provides a corrected paragraph without the factors of 2.

The online version of the original chapter can be found at
http://dx.doi.org/10.1007/978-1-4419-7598-0_3

Table 3.5 Form of third rank tensors for the crystallographic point groups

C_1

$$\begin{bmatrix} d_{111} & d_{121} & d_{131} \\ d_{112} & d_{122} & d_{132} \\ d_{113} & d_{123} & d_{133} \\ d_{211} & d_{221} & d_{231} \\ d_{212} & d_{222} & d_{232} \\ d_{213} & d_{223} & d_{233} \\ d_{311} & d_{321} & d_{331} \\ d_{312} & d_{322} & d_{332} \\ d_{313} & d_{323} & d_{333} \end{bmatrix}$$

C_i, C_{2h}, D_{2h}, C_{4h}, D_{4h}, S_6, D_{3d}, C_{6h}, D_{6h}, T_h, O, O_h

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

C_2

$$\begin{bmatrix} 0 & d_{121} & 0 \\ d_{121} & 0 & d_{132} \\ 0 & d_{132} & 0 \\ d_{211} & 0 & d_{231} \\ 0 & d_{222} & 0 \\ d_{231} & 0 & d_{233} \\ 0 & d_{321} & 0 \\ d_{321} & 0 & d_{332} \\ 0 & d_{332} & 0 \end{bmatrix}$$

C_{2v}

$$\begin{bmatrix} 0 & 0 & d_{131} \\ 0 & 0 & 0 \\ d_{131} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & d_{232} \\ 0 & d_{232} & 0 \\ d_{311} & 0 & 0 \\ 0 & d_{322} & 0 \\ 0 & 0 & d_{333} \end{bmatrix}$$

C_s

$$\begin{bmatrix} d_{111} & 0 & d_{131} \\ 0 & d_{122} & 0 \\ d_{131} & 0 & d_{133} \\ 0 & d_{221} & 0 \\ d_{221} & 0 & d_{232} \\ 0 & d_{232} & 0 \\ d_{311} & 0 & d_{331} \\ 0 & d_{322} & 0 \\ d_{331} & 0 & d_{333} \end{bmatrix}$$

D_2

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & d_{132} \\ 0 & d_{132} & 0 \\ 0 & 0 & d_{231} \\ 0 & 0 & 0 \\ d_{231} & 0 & 0 \\ 0 & d_{321} & 0 \\ d_{321} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

C_4

$$\begin{bmatrix} 0 & 0 & d_{131} \\ 0 & 0 & d_{132} \\ d_{131} & d_{132} & 0 \\ 0 & 0 & -d_{132} \\ 0 & 0 & d_{131} \\ -d_{132} & d_{131} & 0 \\ d_{311} & 0 & 0 \\ 0 & d_{311} & 0 \\ 0 & 0 & d_{333} \end{bmatrix}$$

D_4

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & d_{132} \\ 0 & d_{132} & 0 \\ 0 & 0 & d_{231} \\ 0 & 0 & 0 \\ d_{231} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

S_4

$$\begin{bmatrix} 0 & 0 & d_{131} \\ 0 & 0 & d_{132} \\ d_{131} & d_{132} & 0 \\ -d_{131} & 0 & d_{132} \\ 0 & 0 & 0 \\ d_{132} & 0 & 0 \\ d_{311} & d_{321} & 0 \\ d_{321} & -d_{311} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

C_{4v}

$$\begin{bmatrix} 0 & 0 & d_{131} \\ 0 & 0 & 0 \\ d_{131} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & d_{131} \\ 0 & d_{131} & 0 \\ d_{311} & 0 & 0 \\ 0 & d_{311} & 0 \\ 0 & 0 & d_{333} \end{bmatrix}$$

D_{2d}

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & d_{132} \\ 0 & d_{132} & 0 \\ 0 & 0 & d_{132} \\ 0 & 0 & 0 \\ d_{132} & 0 & 0 \\ 0 & d_{321} & 0 \\ d_{321} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

C_3

$$\begin{bmatrix} d_{111} & d_{211} & d_{131} \\ d_{211} & -d_{111} & d_{132} \\ d_{131} & d_{132} & 0 \\ d_{211} & -d_{111} & -d_{132} \\ -d_{111} & -d_{211} & d_{131} \\ -d_{132} & d_{131} & 0 \\ d_{311} & 0 & 0 \\ 0 & d_{311} & 0 \\ 0 & 0 & d_{333} \end{bmatrix}$$

D_3

$$\begin{bmatrix} d_{111} & 0 & 0 \\ 0 & -d_{111} & d_{132} \\ 0 & d_{132} & 0 \\ 0 & -d_{111} & -d_{132} \\ -d_{111} & 0 & 0 \\ -d_{132} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

C_{3v}

$$\begin{bmatrix} 0 & -d_{222} & d_{131} \\ -d_{222} & 0 & 0 \\ d_{131} & 0 & 0 \\ -d_{222} & 0 & 0 \\ 0 & d_{222} & d_{131} \\ 0 & d_{131} & 0 \\ d_{311} & 0 & 0 \\ 0 & d_{311} & 0 \\ 0 & 0 & d_{333} \end{bmatrix}$$

C_{3h}

$$\begin{bmatrix} d_{111} & -d_{222} & 0 \\ -d_{222} & -d_{111} & 0 \\ 0 & 0 & 0 \\ -d_{222} & d_{111} & 0 \\ d_{111} & d_{222} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

D_{3h}

$$\begin{bmatrix} 0 & -d_{222} & 0 \\ -d_{222} & 0 & 0 \\ 0 & 0 & 0 \\ -d_{222} & 0 & 0 \\ 0 & d_{222} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

C_6

$$\begin{bmatrix} 0 & 0 & d_{131} \\ 0 & 0 & d_{132} \\ d_{131} & d_{132} & 0 \\ 0 & 0 & -d_{132} \\ 0 & 0 & d_{131} \\ -d_{132} & d_{131} & 0 \\ d_{311} & 0 & 0 \\ 0 & d_{311} & 0 \\ 0 & 0 & d_{333} \end{bmatrix}$$

C_{6v}

$$\begin{bmatrix} 0 & 0 & d_{232} \\ 0 & 0 & 0 \\ d_{232} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & d_{232} \\ 0 & d_{232} & 0 \\ d_{311} & 0 & 0 \\ 0 & d_{311} & 0 \\ 0 & 0 & d_{333} \end{bmatrix}$$

D_6

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & d_{132} \\ 0 & d_{132} & 0 \\ 0 & 0 & -d_{132} \\ 0 & 0 & 0 \\ -d_{132} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

T, T_d

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & d_{132} \\ 0 & d_{132} & 0 \\ 0 & 0 & d_{132} \\ 0 & 0 & 0 \\ d_{132} & 0 & 0 \\ 0 & d_{132} & 0 \\ d_{132} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Page 73, second paragraph

As a practical example, consider a quartz crystal that has D_3 symmetry at room temperature. The piezoelectric effect for this case is given by

$$\begin{pmatrix} P_1 \\ P_2 \\ P_3 \end{pmatrix} = \begin{bmatrix} d_{111} & 0 & 0 \\ 0 & -d_{111} & d_{132} \\ 0 & d_{132} & 0 \\ 0 & -d_{111} & -d_{132} \\ -d_{111} & 0 & 0 \\ -d_{132} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{pmatrix},$$

so

$$\begin{aligned} P_1 &= d_{111}\sigma_{11} - d_{111}\sigma_{22} + d_{132}\sigma_{32} + d_{132}\sigma_{23} = (\sigma_{11} - \sigma_{22})d_{111} + (\sigma_{32} + \sigma_{23})d_{132} \\ P_2 &= -d_{111}\sigma_{21} - d_{132}\sigma_{31} - d_{111}\sigma_{12} - d_{132}\sigma_{13} = -d_{111}(\sigma_{21} + \sigma_{12}) - (\sigma_{13} + \sigma_{31})d_{132} \\ P_3 &= 0 \end{aligned}$$

If a uniaxial stress is applied in the σ_{11} direction, $P_1 = d_{111}\sigma_{11}$ and $P_2 = 0$. The same tensile stress applied along σ_{22} also produces a polarization along P_1. The two-fold rotation axis P_1 is the electric axis of quartz. Shear stress can produce polarization along P_2 but no stress conditions can produce a polarization along P_3.