Basu's Work on Likelihood and Information

Joseph B. Kadane

It has been a joy learning from Dev Basu's work on aspects of statistical inference, and especially his deep and often provocative essays on fallacies of common statistical principles. I will limit myself to his epic paper *Statistical Information and Likelihood*.

"Statistical Information and likelihood" is a tour de force in three parts. In the first part, Basu studies the implications of the sufficiency and conditionality principles, and shows that these lead to the likelihood function as the summary of the information in an experiment. His treatment is similar to that of Birnbaum (1962, 1972). His second part reviews non-Bayesian likelihood methods, leaning especially on Fisher's maximum likelihood method (MLE). He criticizes the use of sampling standard errors around the MLE to create confidence intervals in the grounds that they violate the likelihood principle. His third part gives various examples that illuminate what he finds problematic about fiducial arguments, improper Bayesian priors, and simple-null hypothesis testing. Although most of his effort is critical, on the positive side Basu advocates subjective Bayesian analysis with proper priors, and making optimal decisions using a utility (or loss) function.

This essay needs to be understood in the context of its time. It was given in lecture form in 1972, ten years after Fisher's death. Fisher himself vigorously, vociferously, and sometimes with blind fury would attack those who disagreed with him. Basu is speaking from within the Fisherian tradition, and showing, by theorem and by counterexample, that large parts of that tradition simply do not make sense. This took courage and conviction, particularly considering the audience to whom he gave the talk. The discussants, in alphabetical order, were Barnard, Barndorff-Nielsen, Cox, Dempster, Edwards, J.D. Kalbfleisch, Lauritzen, Martin-Lof, and Rasch. Of these, only Dempster had anything supportive to say about Bayesian ideas, and he characterizes himself as a "sometimes Bayesian".

Nonetheless, the discussion is civil and respectful. I am particularly struck by the tone of the exchange of letters between Basu and Barnard. By the end, only a couple of points are still subject to disagreement, and the atmosphere is collegial.

Basu concludes his response to the discussion by writing "The Bayesian and Neyman-Pearson-Wald theories of data analysis are the two poles in current statistical thought. Today I find assembled before me a number of eminent statisticians who are looking for a via media between the two poles. I can only wish you success in an endeavor in which the redoubtable R.A.Fisher failed".

The situation is much the same today. The difficulty lies in what is to be regarded as random and what is to be regarded as fixed. To a classical statistician, the data are random, even after they have been observed, while the parameters are fixed but unknown (whatever that may mean). To a Bayesian, the data, after they are observed, are fixed at the observed values, but the parameters are uncertain,

J.B. Kadane (✉)
University Professor of Statistics, Carnegie Mellon University, Pittsburgh, PA 15213
e-mail: kadane@stat.cmu.edu

and hence random. There are not convenient middle grounds between these two perspectives. Basu has no hesitation about where he stands, writing "with an experiment already planned and performed, and with the sample x already before us, I do not see any point in speculating about all other samples that might have been." This places him solidly in the Bayesian camp.

Reference

[1] Basu, D. (1975). Statistical information and likelihood, with discussion and correspondence between Barnard and Basu, Sankhy$\overline{a}$, Ser. A, 37, 1–71.