ENGINEERING INDEX

Assessing Margin and Time to Failure in Engineered Systems

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Abstract: Inherent in most engineered products is a measure of margin -defined as the amount a product exceeds its functional performance requirements. Often original design and functional performance knowledge is not adequately documented making later uncertainty quantification and margin estimation difficult. This often leads engineers to rely on cultural lore, institutional practices, and product assessments relative to nominal conditions and tolerances to measure quality. Design intent, requirements, and their relationship with a product's intended function often gets lost. The Engineering Index was developed to assess the goodness or quality of a product relative to the margin in the performance requirements.

Key words: Margin, Uncertainty Quantification, Reliability Theory, Probability Theory, and Functional Requirements.

1. PRODUCT SPECIFICATION RELATIVE TO PRODUCT PERFORMANCE

When determining product reliability there generally exists an assumption engineers provide a nominally produced product. The reliability a product functions as intended, (R), depends the probability the product functions given the engineering aspects perform as intended (P(F|E)) times the probability the engineering aspects perform as intended (P(E)). This is written discretely as

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$$R = P(F \mid E) \cdot P(E) \tag{1}$$

If E is continuous, equation 1 becomes

$$R = \int P(F \mid E) g(E) dE \tag{2}$$

Physical products seldom behave as analysis, simulation and judgment predict. Components are never produced as designed, systems never assemble exactly, and products never live idyllic lives, (Dolin et al. 1997). A product's condition needs to be assessed relative to functional performance requirements. A functional performance requirement is a physical condition or configuration required of a product for it to perform its intended function. If a product exceeds a functional requirement it does not necessarily perform better but is perhaps more likely to function.

The Engineering Index (EI) provides a measure of goodness for engineered systems, components, and functions, (Dolin et al. 2001). Goodness measures how well a product exceeds functional performance requirements. The EI assesses the condition of a component or system population at a given time relative to its functional performance requirements and is normalized to the population's initial assessed condition. To compute EI it is necessary to 1) understand a product's functional requirements, 2) assess a product's condition, and 3) quantify uncertainties in both the requirements and assessments.

EI does not measure product reliability, (Upitis et al. 1998). EI measures a product's condition in a domain reliability theory cannot. Reliability is a measure of the probability of success but does not indicate a product's relative goodness. Knowing how close a product is to failure along with its rate of declining margin is paramount to decision making and planning, (Jeffrey 1990; Clemen 1996). Reliability assessments have two limitations. First, reliability is often presented as a continuous metric for the probability of success, while success criteria can be dichotomous (i.e. pass or fail). This all or nothing approach makes partial failures difficult to depict. A product does or does not perform as required and reliability measures the frequency that occurs.

A second limitation of reliability assessment is their basis in probability theory, (Higgins et al. 1995). Probability theory holds the probability of an event occurring cannot exceed one. This is analogous to saying a product's reliability is assessed relative to its performance limit even when the product surpasses this limit. Probability theory used in reliability analysis does not provide a mechanism for measuring how much a product exceeds its requirements. The extra goodness/quality/margin inherent in many aspects of a product is not represented in traditional reliability assessments, (Abbas et al. 1997).

Suppose a product is to be designed to perform a task within some environment. The engineer likely designs the product to over-perform the task in environments more severe than stipulated. In other words, the product is designed to exceed its functional performance requirements. With respect to traditional reliability (R), three conditions exist,

1. $R < 1.0$, when it is probable a product fails to meet the functional performance requirements.
2. $R = 1.0$, when product performance equals the functional performance requirements.
3. $R = 1.0$, when product performance surpasses functional performance requirements.

The EI quantifies the goodness or quality of a product when condition three exists. Within this domain some products perform better than others. For some functional requirements products of varying quality may perform the same, (e.g. have the same life expectancy). Under different conditions (e.g. adverse environments) products of varying quality may perform differently. The goal of the EI is to assess how much a product population exceeds its requirements. The EI can be applied to static design requirements such as geometry, temperature, fit and form, etc. The EI can also be applied to dynamic requirements such as function, dissipation, rate of wear, and aging.

An additional feature of the EI is its ability to project ahead to infer where and when a potential problem/crisis might occur. Unlike process control, (Kane 1986; Singpurwalla 1998) or safety metrics, (LeSage 1990) the EI provides *a priori* knowledge highlighting so called reliability cliffs and warning of potential reliability decrements prior to the reliability change occurring. The EI helps manage scenarios where a product is assessed reliable during the first reporting cycle, reliable after the second reporting cycle, and then suddenly decertified because of lack of confidence in the product's ability to perform as required. By knowing potential product degradations in advance of their occurrence, decision makers can implement mitigation measures.

EI's are normalized from 0.0 to 1.0 allowing comparisons between various parameter and functional assessments. While mathematically possible, in general, EI never equals 1.0 because product populations always have variability, (Dolin et al. 1997). $EI=0.0$ when a product parameter has no margin and any further deterioration may lead to a reliability decrement. During the period of a product's life when $1.0 > EI > 0.0$, reliability is unchanged because while a functional parameter may be deteriorating it has been designed to exceed functional requirements.

2. MARGIN RELATIVE TO PERFORMANCE

It is not appropriate to compute an EI for every design, manufacturing, assembly, and functional requirement. There are several factors to be considered when attempting to determine whether or not an engineering requirement needs an index. Some requirements have no margin (i.e. pass/fail) and cannot be indexed. Other requirements don't impact performance. For example, suppose the wall thickness for a pressure vessel is designed to support some internal loading. The lot-to-lot variability in the vessel thickness is a matter of reliability (i.e. the probability a vessel meets its thickness requirements). The amount a population of vessels supports the required loading allowing for flaws, damage, hidden manufacturing defects, and material inconsistencies is a matter of margin, (Kreuser 2001).

Pressure vessels are generally designed to some thickness with tolerances allowing an acceptable level of variability. This geometric requirement may not be directly related to performance. If the performance requirement for a pressure vessel is to hold gas, the quality or margin in the vessel performance relates to its ability to hold gas. There is no sliding measure of quality in this requirement, the vessel either does or does not hold the gas. If a vessel at minimum thickness holds the gas, a thicker vessel does not hold the gas more effectively. Estimating the portion of a vessel population below the minimum requirement is a variability concern relating to reliability. Estimating how well a vessel holds gas is a matter of margin. While reliability estimates provide no insight into how much margin remains in the subpopulation of acceptable vessels, the EI does.

This leads to an important distinction between various functional requirements. In accordance with the ASME pressure vessel design code, (Upitis et al. 1998), engineers specify vessel thickness with an inherent (3x) factor of safety (FoS). In engineering designs, the vessel's nominal thickness is determined based on nonperformance issues for the vessel itself, such as, desired size, weight, or manufacturability. The design requirement is usually specified as a nominal thickness with upper and lower tolerances such that when the vessel is at minimum thickness it has a (3x)FoS. When the vessel is at nominal and upper tolerance thickness, the FoS is even larger.

If a vessel's thickness is slightly less than the specified minimum tolerance, the vessel is often rejected for not meeting requirements even though the margin is slightly less than (3x)FoS. If the performance requirement for the vessel is to constrain gas under pressure, a vessel at minimum thickness performs the required task as well as a vessel at nominal or maximum thickness. An EI on vessel thickness would indicate margin is not used up until the FoS is less than one, (i.e. EI=0 when FoS=1).

Suppose in addition to constraining gas the vessel needs to provide structural support within a system. Then thickness directly affects functioning performance. With respect to a vessel's ability to support external loads, the nominal thickness provides some (α)FoS and the minimum thickness provides some $(\alpha-\beta)$FoS. Margin becomes the ratio between vessel thickness and the thickness required to support the largest probable loading. How one assigns a probable external load and computes the thickness required to support that load, needs to be quantified.

Three types of engineering requirements have been identified based on how product parameters are constrained. Margin, and hence the EI, is computed differently for each requirement type. Performance requirements are classified by how they constrain some governing aspect of a product to lie within a required domain. The domain can be one or two-sided and may be multi- dimensional. For this discussion, only one-dimensional performance domains are considered. A one-sided requirement has one domain boundary against which a product parameter is measured; a two-sided requirement defines upper and lower domain boundaries. Figure 1 shows the three types of requirements and how the margin domain is bounded. Although systems are generally multi-dimensional requiring the necessity of computing indices in multi-dimensional space, EI's can be mapped back into one-dimensional space for reporting.

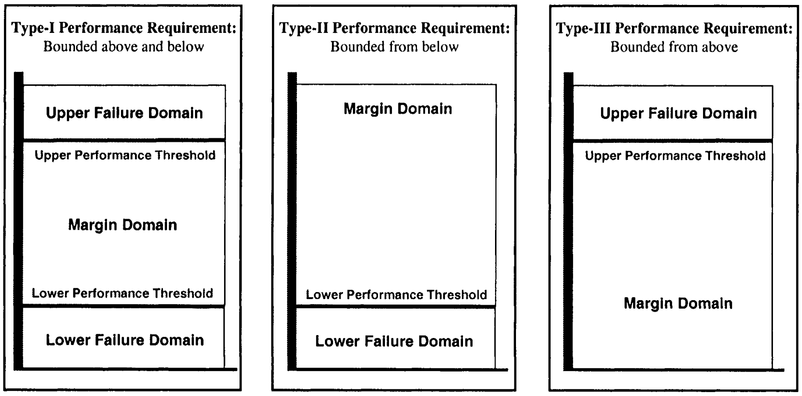


Figure 1. Three types of performance requirements

Type-I performance requirements bound the margin domain from above and below by functional thresholds. An example of a Type-I requirement is a geometry constraint stipulating the length of a component be greater than X units but less than Y units. Mass is another example. Type-I requirements often define a nominal, minimum, and maximum set of conditions. One must be careful to delineate between design, manufacturing, and functional performance requirements. While design and manufacturing requirements are readily available in engineering specifications, functional performance requirements seldom are.

Type-II requirements only have lower limits. For example, consider two materials that must stay bonded together while supporting a load. In this scenario we are concerned about the strength falling below some lower limit but are unconcerned about it exceeding an upper limit. If the goal is to hold two parts together under load, it's hard to imagine being concerned the bond strength is too great. If such an engineering concern did arise, this situation becomes a Type I requirement. Temperature is another example of a requirement that can be bounded from below.

Type-III requirements only have upper limits. For example, the concentration of contaminates a fluid is allowed to contain may be bounded from above. Unless having a zero-contaminants fluid is undesirable, there is no lower limit so the margin domain begins at zero contaminants and extends upward as contaminants are introduced until the upper performance threshold is met. Fluids exceeding this upper threshold level of contaminants fall within the failure domain.

Performance thresholds define the minimum acceptable condition necessary for a product to perform its intended function. While seemingly straight forward, deciding where a performance threshold is drawn depends on criteria far from obvious. There are a whole host of reasons for this difficulty - foremost being engineers often do not defined failure boundaries. On the journey from design to production, many different requirements are specified. In general, design and production requirements do not address performance. Failure boundaries are rarely sought during testing. Establishing performance requirements becomes a difficult process involving data mining, expert elicitation, (Meyer et al. 2001) and analysis simulation, (Dolin 2002).

Figure 2 shows some of the many different requirements that can be specified for a product. Limits are generally defined discretely with upper and lower tolerances. The Performance Thresholds (PT) are displayed discretely but in general may be uncertain having a distribution. In any population of products there will be variability and one can imagine two primary subpopulations being formed. One subpopulation represents acceptable products. The other subpopulation represents unacceptable products. Figure 3, shows such a bimodal distribution. While multi-modal distributions are possible, two primary subpopulations can almost always be formed. If the uncertainty in PT is incorporated into the product assessment distributions, PT becomes a line of demarcation that can be used to express reliability as

$$R = \left[1.0 - \begin{pmatrix} \text{Area of population} \\ \text{density below PT} \end{pmatrix} \right] \tag{3}$$

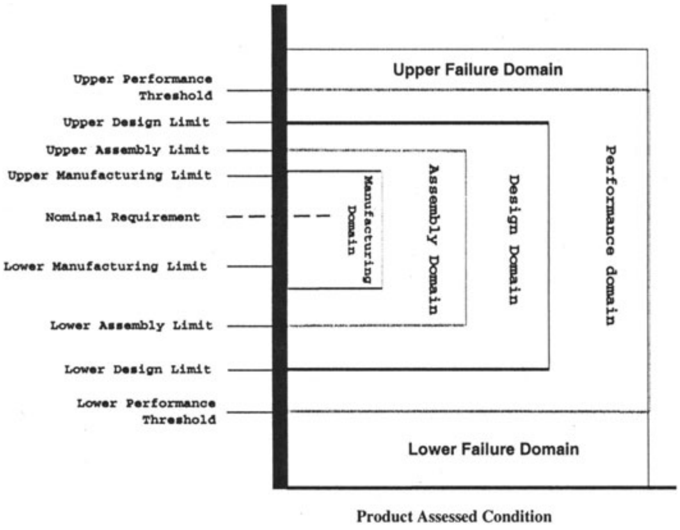


Figure 2. Different requirement domains a product can possess

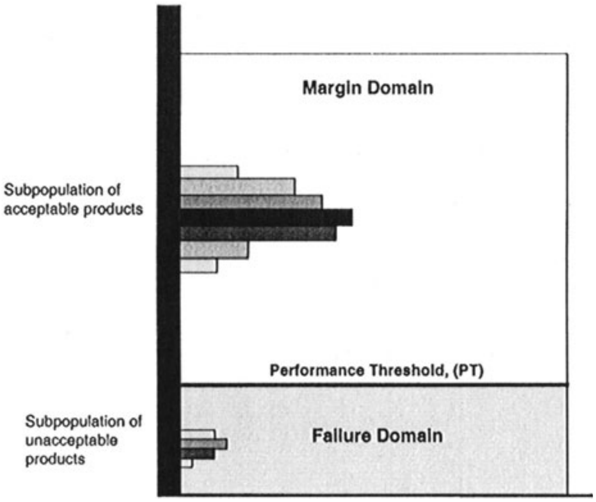


Figure 3. Product variability within a population

Regardless of the shape of the distribution, reliability remains constant until some portion of the acceptable product distribution crosses the performance threshold. The distance from the acceptable product distribution to the performance threshold is the margin. Only after margin goes to zero does reliability change. Figure 4 shows margin defined as the minimum distance from any assessed condition within the subpopulation of acceptable products to the performance threshold. There may be times when the assessed condition of a population of products is not known and must be inferred from sample statistics, analysis simulation, or some synthesis of all available knowledge. In those situations a continuous distribution with tails may result and the lower assessed condition becomes the tip of the tail or the point at which the tail region is truncated.

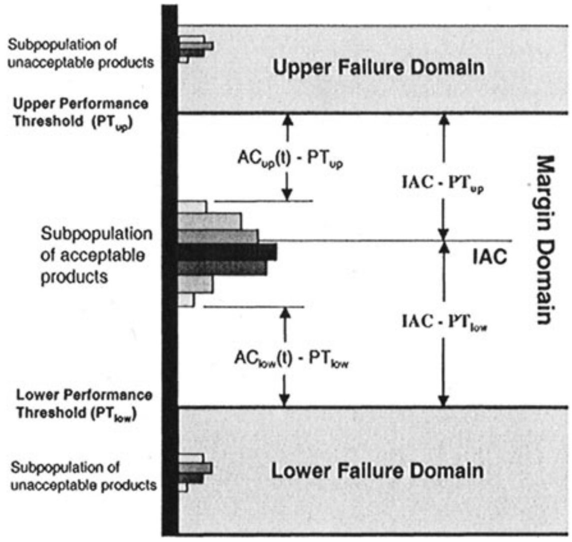


Figure 4. Margin relative to assessed product condition for Type-I requirement

Table 1. Engineering Index for TYPE-I, II, and III Requirements

Requirement Type	**EI Formula**
Type-I	$EI = MIN(EI_{low}, EI_{up})$
Type-II	$EI_{low} = \dfrac{AC_{low} - PT_{low}}{IAC - PT_{low}}$
Type-III	$EI_{up} = \dfrac{AC_{up} - PT_{up}}{IAC - PT_{up}}$

Figure 4 shows the variables of the EI equation. Notice that the EI is normalized with respect to the initial assessed condition (IAC). The IAC is taken as the mean, mode, or median of the acceptable product subpopulation distribution at time zero depending on the shape of the distribution and once determined, remains a constant representation of the initial As-Built condition, (Dolin et al. 1997). Normalizing EI with respect to the IAC provides a way of assessing how a parameter is changing over time relative to its initial condition. Table 1 shows the EI formula for three types of performance requirements.

Uncertainty is defined as variability plus precision plus lack of knowledge. In any product there exist several sources of uncertainty, which is further enhanced from the fact no two products are the same. Engineers often make decisions based on the simulation responses of a single idealized nominal design model even though physical products are never nominal. Even if products were serendipitously manufactured and assembled in a nominal condition, once pressed into service, they have unique life experiences. When As-Built/As-Is engineering practices are used, uncertainties can be quantified and accounted for in margin measurements.

Several sources of uncertainty include, (Dolin et al. 2002) population variability (S_1), aging and degradation (S_2), measurement errors (S_3), statistical inference errors (S_4), extrapolations from data/information and models (S_5), and lack of knowledge (S_6). All quantifiable sources of uncertainty need to be accounted for in some kind of integrated uncertainty estimate. This is a huge undertaking and a methodology for how uncertainty will be represented in EI estimates has not been finalized. However, what ultimately emerges will probably be some kind of functional representation as shown in equation 4

$$S^2_{total} = \mathbf{f}(s_1^2, s_2^2, s_3^2, s_4^2, s_5^2, s_6^2) \qquad (4)$$

3. MEASURING MARGIN CHANGES OVER TIME

The EI is a measure of margin normalized with respect to a product's initial assessed condition. This means reliability can start off less than one for the entire population and remain constant so long as no portion of initial acceptable product subpopulation falls into a failure domain. Over time products degrade and EI's tend toward zero. Figure 5 shows a possible product degradation scenario for a Type-II requirement. Initially the subpopulation of acceptable products is assessed. If sample measurements are taken a sample distribution, such as the histogram shown, can be used and the lower assessed condition is defined as the minimum from any assessed condition to the performance threshold. If analysis, or some synthesized combination, of analysis, measurements, and expert judgment are used to generate a continuous distribution, the lower assessed condition can be found by estimating the extreme quartile (e.g. 0.001 or 0.999).

The assessed condition and the population distributions change with each assessment. Notice the lower assessed condition is steadily moving toward the performance threshold until eventually crossing. While margin is declining in the first three assessments (t=0, 1, and 2), reliability remains constant because the area of the acceptable product distribution is above PT even though the shape of the distribution is changing and margin is diminishing. By the fourth assessment (t=3), there is no margin left in the product population and adjustments to the assessed reliability have to be evaluated.

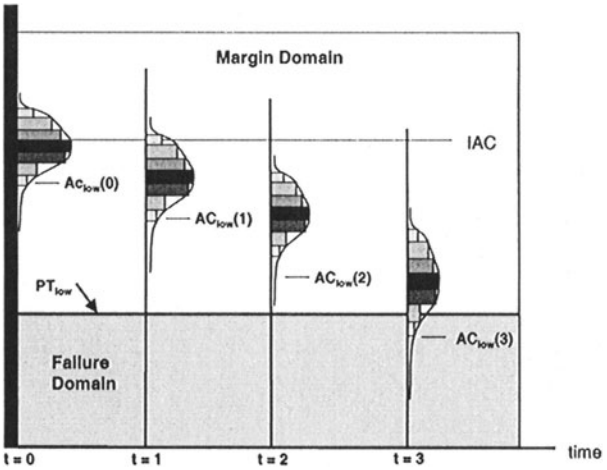


Figure 5. Product degradation scenario for a Type-II requirement

Figure 6 shows how this approach can be used to infer into the future. Once some knowledge is gained on how a subpopulation of acceptable products is declining over time, statistical inference techniques can be used to predict where in the future the EI(t) curve crosses the performance threshold. The further into the future one infers, the larger the uncertainties become. Quantifying this phenomena is key to helping defend knowledge acquisition endeavors such as testing, or surveillance. The general equation for a time dependent EI for a Type-II requirement can be expressed as

$$EI(t) = \frac{AC_{low}(t) - PT}{IAC - PT} \tag{5}$$

Alternatively, a product can be analyzed, tested, and modeled extensively so its behavior is better understood. In that case, uncertainty decreases, which results in improved EI predictions.

Figure 7 shows how the EI can be used as a product watch list. Color is used to indicate how margin changes over time. Blue indicates insufficient knowledge exists to make an assessment but at the same time, there are no immediate concerns. Green conveys a stable margin. Yellow is used when margin is declining over time. Red highlights when a zero margin condition exists. This color-coding scheme provides a snap shot of where and when possible problems are likely to occur.

The watch list indicates subsystem E has a ten-year life. If the subsystem is replaced it has a new life of steady decline. Subsystem F indicates a current and anticipated short-term lack of knowledge. If knowledge is not acquired within ten years the subsystem is considered in a zero-margin state and reliability decrements occur.

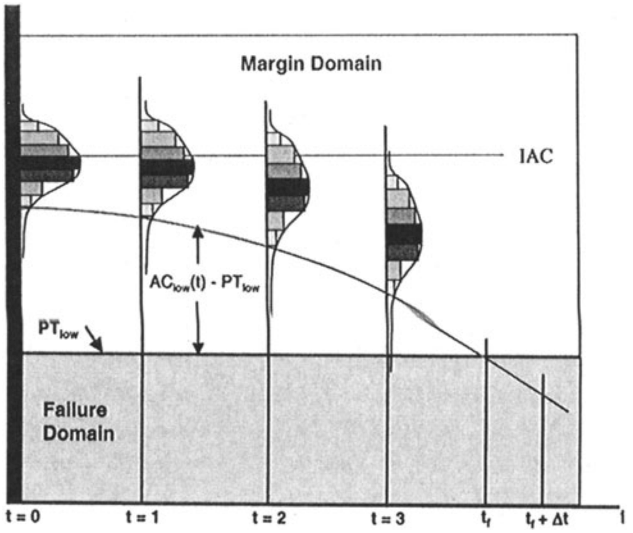
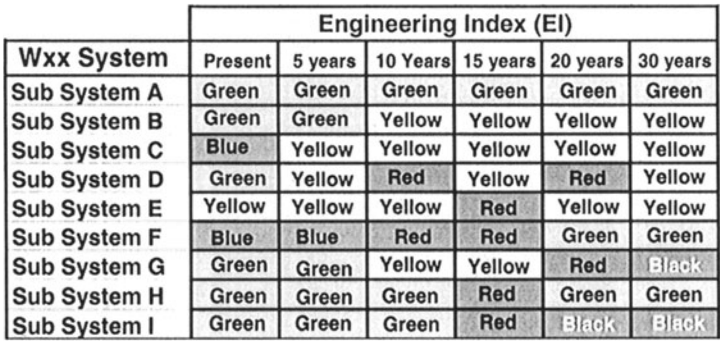


Figure 6. Using EI to infer into the future



	Engineering Index (EI)					
Wxx System	Present	5 years	10 Years	15 years	20 years	30 years
Sub System A	Green	Green	Green	Green	Green	Green
Sub System B	Green	Green	Yellow	Yellow	Yellow	Yellow
Sub System C	Blue	Yellow	Yellow	Yellow	Yellow	Yellow
Sub System D	Green	Yellow	Red	Yellow	Red	Yellow
Sub System E	Yellow	Yellow	Yellow	Red	Yellow	Yellow
Sub System F	Blue	Blue	Red	Red	Green	Green
Sub System G	Green	Green	Yellow	Yellow	Red	Black
Sub System H	Green	Green	Green	Red	Green	Green
Sub System I	Green	Green	Green	Red	Black	Black

Figure 7. Using the EI to provide a watch list

The color index can be modified by mitigating the problem areas by replacing/fixing the subsystem, getting necessary information, or changing the criteria upon which the problem area was assessed. Subsystem G shows a steady margin with no immediate concerns. However, it is believed after ten years the subsystem will begin a steady rate of margin decline. Twenty years into the future it is anticipated the subsystem reaches a zero-margin state and reliability decrements start occurring. If in 30 years the problem is not somehow mitigated, a potential full system decertification must be evaluated.

4. PRESSURE VESSEL EXAMPLE

The EI was first proposed as a means for assessing the goodness of an engineered system, subsystem, component, or function, (Dolin et al. 2001). The EI primarily measures the extent to which a product exceeds its nominal requirements, which is defined as margin, (Abbas et al. 1997). A common metric in statistical process control is margin over uncertainty (MOU). This metric is referred to by many names, including as C_{pk}, (Kane 1986; Singpurwalla 1998), and Figure of Merit (FOM), (Logan 2001). MOU can be described as the amount a product exceeds its requirements relative to the amount of uncertainty in the assessment.

Consider a pressure vessel designed to withstand some maximum operating pressure (MOP). As shown in figure 8 several other qualification pressures are specified during the design process but the MOP is the requirement the vessel must meet. In general a Use Safety Factor is specified as the ratio of the vessel's burst pressure to the MOP. When pressure vessels are built they are proof tested by determining the pressure the vessel holds without exceeding a specified expansion limit.

The maximum allowable working pressure is defined to be 2/3rds of the proof pressure. This becomes the pressure for which the vessel is rated. The rated proof pressure is generally less than the calculated maximum allowable working pressure. The vessel's Safety Factor is defined as the ratio of the burst pressure (P_{BURST}) over the maximum allowable working pressure (P_{MAWP}). The desired Safety factor is

$$SF = \frac{P_{BURST}}{P_{MAWP}} \geq 3:1 \tag{6}$$

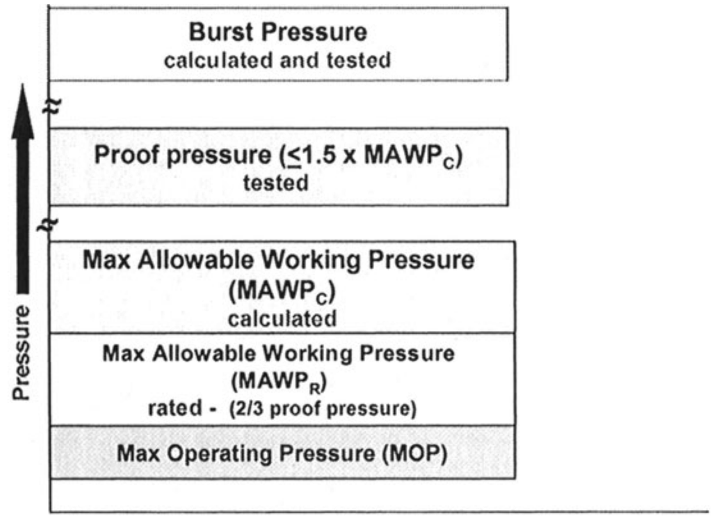


Figure 8. Example of a pressure vessel's requirements

Pressure vessels are built and tested to determine a distribution on burst pressure. The majority of the vessels tested burst around a common/ anticipated pressure. However, it's not uncommon for a subpopulation to burst at much lower than anticipated pressures. Vessels in this subpopulation are deemed failures resulting in a bimodal distribution. One region represents the burst pressures of acceptable vessels and the other represents the subpopulation of failures. This ratio of good to bad vessels is used to establish an initial reliability (R) for the inventory.

Uncertainties exist in these conclusions. For example, there is uncertainty in the pressure measurements, in using a limited number of tests to generate a distribution, and in assuming an entire population behaves in a manner consistent with the response characteristics of a limited test suite. For these reasons, there is uncertainty in the estimated distribution of the population of acceptable vessels. However, as shown in figure 9, the initial EI is determined based on the minimum assessed condition of the distribution of the subpopulation of acceptable vessels.

Notice, even though the performance metric is vessel pressure, the component variable measured is wall thickness. This is because wall thickness is a measurable variable that can be assessed without destroying the vessel. Notice the performance threshold is not related to proof pressure thickness, nor the maximum allowable working pressure (MAWP) thickness, but rather, it is related to the maximum operating pressure thickness. This demonstrates how confusing it can be to ascertain performance thresholds. The zero-margin condition is not reached until the thickness of the vessel is such that it will not contain the maximum operating pressure.

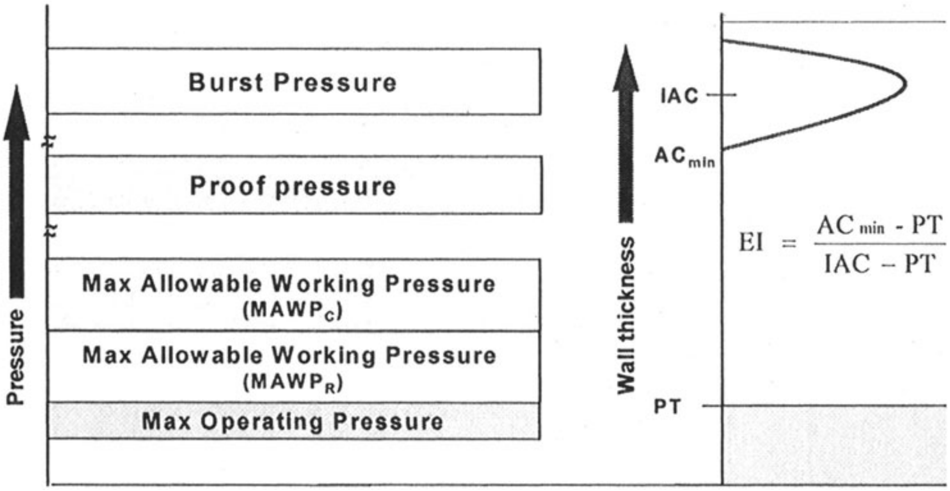


Figure 9. Calculating EI based on vessel wall thickness

The performance threshold is not an absolute line of demarcation below which the vessel fails to hold the maximum operating pressure. Rather, the performance threshold is the minimum vessel thickness for which it is confidently believed the vessel is able to hold MOP. For any vessel whose thickness is less than the PT thickness confidence it will hold the MOP is diminished. For example, during the lifetime of these vessels both corrosion and embrittlement take their toll. The net result of these aging mechanisms is a reduction in the effective vessel wall thickness. As shown in figure 10, ten years into the future there is still a high and fairly stable estimated margin. By inferring twenty years into the future it is anticipated the margin starts dropping. By the time the vessels are 30 years old they are no longer able to support their rated pressure but still have margin relative to their maximum operating pressure. By the time the vessels are 40 years old though they have surpassed their zero-margin state and can no longer be safely used.

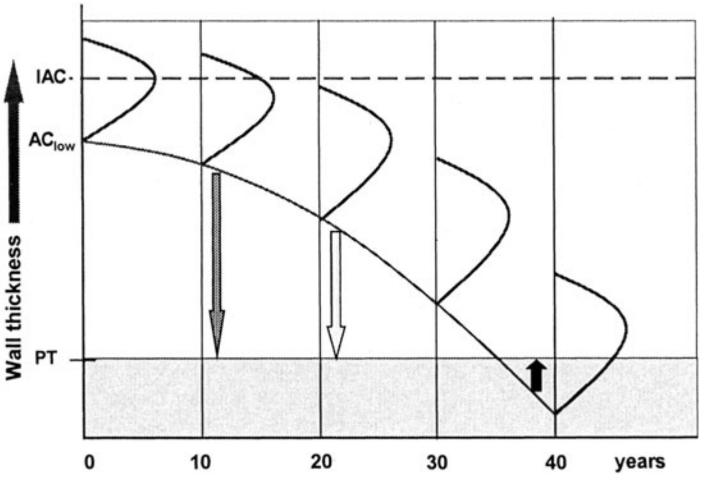


Figure 10. Inferring EI on future pressure vessel wall thickness margin

5. CONCLUSIONS

The Engineering Index is a simple metric for assessing the amount of margin a product has before it becomes non-useable. Key aspects in assessing an Engineering Index include quantifying uncertainty, assessing a product's initial and current states, and understanding the product's functional performance requirements. It is difficult at times to delineate the many different product requirements. While design and manufacturing requirements are usually readily available in engineering specifications, functional performance requirements are not. Even when performance requirements exist, they are often ambiguous and it is difficult to determine an engineer's exact intent. As challenging as it is to compute and Engineering Index it is a necessary part of product assessment, qualification, and certification. In the future, knowledge intensive engineering tools should include performance requirements and measures of margin.

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