THE 7-PIECE PERFECT PLAY LOOKUP DATABASE FOR THE GAME OF CHECKERS

E. Trice, G. Dodgen

Gothic Chess Federation

GothicChessInfo@aol.com; GilDodgen@cox.net, http://www.GothicChess.org

Abstract Many research teams and individuals have computed endgame databases for the game of chess which use the distance-to-mate metric, enabling their software to forecast the number of moves remaining until the game is over. This is not the case for the game of checkers. Only one programming team has generated a checkers database capable of announcing the distance to the terminal position. This paper examines the benefits and detriments associated with computing three different types of checkers endgames databases, demonstrates the solutions to the longest wins in the 7-piece checkers database, presents tables of longest wins for positions including all permutations of four pieces and fewer against three pieces and fewer, and offers major improvements to some previously published play.

Keywords: checkers, database, endgame, move to win, perfect play

1. Introduction

It is a widespread misconception that since the rules of the game of checkers are simple, so is the playing of the game. This misperception is not limited to just the general public. Several reputable scientific sources have disseminated inaccurate information regarding the state of computer checkers (Gibson, 1993; Schaeffer, 1997 pp. 101-102).

"Computers became unbeatable in checkers several years ago." Thomas Hoover, "Intelligent Machines," *Omni* magazine, 1979, p. 162.

"...an improved model of Samuel's checkers-playing computer today is virtually unbeatable, even defeating checkers champions foolhardy enough to 'challenge' it to a game." Richard Restak, *The Brain, The Last Frontier*, 1979, p. 336.

"Although computers had long since been unbeatable at such basic games as checkers..." Clark Whelton, *Horizon*, February 1978.

"So whereas computers can 'crunch' tic-tac-toe, and even checkers, by looking all the way to the end of the game, they cannot do this with chess." Lynn Steen, "Computer Chess: Mind vs. Machine" *Science News*, November 29, 1975.

On August 29, 1992, World Checkers Champion Dr. Marion Tinsley defeated the world's strongest checkers program, CHINOOK (Schaeffer, 1997 pp. 328-332). The score of their match was 4 wins for Tinsley, 2 wins for CHINOOK, and 33 draws each. Tinsley's four wins disproved the notion that checker programs were "unbeatable".

The CHINOOK team experienced a great deal of success while ascending the competition rungs, which allowed them to challenge Dr. Tinsley for the title of "Man vs. Machine" World Champion in 1992. One of the key factors that made CHINOOK such a strong program was the size of its endgame databases. The program eventually had access to 443,748,401,247 precomputed positions that were known to be either wins, losses, or draws (Lake, Schaeffer, and Lu, 1994). These data were available at runtime during the look-ahead search, which allowed CHINOOK to enter into lines of play that would avoid losses (within its horizon of search) as well discover deep, subtle wins.

Having such game-theoretical values (GTV) available for the search engine at runtime is extremely valuable, but in certain cases it is not enough information to procure the win. Section 3.2 showcases some 7-piece positions that are wins for the side to move but cannot be won using only a database with the game-theoretical values stored. While such a database recognizes the wins as it builds its tree during the search, it cannot determine the winning sequence. A database with information associated with the distance until a conversion (capture of a piece, or promotion of a checker to a king) takes place is of some help. Examples are presented that demonstrate the power of such a Distance To Conversion database, as well as some of the weaknesses.

This paper is organized as follows. Section 2 presents an overview of the three different types of checkers endgame databases, and briefly tabulates the pros and cons of each category. Section 3 contains the solution to the longest 7-piece database win and a comprehensive listing of all of the data collected in the 2-to-7-piece Perfect Play Lookup (PPL) databases. The longest wins are presented in each sub-database grouping. Section 4 demonstrates several improvements to a very common checkers endgame known as Fourth Position. This ending was first published in 1756 and has been studied by the tournament checkers playing community ever since. It should be noted that the PPL database solution begins in such an unorthodox fashion that it is worthy of special attention. Section 5 offers a brief conclusion regarding what has been learned, particularly about the complexity of the game of checkers.

By improving upon the play of a common checkers endgame first published 247 years ago that has since been studied and analyzed by the strongest human players in the history of checkers, this paper asserts that the PPL database is capable of outperforming the world's best human players from any time period.

2. Overview of Different Types of Databases

There are three different ways to catalog checkers information that can be useful to a program. Each type of database has benefits and drawbacks, which are summarized in Table 1.

Database	Benefits	Drawbacks
Game	1) Easiest type of database to	1) Once reaching a database
Theoretical	compute.	position over the board, no
Value	2) Can be generated quickly.	information is available
(GTV)	3) A post-process routine can	regarding the best way to
	compress the data efficiently	proceed.
	allowing for runtime probing	2) Positions that are theoretically
	to assist the Alpha-Beta	won can in practice be drawn by
	search engine.	repetition since the winning path
		cannot be found.
Distance	1) Can be computed about as	1) Requires much more RAM and
То	easily as a GTV database.	disk space to compute compared
Conversion	2) In King-heavy endings the	to a GTV database.
(DTC)	play can mirror PPL	2) The database prefers a known
	database results.	conversion path which may take
	3) A win can always be	longer to win than a potentially
	achieved, even if it takes	much shorter path to victory
	much longer than a PPL	known by a PPL database.
	database's solution.	3) With fewer kings on the board,
	4) Positions with draws as the	playing precision is much lower
	result can be removed from	than that of a PPL database.
	the database.	4) Post-process compression is not
		nearly as good as that of a GTV
		database.
Perfect	1) Always wins by selecting the	1) Difficult to compute both in
Play	shortest possible route.	algorithm complexity and time
Lookup	2) Always capable of	requirements.
(PPL)	postponing losses for as long	2) Requires much more RAM and
	as possible.	disk space to compute compared
	3) Positions with draws as the	to a GTV database.
	result can be removed from	3) Post-process compression is not
	the database.	nearly as good as that of a GTV
	4) The verification routine	database (but can be as good as
	virtually guarantees that the	that of a DTC database).
	GTV database used in the	
	process is correct.	

Table 1. Benefits and drawbacks of the different types of checkers databases.

2.1 Application of a GTV Database

Typically, a large database of game-theoretical-value information can be probed at run time, greatly reducing the number of nodes that need to be evaluated by a search engine. Once a database position is encountered in RAM, no additional move generation or merit assignments need to be invoked. The GTV score is returned, and that particular leaf node has perfect information attached to it. Schaeffer, Lake, Lu, and Bryant (1996) have demonstrated the benefits of this approach with their program CHINOOK, which won the World "Man versus Machine" Championship in 1994 and successfully defended its title in 1995. In probing such a database, the search tree can return valuable information while still a great distance away. The examples in Figures 1 to 3 demonstrate how a 12-piece position can be evaluated as a forced win with only a six-piece database accessible in RAM.

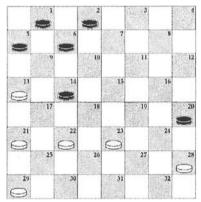


Figure 1. Black to move wins by forcing a trade into a won six-piece database position.

From the position in Figure 1, a program with the black pieces will generate not only moves for evaluation such as 6-10 and 2-7, but those that seem to toss away material as well, such as 20-24 and 14-17. The program will arrive at Figure 2 after Black initiated the sequence 14-17 21x14, 20-24 28x19, 6-9 13x6, 2x9x18x27. With White to move, the 6-piece database result is a draw, so the score for *Black* to move from the parent 12-piece position is backed up as a draw.

As other jump paths are examined, the position shown in Figure 3 will be reached after 14-17 21x14, 20-24 28x19, 6-9 13x6, 1x10x17x26. The 6-piece database position shown in Figure 3 is a loss for *White* to move, so the score for *Black* to move from the parent position is returned as a win. Since all captures are forced in the game of checkers, the program will elect to enter into the inescapable line of play leading to Figure 3. In so doing, the program will properly announce a win from the 12-piece position in Figure 1, and play the move 14-17.

A quick glance at Figures 2 and 3 will not reveal anything obvious to the casual player, but after conducting a search the correct result becomes evident. The most outstanding feature of the GTV information is that *no* such search ever needs to be performed once a position in the database is

found. This is the equivalent of "extending the search" from the database position to the terminal node many plies distant.

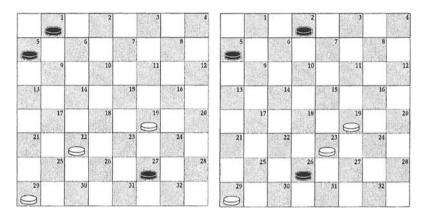


Figure 2. White to move draws. Figure 3. W

Figure 3. White to move loses.

In the case of Figure 3, the PPL database indicates that White loses after a total of 102 plies. So, a GTV database, by identifying that position as a theoretical win, performed the functional equivalent of searching 102 plies and returning a score indicating a loss would occur. Notice that the number of pieces in a GTV database can be relatively small yet it can still effectively direct the search from a position with many more pieces. However, as will be shown in Subsection 3.2, once a program with only a GTV database is actually in a won position, in certain cases it can be sufficiently difficult to converge on the win.

2.2 Creating a DTC Database

A DTC database will store the number of plies for each position until either a checker crowns or a piece is captured. It does not have any information to distinguish which result is achieved as the goal; it only knows it is heading for one or the other. Unlike a GTV database, which can represent four positions in each byte *during the computation* (and five positions per byte after the computation prior to compression) the DTC database needs one entire byte for each position in order to store a conversion range from 0 to 255 plies.

The process of creating the DTC database is nearly identical to the GTV database. If the DTC database is also being used as a GTV database, then two of the eight bits in the byte must be reserved for the win-loss-draw assessment, leaving only six bits (0-63) available for the maximum depth to conversion. There is a way to double this ply count if you divide the actual

depth by two, and take note that if the side to move wins, the plycount must be an odd number, since that side makes the last move. If the side to move loses, it makes the second to last move, which always must be an even number. Therefore, to "decompress" the true ply count, double the number that is stored, then add one to it if the position is a win. If it is a loss, there is no need to change the number. If it is a draw, the depth until conversion is meaningless. If the DTC database is created as a separate post-process, and the GTV database is used to determine the win-loss-draw status of a position, then all eight bits can be used to store conversion information. Using the aforementioned division-by-two schema a maximum conversion depth of 511 plies can be stored.

The iteration process for the DTC database begins with the jump pass, which is the same as would be performed with a GTV database, with one notable difference. The DTC database stores the result for each jumping win as a "1-ply" conversion. Therefore the counter for each win in which a jump exists will be set to 1. Next, any crowning moves are generated, and independent of the win or loss result, these are stored as a conversion in 1 ply as well. Thereafter, as each pass over the database is made, whenever a win or loss result is able to be determined, the iteration number is stored in the database. The idea is that more difficult positions (presumably) will have conversion iteration counts greater than positions that are near the conversion horizon.

When the computation is completed, one goes on to the next slice, but the conversion information for the crowning moves or jumps is not inherited. Each database slice is computed independently of all others. No conversion information is shared across databases.

2.3 Weaknesses of DTC Databases

In difficult positions where there is a majority of Kings, a DTC database is most valuable. As more Checkers are introduced into a position, the probability that a DTC database will make the same, highly accurate move as the PPL database diminishes. Even in elementary positions like the one shown below in Figure 4, a DTC database will not play a move that is obvious to any ordinary player.

Even novice checker players will make the move 18-23 in Figure 4, winning after White makes any move with the King, but a DTC database will not.

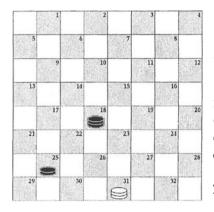


Figure 4. A DTC database will not play 18-23, the move that wins most quickly.

The glaring weakness of a DTC database centers on a potential "one ply conversion horizon" that can arise during a computation cycle. A move that converts in fewer plies but takes much longer to win could be preferred over a shorter win that takes longer to convert. With a quick glance anyone can see that in Figure 4 the sequence 18-23 31-26 23x30 wins, as does 18-23 31-27 23x32. This requires three plies of information, which the PPL database does have, but which the DTC database *does not* have. The

DTC database will "know" that 25-29 converts in 1 ply, and 25-30 converts in 1 ply. When the move 18-23 is examined, it will *not* lead to an immediate conversion, and therefore will be deemed to be inferior. The DTC database will have stored a value indicating that the position after 18-23 results in a conversion in 2 more plies. This conversion into the "2 against 0" database will win, of course, but the DTC has no information regarding the "distance to win".

2.4 Creating a PPL Database

Unlike a DTC database, a PPL database stores complete information about the line of play leading all the way to the terminal (lost) position. It does this by backing up and storing the number of plies to win or lose for every won or lost position during the database generation process, starting with the 2-piece database. Like the DTC database, an entire byte is required for each position in order to store this move-to-win (MTW) information.

Computation of a PPL database is much more difficult, both algorithmically and in terms of the amount of calculation required, than that of either a GTV or DTC database. This is due to the fact that one is not necessarily done once an MTW value has been assigned to a position. During the computation of a GTV or DTC database, once a GTV (win/loss/draw) or DTC (iteration count) value has been assigned to a position, no further computation is required. During the computation of a PPL database, the MTW values are subject to change from one iteration to the next. The MTV values are backed up through a tree of possible lines of play, and this tree dynamically changes as a function of the iteration depth. This process essentially amounts to a complex sorting procedure which cannot be terminated until a pass is made over the entire database that produces no changes in any of the MTW values.

The sorting procedure works as follows. In a won position all moves are generated, and the resulting positions that lose for the other side are queried for their MTW values (which may not yet exist or be reliable, depending on the position and the current iteration). The smallest value is then backed up into the parent position. This represents the move that will result in the quickest win. In a lost position all moves are generated, and the largest resulting MTW value is backed up into the parent position. (Recall that in a lost position all moves must lead to wins for the other side.) This represents the move that will result in the longest, most drawn-out loss. The database generation program makes repeated passes over the database slice being computed, and the sorting process continues until none of the MTW values changes.

It should be noted that there are some lines of play that lead to wins with no "conversion" taking place, and this must be taken into account during the generation of the database. This situation occurs when a move is made that blocks the opponent so he cannot move. A loss in the game of checkers occurs when one side cannot move, due to not having any pieces remaining, or having pieces on the board that are blocked in such a way that no moves are available. The blocking move leading to the win may or may not involve a capture or a promotion (i.e., a conversion).

3. The Longest 7-Piece Database Win

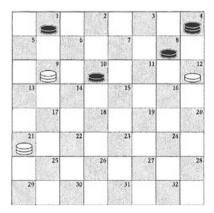


Figure 5. Black to move wins in 253 plies with 8-11.

Figure 5 contains the longest 7piece database win. Below we provide the PPL database solution.

Listing 1. The PPL database solution to the longest 7-piece win.

8-11, 12-8, 1-5, 9-13, 10-14, 8-3, 11-16, 13-9, 14-18, 9-14, 18-23, 14-18, 23-27, 18-23, 27-32, 21-17, 32-28, 17-22, 28-24, 22-26, 24-20, 23-27, 16-19, 26-31, 19-24, 27-32, 20-16, 32-28, 16-19, 3-7, 4-8, 28-32, 19-16, 31-26, 16-20, 7-10, 8-11, 26-31, 5-9, 10-6, 9-13, 6-10, 20-16, 32-27, 24-28, 27-32, 16-19, 31-27, 11-8, 27-24, 19-23, 10-14, 8-11, 24-20, 11-15, 20-24, 23-26, 24-27, 26-30, 27-31, 15-11, 31-27, 11-16, 27-23, 30-25, 14-18, 13-17, 18-14, 25-21, 23-26, 16-19,

26-31, 17-22, 31-27, 19-15, 27-31, 21-25, 32-27, 15-19, 27-32, 25-30, 31-27, 19-16, 14-10, 30-26, 27-24, 22-25, 10-15, 26-22, 24-20, 16-12, 15-19, 25-30, 19-15, 30-26, 20-24, 22-17, 15-10, 12-16, 10-15, 17-13, 24-20, 16-12, 15-11, 13-9, 20-24, 26-31, 24-20, 9-6, 11-15, 6-2, 15-11, 31-26, 11-15, 2-7, 15-18, 7-10,

 $\begin{array}{l} 20-24,\ 26-31,\ 24-19,\ 12-8,\ 18-15,\ 10-7,\ 15-18,\ 8-11,\ 18-14,\ 31-26,\ 14-18,\ 7-10,\ 19-24,\ 10-15,\ 18-14,\ 26-23,\ 14-9,\ 23-19,\ 9-5,\ 11-7,\ 5-9,\ 7-10,\ 24-20,\ 15-11,\ 20-24,\ 19-15,\ 9-5,\ 10-6,\ 5-1,\ 6-2,\ 24-20,\ 15-19,\ 1-5,\ 19-16,\ 20-24,\ 16-20,\ 24-27,\ 2-6,\ 5-1,\ 6-10,\ 1-5,\ 11-16,\ 5-9,\ 10-15,\ 9-14,\ 15-19,\ 27-31,\ 20-24,\ 31-26,\ 19-15,\ 26-22,\ 24-20,\ 14-9,\ 16-19,\ 9-13,\ 20-24,\ 22-17,\ 19-23,\ 17-21,\ 24-19,\ 13-17,\ 23-18,\ 17-13,\ 18-22,\ 21-17,\ 22-26,\ 13-9,\ 19-23,\ 9-5,\ 23-27,\ 32-23,\ 26-19,\ 5-9,\ 30-26,\ 1-5,\ 26-22,\ 9-14,\ 28-32,\ 5-1,\ 32-27,\ 1-5,\ 27-23,\ 5-1,\ 22-18,\ 14-10,\ 18-15,\ 10-6,\ 23-26,\ 6-2,\ 19-23,\ 2-6,\ 26-22,\ 1-5,\ 23-18,\ 6-9,\ 15-10,\ 9-13,\ 18-14,\ 13-9,\ 22-18,\ 9-13,\ 10-6,\ 5-1,\ 14-10,\ 13-17,\ 10-14,\ 17-10,\ 6-15,\ 1-5,\ 18-14,\ 5-1,\ 14-10,\ 1-5,\ 10-6,\ 5-1,\ 15-10,\ 1-5,\ 6-1,\ 5-9,\ 1-5,\ 9-13,\ 10-15,\ 13-17,\ 15-18,\ 17-13,\ 18-22,\ 13-9,\ 5-14\\ \end{array}$

3.1 Perfect Play Data

Table 2 lists statistics on all of the 2-to-7 piece database slices that were solved with four or fewer pieces for one side and Black to move. In it is shown the total number of positions as a function of the database slice, the number of plies associated with the longest win and loss for each slice, and one position for a longest win for Black to move. In some cases, as the material distribution becomes more dominant for one side, the longest win features a position with a forced jump for the weaker side to move. After a bit of reflection, this result makes sense. With the strong side to move, the win will precipitate very quickly. The weak side to move will execute a jump, perhaps equalizing or even surpassing the material of the former strong side of the board, then the total number of plies to win from the resulting sub-database will substantially add to the length of the game. In the "Position" column, BK = Black King, WK = White King, BC = Black Checker, WC = White Checker.

Material Distribution	Total Fosition	s Longest wi	II/Loss Position
1K + 0C vs. $1K + 0C$	992	11/10	BK: 4; WK: 29
1K + 0C vs. 0K + 1C	868	11/10	BK: 32; WC: 20
0K + 1C vs. $1K + 0C$	868	5/12	BC: 14; WK: 26
0K + 1C vs. 0K + 1C	760	13/12	BC: 25; WC: 30
2K + 0C vs. $1K + 0C$	14,880	33/34	BK: 1,2; WK: 19
2K + 0C vs. 0K + 1C	13,020	33/34	BK: 1,2; WC: 19
1K + 1C vs. 1K + 0C	26,040	47/48	BK: 32; BC: 4; WK: 23
1K + 1C vs. 0K + 1C	22,800	47/48	BK: 32; BC: 4; WC: 15
0K + 2C vs. 1K + 0C	11,340	61/62	BC: 3,4; WK: 26
0K + 2C vs. 0K + 1C	9,936	61/62	BC: 3,4; WC: 26
2K + 0C vs. $2K + 0C$	215,760	49/48	BK: 26,30; WK: 29,31
2K + 0C vs. 1K + 1C	377,580	95/94	BK: 2,3; WK: 21; WC: 25
2K + 0C vs. 0K + 2C	164,430	89/92	BK: 28,31; WC: 6,30
1K + 1C vs. 1K + 1C	661,200	103/102	BK: 28; BC: 18; WK: 3; WC: 29
1K + 1C vs. 0K + 2C	288,144	107/108	BK: 28; BC: 4; WC: 27,30
0K + 2C vs. $0K + 2C$	125,664	109/108	BC: 4,24; WC: 29,30

Material Distribution Total Positions Longest Win/Loss Position

$\begin{array}{rrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrr$		· ·		
$\begin{array}{llllllllllllllllllllllllllllllllllll$	3K + 0C vs. $1K + 0C$	143,840	29/30	BK: 7,16,29; WK: 11
$\begin{array}{l c c c c c c c c c c c c c c c c c c c$	3K + 0C vs. 0K + 1C	125,860	27/28	BK: 28,31,32; WC: 19
$\begin{array}{rrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrr$	2K + 1C vs. 1K + 0C	377,580	41/38	BK: 7,16; BC: 8; WK: 3
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	2K + 1C vs. 0K + 1C	330,600	37/32	BK: 29,30; BC: 25; WC: 31
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	1K + 2C vs. $1K + 0C$	328,860	53/54	BK: 19; BC: 9,10; WK: 15
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	1K + 2C vs. 0K + 1C	288,144	41/42	BK: 32; BC: 9,14; WC: 13
$\begin{array}{l c c c c c c c c c c c c c c c c c c c$	0K + 3C vs. 1K + 0C	95,004	59/58	BC: 4,7,8; WK: 3
$\begin{array}{l c c c c c c c c c c c c c c c c c c c$	0K + 3C vs. 0K + 1C	83,304	55/56	BC: 7,8,11; WC: 12
$\begin{array}{llllllllllllllllllllllllllllllllllll$	3K + 0C vs. 2K + 0C	2,013,760	67/68	BK: 8,29,30; WK: 12,18
$\begin{array}{llllllllllllllllllllllllllllllllllll$	3K + 0C vs. 1K + 1C	3,524,080	89/90	BK: 24,16,12; WK: 20; WC: 29
$\begin{array}{llllllllllllllllllllllllllllllllllll$	3K + 0C vs. 0K + 2C	1,534,680	81/62	BK: 25,26,29; WC: 9,30
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	2K + 1C vs. 2K + 0C	5,286,120	147/148	BK: 4,29; BC: 5; WK: 26,30
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	2K + 1C vs. 1K + 1C	9,256,800	139/140	BK: 4,30; BC: 5; WK: 22; WC: 10
1K + 2C vs. 1K + 1C $8,068,032$ $159/160$ $BK: 8; BC: 5,9; WK: 10; WC: 31$ $1K + 2C vs. 0K + 2C$ $3,518,592$ $111/140$ $BK: 28; BC: 4,8; WC: 7,12$ $0K + 3C vs. 2K + 0C$ $1,330,056$ $155/154$ $BC: 1,3,4; WK: 5,26$ $0K + 3C vs. 1K + 1C$ $2,332,512$ $161/162$ $BC: 5,79; WC: 6,26$ $4K + 0C vs. 1K + 0C$ $1,080,056$ $155/160$ $BC: 5,79; WC: 6,26$ $4K + 0C vs. 1K + 0C$ $1,006,880$ $29/30$ $BK: 9,17,26,27; WK: 22$ $4K + 0C vs. 0K + 1C$ $881,020$ $23/24$ $BK: 428,29,32; WC: 23$ $3K + 1C vs. 0K + 1C$ $3,085,600$ $25/26$ $BK: 28,31,32; BC: 24; WC: 19$ $2K + 2C vs. 1K + 0C$ $4,604,040$ $37/38$ $BK: 31,32; BC: 27,28; WK: 24$ $2K + 2C vs. 1K + 0C$ $4,604,040$ $37/38$ $BK: 31,32; BC: 27,28; WC: 30$ $1K + 3C vs. 1K + 0C$ $2,660,112$ $43/44$ $BK: 26; BC: 4,11,19; WK: 23$ $1K + 3C vs. 0K + 1C$ $2,332,512$ $39/40$ $BK: 4; BC: 7,8,11; WC: 12$ $0K + 4C vs. 0K + 1C$ $503,100$ $49/50$ $BC: 4,7,8,11; WC: 12$ $0K + 4C vs. 0K + 1C$ $503,100$ $49/50$ $BC: 4,7,8,11; WC: 12$ $3K + 0C vs. 3K + 0C$ $18,123,840$ $73/74$ $BK: 31,22,3; WK: 16,31,32$ $3K + 0C vs. 1K + 2C$ $41,436,360$ $151/150$ $BK: 18,25,30$ $2K + 1C vs. 3K + 0C$ $47,575,080$ $147/146$ $BK: 12,6; BC: 5; WK: 18,25,30$ $2K + 1C vs. 3K + 0C$ $41,436,360$ $151/150$ $BK: 12,27; BC: 22; WC: 23,28,32$ $2K + 1C vs. 3K + 0C$ $41,436,360$	2K + 1C vs. 0K + 2C	4,034,016	93/88	BK: 7,26; BC: 16; WC: 11,30
1K + 2C vs. 0K + 2C $3,518,592$ $111/140$ $BK: 28; BC: 4,8; WC: 7,12$ $0K + 3C vs. 2K + 0C$ $1,330,056$ $155/154$ $BC: 1,3,4; WK: 5,26$ $0K + 3C vs. 1K + 1C$ $2,332,512$ $161/162$ $BC: 1,4,5; WK: 14; WC: 24$ $0K + 3C vs. 0K + 2C$ $1,018,056$ $155/160$ $BC: 5,7,9; WC: 6,26$ $4K + 0C vs. 1K + 0C$ $1,006,880$ $29/30$ $BK: 9,17,26,27; WK: 22$ $4K + 0C vs. 0K + 1C$ $881,020$ $23/24$ $BK: 4,28,29,32; WC: 23$ $3K + 1C vs. 1K + 0C$ $3,524,080$ $29/30$ $BK: 17,26,27; BC: 9; WK: 22$ $3K + 1C vs. 0K + 1C$ $3,085,600$ $25/26$ $BK: 28,31,32; BC: 24; WC: 19$ $2K + 2C vs. 0K + 1C$ $4,034,016$ $31/28$ $BK: 31,32; BC: 27,28; WK: 24$ $2K + 2C vs. 0K + 1C$ $4,034,016$ $31/28$ $BK: 31,32; BC: 27,28; WC: 30$ $1K + 3C vs. 1K + 0C$ $2,660,112$ $43/44$ $BK: 26; BC: 4,11,19; WK: 23$ $1K + 3C vs. 0K + 1C$ $2,332,512$ $39/40$ $BK: 41,20; WC: 12$ $0K + 4C vs. 0K + 1C$ $503,100$ $49/50$ $BC: 4,7,8,11; WC: 12$ $0K + 4C vs. 0K + 1C$ $503,100$ $49/50$ $BC: 4,7,8,11; WC: 12$ $3K + 0C vs. 3K + 0C$ $18,123,840$ $73/74$ $BK: 13,26,28; WC: 5,14,29$ $3K + 0C vs. 0K + 3C$ $11,970,504$ $149/150$ $BK: 13,26,28; WC: 5,14,29$ $2K + 1C vs. 3K + 0C$ $47,575,080$ $147/146$ $BK: 10,19; BC: 1; WK: 17; WC: 28,29$ $3K + 0C vs. 0K + 3C$ $11,970,504$ $149/150$ $BK: 15,27; BC: 22; WC: 23,28,32$ $1K + 2C vs. 3K + 0C$	1K + 2C vs. $2K + 0C$	4,604,040	149/148	BK: 4; BC: 5,25; WK: 30,31
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	1K + 2C vs. $1K + 1C$	8,068,032	159/160	BK: 8; BC: 5,9; WK: 10; WC: 31
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	1K + 2C vs. $0K + 2C$	3,518,592	111/140	BK: 28; BC: 4,8; WC: 7,12
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	0K + 3C vs. $2K + 0C$	1,330,056	155/154	BC: 1,3,4; WK: 5,26
$\begin{array}{r c c c c c c c c c c c c c c c c c c c$	0K + 3C vs. $1K + 1C$	2,332,512	161/162	BC: 1,4,5; WK: 14; WC: 24
4K + 0C vs. 0K + 1C $881,020$ $23/24$ $BK: 4,28,29,32; WC: 23$ $3K + 1C vs. 1K + 0C$ $3,524,080$ $29/30$ $BK: 17,26,27; BC: 9; WK: 22$ $3K + 1C vs. 0K + 1C$ $3,085,600$ $25/26$ $BK: 28,31,32; BC: 24; WC: 19$ $2K + 2C vs. 1K + 0C$ $4,604,040$ $37/38$ $BK: 31,32; BC: 27,28; WK: 24$ $2K + 2C vs. 0K + 1C$ $4,034,016$ $31/28$ $BK: 31,32; BC: 27,28; WC: 30$ $1K + 3C vs. 0K + 1C$ $2,660,112$ $43/44$ $BK: 26; BC: 4,11,19; WK: 23$ $1K + 3C vs. 0K + 1C$ $2,332,512$ $39/40$ $BK: 4; BC: 7,8,11; WC: 12$ $0K + 4C vs. 1K + 0C$ $573,300$ $51/52$ $BC: 4,7,8,11; WC: 12$ $0K + 4C vs. 0K + 1C$ $503,100$ $49/50$ $BC: 4,7,8,11; WC: 12$ $3K + 0C vs. 3K + 0C$ $18,123,840$ $73/74$ $BK: 3,8,15; WK: 7,22; WC: 28$ $3K + 0C vs. 1K + 2C$ $41,436,360$ $151/150$ $BK: 13,26,28; WC: 5,14,29$ $3K + 0C vs. 0K + 3C$ $11,970,504$ $149/150$ $BK: 11,26; BC: 5; WK: 18,25,30$ $2K + 1C vs. 3K + 0C$ $47,575,080$ $147/146$ $BK: 12,27; BC: 22; WC: 23,28,32$ $2K + 1C vs. 0K + 3C$ $31,488,912$ $155/160$ $BK: 15,27; BC: 22; WC: 23,28,32$ $1K + 2C vs. 3K + 0C$ $41,436,360$ $151/150$ $BK: 26; BC: 4,5; WK: 18,25,32$ $1K + 2C vs. 3K + 0C$ $41,436,360$ $151/150$ $BK: 25,18; BC: 1; WK: 17; WC: 24,28$ $2K + 1C vs. 0K + 3C$ $31,488,912$ $155/160$ $BK: 15,27; BC: 22; WC: 23,28,32$ $1K + 2C vs. 1K + 2C$ $95,001,984$ $167/166$ $BK: 25, BC: 5,9;$	0K + 3C vs. 0K + 2C	1,018,056	155/160	BC: 5,7,9; WC: 6,26
$\begin{array}{rrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrr$	4K + 0C vs. $1K + 0C$	1,006,880	29/30	BK: 9,17,26,27; WK: 22
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	4K + 0C vs. 0K + 1C	881,020	23/24	BK: 4,28,29,32; WC: 23
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	3K + 1C vs. $1K + 0C$	3,524,080	29/30	BK: 17,26,27; BC: 9; WK: 22
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	3K + 1C vs. 0K + 1C	3,085,600	25/26	BK: 28,31,32; BC: 24; WC: 19
1K + 3C vs. 1K + 0C $2,660,112$ $43/44$ $BK: 26; BC: 4,11,19; WK: 23$ $1K + 3C vs. 0K + 1C$ $2,332,512$ $39/40$ $BK: 4; BC: 7,8,11; WC: 12$ $0K + 4C vs. 1K + 0C$ $573,300$ $51/52$ $BC: 7,8,11; WC: 12$ $0K + 4C vs. 0K + 1C$ $503,100$ $49/50$ $BC: 4,7,8,11; WC: 12$ $3K + 0C vs. 3K + 0C$ $18,123,840$ $73/74$ $BK: 3,12,23; WK: 16,31,32$ $3K + 0C vs. 2K + 1C$ $47,575,080$ $147/146$ $BK: 3,8,15; WK: 7,22; WC: 28$ $3K + 0C vs. 1K + 2C$ $41,436,360$ $151/150$ $BK: 13,26,28; WC: 5,14,29$ $2K + 1C vs. 0K + 3C$ $11,970,504$ $149/150$ $BK: 11,26; BC: 5; WK: 18,25,30$ $2K + 1C vs. 3K + 0C$ $47,575,080$ $147/146$ $BK: 11,26; BC: 1; WK: 2,6; WC: 31$ $2K + 1C vs. 3K + 0C$ $47,575,080$ $147/146$ $BK: 11,26; BC: 1; WK: 17; WC: 24,28$ $2K + 1C vs. 3K + 0C$ $41,436,360$ $153/152$ $BK: 10,19; BC: 1; WK: 17; WC: 24,28$ $2K + 1C vs. 0K + 3C$ $31,488,912$ $155/160$ $BK: 25,18; BC: 1; WK: 18,25,32$ $1K + 2C vs. 3K + 0C$ $41,436,360$ $151/150$ $BK: 26; BC: 4,5; WK: 18,25,32$ $1K + 2C vs. 3K + 0C$ $41,436,360$ $151/150$ $BK: 26; BC: 5,9; WK: 8,15; WC: 321K + 2C vs. 0K + 3C27,487,512163/164BK: 14; BC: 5,6; WC: 25,19,120K + 3C vs. 3K + 0C11,970,504149/150BC: 4,19,28; WK: 5,7,200K + 3C vs. 3K + 0C11,970,504149/150BC: 4,19,28; WK: 5,7,200K + 3C vs. 3K + 0C11,970,504149/1502K + 2C$ vs. $1K + 0C$	2K + 2C	4,604,040	37/38	BK: 31,32; BC: 27,28; WK: 24
1K + 3C vs. 0K + 1C $2,332,512$ $39/40$ $BK: 4; BC: 7,8,11; WC: 12$ $0K + 4C vs. 1K + 0C$ $573,300$ $51/52$ $BC: 7,8,11,15; WK: 12$ $0K + 4C vs. 0K + 1C$ $503,100$ $49/50$ $BC: 4,7,8,11; WC: 12$ $3K + 0C vs. 3K + 0C$ $18,123,840$ $73/74$ $BK: 3,12,23; WK: 16,31,32$ $3K + 0C vs. 3K + 0C$ $18,123,840$ $73/74$ $BK: 3,12,23; WK: 16,31,32$ $3K + 0C vs. 3K + 0C$ $18,123,840$ $73/74$ $BK: 3,12,23; WK: 16,31,32$ $3K + 0C vs. 2K + 1C$ $47,575,080$ $147/146$ $BK: 3,8,15; WK: 7,22; WC: 28$ $3K + 0C vs. 1K + 2C$ $41,436,360$ $151/150$ $BK: 13,26,28; WC: 5,14,29$ $2K + 1C vs. 3K + 0C$ $47,575,080$ $147/146$ $BK: 11,26; BC: 5; WK: 18,25,30$ $2K + 1C vs. 3K + 0C$ $47,575,080$ $147/146$ $BK: 10,19; BC: 1; WK: 2,6; WC: 31$ $2K + 1C vs. 3K + 0C$ $124,966,800$ $153/152$ $BK: 10,19; BC: 1; WK: 2,6; WC: 31$ $2K + 1C vs. 3K + 0C$ $124,966,800$ $153/152$ $BK: 10,19; BC: 1; WK: 2,6; WC: 31$ $2K + 1C vs. 0K + 3C$ $31,488,912$ $155/160$ $BK: 15,27; BC: 22; WC: 23,28,32$ $1K + 2C vs. 3K + 0C$ $41,436,360$ $151/150$ $BK: 26; BC: 4,5; WK: 18,25,32$ $1K + 2C vs. 3K + 0C$ $41,436,360$ $151/150$ $BK: 16; BC: 5,9; WK: 8,15; WC: 32$ $1K + 2C vs. 0K + 3C$ $27,487,512$ $163/164$ $BK: 14; BC: 5,6; WC: 25,19,12$ $0K + 3C vs. 3K + 0C$ $11,970,504$ $149/150$ $BC: 4,19,28; WK: 5,7,20$ $0K + 3C vs. 3K + 0C$ $11,970,504$ $149/150$	2K + 2C vs. $0K + 1C$	4,034,016	31/28	BK: 31,32; BC: 27,28; WC: 30
0K + 4C vs. 1K + 0C $573,300$ $51/52$ $BC: 7,8,11,15; WK: 12$ $0K + 4C vs. 0K + 1C$ $503,100$ $49/50$ $BC: 4,7,8,11; WC: 12$ $3K + 0C vs. 3K + 0C$ $18,123,840$ $73/74$ $BK: 3,12,23; WK: 16,31,32$ $3K + 0C vs. 2K + 1C$ $47,575,080$ $147/146$ $BK: 3,8,15; WK: 7,22; WC: 28$ $3K + 0C vs. 1K + 2C$ $41,436,360$ $151/150$ $BK: 1,8,15; WK: 7, WC: 28,29$ $3K + 0C vs. 0K + 3C$ $11,970,504$ $149/150$ $BK: 13,26,28; WC: 5,14,29$ $2K + 1C vs. 3K + 0C$ $47,575,080$ $147/146$ $BK: 11,26; BC: 5; WK: 18,25,30$ $2K + 1C vs. 3K + 0C$ $47,575,080$ $147/146$ $BK: 10,19; BC: 1; WK: 2,6; WC: 31$ $2K + 1C vs. 3K + 0C$ $47,575,080$ $147/146$ $BK: 10,19; BC: 1; WK: 2,6; WC: 31$ $2K + 1C vs. 3K + 0C$ $47,575,080$ $153/152$ $BK: 10,19; BC: 1; WK: 2,6; WC: 31$ $2K + 1C vs. 3K + 0C$ $11,970,504$ $161/162$ $BK: 25,18; BC: 1; WK: 17; WC: 24,28$ $2K + 1C vs. 0K + 3C$ $31,488,912$ $155/160$ $BK: 15,27; BC: 22; WC: 23,28,32$ $1K + 2C vs. 3K + 0C$ $41,436,360$ $151/150$ $BK: 26; BC: 4,5; WK: 18,25,32$ $1K + 2C vs. 3K + 0C$ $41,436,360$ $151/150$ $BK: 16; BC: 5,9; WK: 8,15; WC: 32$ $1K + 2C vs. 0K + 3C$ $27,487,512$ $163/164$ $BK: 14; BC: 5,6; WC: 25,19,12$ $0K + 3C vs. 3K + 0C$ $11,970,504$ $149/150$ $BC: 4,19,28; WK: 5,7,20$ $0K + 3C vs. 1K + 2C$ $27,487,512$ $163/164$ $BC: 1,5,10; WK:6,18; WC: 11$ $0K + 3C vs. 1K + 2C$ $27,487,512$ <td>1K + 3C vs. $1K + 0C$</td> <td>2,660,112</td> <td>43/44</td> <td>BK: 26; BC: 4,11,19; WK: 23</td>	1K + 3C vs. $1K + 0C$	2,660,112	43/44	BK: 26; BC: 4,11,19; WK: 23
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	1K + 3C vs. 0K + 1C	2,332,512	39/40	BK: 4; BC: 7,8,11; WC: 12
3K + 0C vs. 3K + 0C $18,123,840$ $73/74$ $BK: 3,12,23; WK: 16,31,32$ $3K + 0C vs. 2K + 1C$ $47,575,080$ $147/146$ $BK: 3,8,15; WK: 7,22; WC: 28$ $3K + 0C vs. 1K + 2C$ $41,436,360$ $151/150$ $BK: 1,8,15; WK: 7, WC: 28,29$ $3K + 0C vs. 0K + 3C$ $11,970,504$ $149/150$ $BK: 13,26,28; WC: 5,14,29$ $2K + 1C vs. 3K + 0C$ $47,575,080$ $147/146$ $BK: 11,26; BC: 5; WK: 18,25,30$ $2K + 1C vs. 3K + 0C$ $47,575,080$ $147/146$ $BK: 10,19; BC: 1; WK: 2,6; WC: 31$ $2K + 1C vs. 2K + 1C$ $124,966,800$ $153/152$ $BK: 10,19; BC: 1; WK: 2,6; WC: 31$ $2K + 1C vs. 1K + 2C$ $108,918,432$ $161/162$ $BK: 25,18; BC: 1; WK: 17; WC: 24,28$ $2K + 1C vs. 0K + 3C$ $31,488,912$ $155/160$ $BK: 15,27; BC: 22; WC: 23,28,32$ $1K + 2C vs. 3K + 0C$ $41,436,360$ $151/150$ $BK: 26; BC: 4,5; WK: 18,25,32$ $1K + 2C vs. 2K + 1C$ $108,918,432$ $161/162$ $BK: 25; BC: 5,9; WK: 8,15; WC: 32$ $1K + 2C vs. 1K + 2C$ $95,001,984$ $167/166$ $BK: 25; BC: 5,9; WK: 18; WC: 17,30$ $1K + 2C vs. 0K + 3C$ $27,487,512$ $163/164$ $BK: 14; BC: 5,6; WC: 25,19,12$ $0K + 3C vs. 3K + 0C$ $11,970,504$ $149/150$ $BC: 4,19,28; WK: 5,7,20$ $0K + 3C vs. 1K + 2C$ $27,487,512$ $163/164$ $BC: 1,5,10; WK:6,18; WC: 11$ $0K + 3C vs. 1K + 2C$ $27,487,512$ $163/164$ $BC: 1,5,10; WK:6,18; WC: 11$ $0K + 3C vs. 1K + 2C$ $27,487,512$ $163/164$ $BC: 8,14,21; WK: 19; WC: 27,28$	0K + 4C vs. 1K + 0C	573,300	51/52	BC: 7,8,11,15; WK: 12
3K + 0C vs. 2K + 1C $47,575,080$ $147/146$ $BK: 3,8,15; WK: 7,22; WC: 28$ $3K + 0C vs. 1K + 2C$ $41,436,360$ $151/150$ $BK: 1,8,15; WK: 7, 22; WC: 28,29$ $3K + 0C vs. 0K + 3C$ $11,970,504$ $149/150$ $BK: 13,26,28; WC: 5,14,29$ $2K + 1C vs. 3K + 0C$ $47,575,080$ $147/146$ $BK: 11,26; BC: 5; WK: 18,25,30$ $2K + 1C vs. 2K + 1C$ $124,966,800$ $153/152$ $BK: 10,19; BC: 1; WK: 2,6; WC: 31$ $2K + 1C vs. 1K + 2C$ $108,918,432$ $161/162$ $BK: 25,18; BC: 1; WK: 17; WC: 24,28$ $2K + 1C vs. 0K + 3C$ $31,488,912$ $155/160$ $BK: 15,27; BC: 22; WC: 23,28,32$ $1K + 2C vs. 3K + 0C$ $41,436,360$ $151/150$ $BK: 26; BC: 4,5; WK: 18,25,32$ $1K + 2C vs. 2K + 1C$ $108,918,432$ $161/162$ $BK: 16; BC: 5,9; WK: 8,15; WC: 32$ $1K + 2C vs. 2K + 1C$ $108,918,432$ $161/162$ $BK: 16; BC: 5,9; WK: 8,15; WC: 32$ $1K + 2C vs. 0K + 3C$ $27,487,512$ $163/164$ $BK: 14; BC: 5,6; WC: 25,19,12$ $0K + 3C vs. 3K + 0C$ $11,970,504$ $149/150$ $BC: 4,19,28; WK: 5,7,20$ $0K + 3C vs. 1K + 2C$ $27,487,512$ $163/164$ $BC: 1,5,10; WK:6,18; WC: 11$ $0K + 3C vs. 1K + 2C$ $27,487,512$ $163/164$ $BC: 1,5,10; WK:6,18; WC: 11$	0K + 4C vs. $0K + 1C$	503,100	49/50	BC: 4,7,8,11; WC: 12
3K + 0C vs. 1K + 2C $41,436,360$ $151/150$ $BK: 1,8,15; WK: 7; WC: 28,29$ $3K + 0C vs. 0K + 3C$ $11,970,504$ $149/150$ $BK: 13,26,28; WC: 5,14,29$ $2K + 1C vs. 3K + 0C$ $47,575,080$ $147/146$ $BK: 11,26; BC: 5; WK: 18,25,30$ $2K + 1C vs. 2K + 1C$ $124,966,800$ $153/152$ $BK: 10,19; BC: 1; WK: 2,6; WC: 31$ $2K + 1C vs. 1K + 2C$ $108,918,432$ $161/162$ $BK: 25,18; BC: 1; WK: 17; WC: 24,28$ $2K + 1C vs. 0K + 3C$ $31,488,912$ $155/160$ $BK: 16; BC: 5,9; WK: 18,25,32$ $1K + 2C vs. 3K + 0C$ $41,436,360$ $151/150$ $BK: 26; BC: 4,5; WK: 18,25,32$ $1K + 2C vs. 2K + 1C$ $108,918,432$ $161/162$ $BK: 16; BC: 5,9; WK: 8,15; WC: 32$ $1K + 2C vs. 2K + 1C$ $108,918,432$ $161/162$ $BK: 16; BC: 5,9; WK: 8,15; WC: 32$ $1K + 2C vs. 0K + 3C$ $27,487,512$ $163/164$ $BK: 14; BC: 5,6; WC: 25,19,12$ $0K + 3C vs. 3K + 0C$ $11,970,504$ $149/150$ $BC: 4,19,28; WK: 5,7,20$ $0K + 3C vs. 1K + 2C$ $27,487,512$ $163/164$ $BC: 1,5,10; WK:6,18; WC: 11$ $0K + 3C vs. 1K + 2C$ $27,487,512$ $163/164$ $BC: 1,5,10; WK:6,18; WC: 11$	3K + 0C vs. $3K + 0C$	18,123,840	73/74	BK: 3,12,23; WK: 16,31,32
3K + 0C vs. 0K + 3C $11,970,504$ $149/150$ $BK: 13,26,28; WC: 5,14,29$ $2K + 1C vs. 3K + 0C$ $47,575,080$ $147/146$ $BK: 11,26; BC: 5; WK: 18,25,30$ $2K + 1C vs. 2K + 1C$ $124,966,800$ $153/152$ $BK: 10,19; BC: 1; WK: 2,6; WC: 31$ $2K + 1C vs. 1K + 2C$ $108,918,432$ $161/162$ $BK: 25,18; BC: 1; WK: 17; WC: 24,28$ $2K + 1C vs. 0K + 3C$ $31,488,912$ $155/160$ $BK: 15,27; BC: 22; WC: 23,28,32$ $1K + 2C vs. 3K + 0C$ $41,436,360$ $151/150$ $BK: 26; BC: 4,5; WK: 18,25,32$ $1K + 2C vs. 2K + 1C$ $108,918,432$ $161/162$ $BK: 16; BC: 5,9; WK: 8,15; WC: 32$ $1K + 2C vs. 1K + 2C$ $95,001,984$ $167/166$ $BK: 25; BC: 5,9; WK: 18; WC: 17,30$ $1K + 2C vs. 0K + 3C$ $27,487,512$ $163/164$ $BK: 14; BC: 5,6; WC: 25,19,12$ $0K + 3C vs. 3K + 0C$ $11,970,504$ $149/150$ $BC: 4,19,28; WK: 5,7,20$ $0K + 3C vs. 1K + 2C$ $27,487,512$ $163/164$ $BC: 1,5,10; WK:6,18; WC: 11$ $0K + 3C vs. 1K + 2C$ $27,487,512$ $163/164$ $BC: 1,5,10; WK:6,18; WC: 11$	3K + 0C vs. $2K + 1C$	47,575,080	147/146	BK: 3,8,15; WK: 7,22; WC: 28
2K + 1C vs. 3K + 0C $47,575,080$ $147/146$ $BK: 11,26; BC: 5; WK: 18,25,30$ $2K + 1C vs. 2K + 1C$ $124,966,800$ $153/152$ $BK: 10,19; BC: 1; WK: 2,6; WC: 31$ $2K + 1C vs. 1K + 2C$ $108,918,432$ $161/162$ $BK: 25,18; BC: 1; WK: 17; WC: 24,28$ $2K + 1C vs. 0K + 3C$ $31,488,912$ $155/160$ $BK: 15,27; BC: 22; WC: 23,28,32$ $1K + 2C vs. 3K + 0C$ $41,436,360$ $151/150$ $BK: 26; BC: 4,5; WK: 18,25,32$ $1K + 2C vs. 2K + 1C$ $108,918,432$ $161/162$ $BK: 16; BC: 5,9; WK: 8,15; WC: 32$ $1K + 2C vs. 1K + 2C$ $95,001,984$ $167/166$ $BK: 25; BC: 5,9; WK: 18; WC: 17,30$ $1K + 2C vs. 0K + 3C$ $27,487,512$ $163/164$ $BK: 14; BC: 5,6; WC: 25,19,12$ $0K + 3C vs. 3K + 0C$ $11,970,504$ $149/150$ $BC: 4,19,28; WK: 5,7,20$ $0K + 3C vs. 1K + 2C$ $27,487,512$ $163/164$ $BC: 1,5,10; WK:6,18; WC: 11$ $0K + 3C vs. 1K + 2C$ $27,487,512$ $163/164$ $BC: 1,5,10; WK:6,18; WC: 11$	3K + 0C vs. $1K + 2C$	41,436,360	151/150	BK: 1,8,15; WK: 7; WC: 28,29
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	3K + 0C vs. $0K + 3C$	11,970,504	149/150	BK: 13,26,28; WC: 5,14,29
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	2K + 1C vs. $3K + 0C$	47,575,080	147/146	BK: 11,26; BC: 5; WK: 18,25,30
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	2K + 1C vs. $2K + 1C$	124,966,800	153/152	BK: 10,19; BC: 1; WK: 2,6; WC: 31
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	2K + 1C vs. $1K + 2C$	108,918,432	161/162	BK: 25,18; BC: 1; WK: 17; WC: 24,28
1K + 2C vs. 2K + 1C 108,918,432 161/162 BK: 16; BC: 5,9; WK: 8,15; WC: 32 1K + 2C vs. 1K + 2C 95,001,984 167/166 BK: 25; BC: 5,9; WK: 18; WC: 17,30 1K + 2C vs. 0K + 3C 27,487,512 163/164 BK: 14; BC: 5,6; WC: 25,19,12 0K + 3C vs. 3K + 0C 11,970,504 149/150 BC: 4,19,28; WK: 5,7,20 0K + 3C vs. 2K + 1C 31,488,912 155/160 BC: 1,5,10; WK:6,18; WC: 11 0K + 3C vs. 1K + 2C 27,487,512 163/164 BC: 8,14,21; WK: 19; WC: 27,28	2K + 1C vs. $0K + 3C$	31,488,912	155/160	BK: 15,27; BC: 22; WC: 23,28,32
1K + 2C vs. 1K + 2C 95,001,984 167/166 BK: 25; BC: 5,9; WK: 18; WC: 17,30 1K + 2C vs. 0K + 3C 27,487,512 163/164 BK: 14; BC: 5,6; WC: 25,19,12 0K + 3C vs. 3K + 0C 11,970,504 149/150 BC: 4,19,28; WK: 5,7,20 0K + 3C vs. 2K + 1C 31,488,912 155/160 BC: 1,5,10; WK:6,18; WC: 11 0K + 3C vs. 1K + 2C 27,487,512 163/164 BC: 8,14,21; WK: 19; WC: 27,28	1K + 2C vs. $3K + 0C$	41,436,360	151/150	BK: 26; BC: 4,5; WK: 18,25,32
1K + 2C vs. 0K + 3C 27,487,512 163/164 BK: 14; BC: 5,6; WC: 25,19,12 0K + 3C vs. 3K + 0C 11,970,504 149/150 BC: 4,19,28; WK: 5,7,20 0K + 3C vs. 2K + 1C 31,488,912 155/160 BC: 1,5,10; WK:6,18; WC: 11 0K + 3C vs. 1K + 2C 27,487,512 163/164 BC: 8,14,21; WK: 19; WC: 27,28	1K + 2C vs. $2K + 1C$	108,918,432	161/162	BK: 16; BC: 5,9; WK: 8,15; WC: 32
0K + 3C vs. 3K + 0C 11,970,504 149/150 BC: 4,19,28; WK: 5,7,20 0K + 3C vs. 2K + 1C 31,488,912 155/160 BC: 1,5,10; WK:6,18; WC: 11 0K + 3C vs. 1K + 2C 27,487,512 163/164 BC: 8,14,21; WK: 19; WC: 27,28	1K + 2C vs. $1K + 2C$	95,001,984	167/166	BK: 25; BC: 5,9; WK: 18; WC: 17,30
0K + 3C vs. 2K + 1C 31,488,912 155/160 BC: 1,5,10; WK:6,18; WC: 11 0K + 3C vs. 1K + 2C 27,487,512 163/164 BC: 8,14,21; WK: 19; WC: 27,28	1K + 2C vs. 0K + 3C	27,487,512	163/164	
0K + 3C vs. 1K + 2C 27,487,512 163/164 BC: 8,14,21; WK: 19; WC: 27,28	0K + 3C vs. 3K + 0C	11,970,504	149/150	
	0K + 3C vs. 2K + 1C	31,488,912	155/160	
0K + 3C vs. 0K + 3C 79,59,904 161/162 BC: 1,2,3; WC: 14,17,19	0K + 3C vs. 1K + 2C	27,487,512	163/164	BC: 8,14,21; WK: 19; WC: 27,28
	0K + 3C vs. 0K + 3C	79,59,904	161/162	BC: 1,2,3; WC: 14,17,19

4K + 0C vs. 2K + 0C	13,592,880	67/68	BK: 4,12,29,30; WK: 11,22
4K + 0C vs. 1K + 1C	23,787,540	87/88	BK: 9,10,19,27; WK: 15; WC: 30
4K + 0C vs. $0K + 2C$	10,359,090	51/44	BK: 17,19,26,27; WC: 20,31
3K + 1C vs. $2K + 0C$	47,575,080	135/114	BK: 9,17,29; BC: 5; WK: 13,17
3K + 1C vs. 1K + 1C	83,311,200	91/88	BK: 17,25,26; BC: 19; WK: 20; WC: 29
3K + 1C vs. 0K + 2C	36,306,144	95/66	BK: 19,25,27; BC: 17; WC: 30,31
2K + 2C vs. $2K + 0C$	62,154,540	147/138	BK: 14,29; BC: 5,26; WK: 30,31
2K + 2C vs. $1K + 1C$	108,918,432	143/140	BK: 26,27; BC: 5,13; WK: 14; WC: 31
2K + 2C vs. 0K + 2C	47,500,992	99/80	BK: 17,19; BC: 4,26; WC: 30,31
1K + 3C vs. 2K + 0C	35,911,512	149/150	BK: 26; BC: 5,6,7; WK: 1,31
1K + 3C vs. 1K + 1C	62,977,824	153/154	BK: 9; BC: 4,5,8; WK: 10; WC: 13
1K + 3C vs. $0K + 2C$	27,487,512	109/146	BK: 28; BC: 4,8,11; WC: 7,12
0K + 4C vs. 2K + 0C	7,739,550	155/156	BC: 1,4,8,18; WK: 10,26
0K + 4C vs. 1K + 1C	13,583,700	153/154	BC: 1,6,16,19; WK: 14; WC: 20
0K + 4C vs. 0K + 2C	5,933,850	153/148	BC: 5,7,10,14; WC: 15,19
0K + 2C vs. $0K + 4C$	5,933,850	153/148	BC: 5,7,10,14; WC: 15,19
4K + 0C vs. $3K + 0C$	117,804,960	113/114	BK: 3,4,5,26; WK: 1,11,15
4K + 0C vs. 2K + 1C	309,238,020	149/142	BK: 2,29; BC: 5; WK: 6,7,8,30
4K + 0C vs. $1K + 2C$	269,336,340	149/148	BK: 3; BC: 5,15; WK: 6,18,25,30
4K + 0C vs. 0K + 3C	77,808,276	151/150	BC: 5,9,11; WK: 16,24,28,29
3K + 1C vs. 3K + 0C	412,317,360	207/208	BK: 21,28,30; BC: 3; WK: 1,22,32
3K + 1C vs. 2K + 1C	1,083,045,600	201/202	BK: 1,29,30; BC: 24; WK: 31,27; WC: 11
3K + 1C vs. 1K + 2C	943,959,744	153/144	BK: 10; BC: 4,5; WK: 14,18,25; WC: 9
3K + 1C vs. 0K + 3C	272,903,904	159/158	BC: 3,5,9; WK: 8,16,18; WC: 31
2K + 2C vs. 3K + 0C	538,672,680	245/246	BK: 4,11; BC: 2,5; WK: 3,10,29
2K + 2C vs. 2K + 1C	1,415,939,616	241/240	BK: 4,32; BC: 5,8; WK: 17,23; WC:12
2K + 2C vs. 1K + 2C	1,235,025,792	191/192	BK: 5,27; BC: 12,20; WK: 19; WC: 11,32
2K + 2C vs. 0K + 3C	357,337,656	161/166	BC: 2,5,12; WK: 3,16; WC: 9,31
1K + 3C vs. 3K + 0C	311,233,104	249/248	BK: 6; BC: 1,18,15; WK: 5,14,16
1K + 3C vs. 2K + 1C	818,711,712	253/252	BK: 4; BC: 1,8,10; WK: 9,21; WC: 12
1K + 3C vs. 1K + 2C	714,675,312	237/238	BK: 5; BC: 7,8,9; WK: 27; WC: 6,19
1K + 3C vs. 0K + 3C	206,957,504	183/198	BK: 29; BC: 5,7,8; WC: 12,24,30
0K + 4C vs. 3K + 0C	67,076,100	233/230	BC: 2,4,15,27; WK: 5,16,32
0K + 4C vs. $2K + 1C$	176,588,100	249/248	BC: 1,2,4,6; WK: 28,32; WC: 27
0K + 4C vs. 1K + 2C	154,280,100	243/242	BC: 4,5,6,8; WK: 28; WC: 12,27
0K + 4C vs. 0K + 3C	44,717,500	209/210	BC: 4,5,7,11; WC: 6,19,32

Table 2. Positions with the longest solutions for the 2- to-7-piece databases.

3.2 Difficult Theoretical Wins

It is possible to be in a position that is a theoretical win that is too difficult for a program to win even while consulting a GTV database (see Figures 6 and 7). This is due to several factors; we mention three of them.

1. The program will never make a move that loses or gives away the draw on any given turn as it consults the GTV databases, but sometimes every move in a position will win. The GTV database is of no help in reducing the size of the game tree in these instances.

- 2. The complete path to the win might not require the use of any of the intermediate goals in the evaluation function of the checkers program, so as long as the solution is beyond the horizon of the search, the win could be postponed.
- 3. King-heavy positions will produce principal variations in which the Kings tend to wander and gain nothing if forced trades can be avoided.

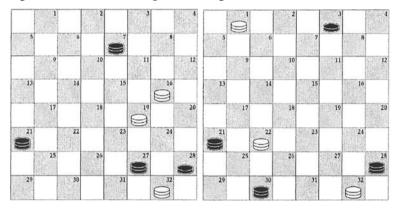


Figure 6. A longest conversion win with 3 Kings and 1 Checker versus 3 Kings. Black to move can force a trade after 149 plies, starting with the move 27-24.

Figure 7. A longest win with 3 Kings and 1 Checker versus 3 Kings. Moving the Checker on square 3 will result in a Draw. Only 28-24 will win.

Figure 6 comes from the DTC database of Murray Cash (Cash and Miller, 2002) and is listed as the longest conversion win in the 7-piece database in which a Checker remains unmoved. Comparing this position to Figure 19, page 248 from Schaeffer (1997) we note that Schaeffer had two of the white Kings on squares 12 and 15 instead of 16 and 19. The Schaeffer position and the Cash position both require 207 plies to win.

Figure 7 is from the Dodgen-Trice PPL database, showing another "longest win" possible in the same database slice as the position in Figure 6. Listings 2 and 3 show how the PPL database will play each position.

Listing 2. The PPL database solution to Figure 6.

27-24, 19-15, 24-20, 16-19, 21-25, 15-18, 25-30, 18-15, 20-24, 19-23, 7-2, 15-10, 30-25, 10-14, 25-22, 14-10, 24-20, 23-19, 22-25, 19-15, 20-16, 15-18, 16-12, 10-15, 2-6, 15-19, 6-9, 19-23, 9-13, 18-14, 25-21, 14-10, 13-17, 10-15, 21-25, 15-19, 25-30, 19-24, 17-22, 24-19, 22-26, 23-27, 26-31, 27-24, 30-26, 19-15, 26-22, 24-20, 31-26, 20-24, 22-17, 15-10, 12-16, 10-15, 17-13, 24-20, 16-12, 15-11, 13-9, 20-24, 26-31, 24-20, 9-6, 11-15, 6-2, 15-11, 31-26, 11-15, 2-7, 15-18, 7-10, 20-24, 26-31, 24-19, 12-8, 18-15, 10-7, 15-18, 8-11, 18-14, 31-26, 14-18, \rangle r-10, 19-24, 10-15, 18-14, 26-23, 14-9, 23-19, 9-5, 11-7, 5-9, 7-10, 24-10, 12-16, 10-15, 18-14, 26-23, 14-9, 23-19, 9-5, 11-7, 5-9, 7-10, 24-10, 12-16, 10-15, 18-14, 26-23, 14-9, 23-19, 9-5, 11-7, 5-9, 7-10, 24-10, 12-18, 18-14, 26-23, 14-9, 23-19, 9-5, 11-7, 5-9, 7-10, 24-10, 12-18, 10-15, 18-14, 26-23, 14-9, 23-19, 9-5, 11-7, 5-9, 7-10, 24-10, 15-18, 12-16, 10-15, 18-14, 26-23, 14-9, 23-19, 9-5, 11-7, 5-9, 7-10, 24-10, 15-18, 12-16, 10-15, 18-14, 26-23, 14-9, 23-19, 9-5, 11-7, 5-9, 7-10, 24-10, 15-18, 12-16, 10-15, 18-14, 26-23, 14-9, 23-19, 9-5, 11-7, 5-9, 7-10, 24-10, 15-18, 14-18, \rangle r-10, 12-16, 10-15, 18-14, 26-23, 14-9, 23-19, 9-5, 11-7, 5-9, 7-10, 24-10, 15-18, 14-18, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14, 14-14,

 $\begin{array}{l} 20, 15\text{-}11, 20\text{-}24, 19\text{-}15, 9\text{-}5, 10\text{-}6, 5\text{-}1, 6\text{-}2, 24\text{-}20, 15\text{-}19, 1\text{-}5, 19\text{-}16, 20\text{-}24, 16\text{-}20, 24\text{-}27, 2\text{-}6, 5\text{-}1, 6\text{-}10, \\ 1\text{-}5, 11\text{-}16, 5\text{-}9, 10\text{-}15, 9\text{-}14, 15\text{-}19, 27\text{-}31, 20\text{-}24, 31\text{-}26, 19\text{-}15, 26\text{-}22, 24\text{-}20, 14\text{-}9, 16\text{-}19, 9\text{-}13, 20\text{-}24, \\ 22\text{-}17, 19\text{-}23, 17\text{-}21, 24\text{-}19, 13\text{-}17, 23\text{-}18, 17\text{-}13, 18\text{-}22, 21\text{-}17, 22\text{-}26, 13\text{-}9, 19\text{-}23, 9\text{-}5, 26\text{-}30, 5\text{-}1, 23\text{-}26, 17\text{-}13, 15\text{-}19, 13\text{-}9, 19\text{-}23, 9\text{-}5, 23\text{-}27, 32\text{-}23, 26\text{-}19, 5\text{-}9, 30\text{-}26, 1\text{-}5, 26\text{-}22, 9\text{-}14, 28\text{-}32, 5\text{-}1, 32\text{-}27, \\ 1\text{-}5, 27\text{-}23, 5\text{-}1, 22\text{-}18, 14\text{-}10, 18\text{-}15, 10\text{-}6, 23\text{-}26, 6\text{-}2, 19\text{-}23, 2\text{-}6, 26\text{-}22, 1\text{-}5, 23\text{-}18, 6\text{-}9, 15\text{-}10, 9\text{-}13, 18\text{-}14, 13\text{-}9, 22\text{-}18, 9\text{-}13, 10\text{-}6, 5\text{-}1, 14\text{-}10, 13\text{-}17, 10\text{-}14, 17\text{-}10, 6\text{-}15, 1\text{-}5, 18\text{-}14, 5\text{-}1, 14\text{-}10, 1\text{-}5, 10\text{-}6, 5\text{-}1, \\ 15\text{-}10, 1\text{-}5, 6\text{-}1, 5\text{-}9, 1\text{-}5, 9\text{-}13, 10\text{-}15, 13\text{-}17, 15\text{-}18, 17\text{-}13, 18\text{-}22, 13\text{-}9, 5\text{-}14 \end{array}$

Listing 3. The PPL database solution to Figure 7.

 $\begin{aligned} & 28-24, 1-6, 24-19, 6-10, 19-23, 10-6, 3-7, 6-2, 7-11, 2-7, 11-15, 22-26, 15-19, 26-22, 19-24, 22-26, 23-19, \\ & 26-31, 24-28, 7-11, 19-24, 32-27, 24-20, 27-32, 21-17, 31-27, 30-25, 27-31, 17-14, 11-15, 20-16, 31-27, \\ & 14-17, 15-18, 25-30, 18-23, 17-22, 27-31, 16-12, 31-27, 22-26, 23-19, 26-31, 27-24, 30-26, 19-15, 26-22, \\ & 24-20, 31-26, 20-24, 22-17, 15-10, 12-16, 10-15, 17-13, 24-20, 16-12, 15-11, 13-9, 20-24, 26-31, 24-20, \\ & 9-6, 11-15, 6-2, 15-11, 31-26, 11-15, 2-7, 15-18, 7-10, 20-24, 26-31, 24-19, 12-8, 18-15, 10-7, 15-18, 8-11, 18-14, 31-26, 14-18, 7-10, 19-24, 10-15, 18-14, 26-23, 14-9, 23-19, 9-5, 11-7, 5-9, 7-10, 24-20, 15-11, 20-24, 19-15, 9-5, 10-6, 5-1, 6-2, 24-20, 15-19, 1-5, 19-16, 20-24, 16-20, 24-27, 2-6, 5-1, 6-10, 1-5, \\ & 11-16, 5-9, 10-15, 9-14, 15-19, 27-31, 20-24, 31-26, 19-15, 26-22, 24-20, 14-9, 16-19, 9-13, 20-24, 22-17, 19-23, 17-21, 24-19, 13-17, 23-18, 17-13, 18-22, 21-17, 22-26, 13-9, 19-23, 9-5, 26-30, 5-1, 23-26, 17-13, 15-19, 13-9, 19-23, 9-5, 23-27, 32-23, 26-19, 5-9, 30-26, 1-5, 26-22, 9-14, 28-32, 5-1, 32-27, 1-5, 27-23, 5-1, 22-18, 14-10, 18-15, 10-6, 23-26, 6-2, 19-23, 2-6, 26-22, 1-5, 23-18, 6-9, 15-10, 9-13, 18-14, 13-9, 22-18, 9-13, 10-6, 5-1, 14-10, 13-17, 10-14, 17-10, 6-15, 1-5, 18-14, 5-1, 14-10, 1-5, 10-6, 5-1, 15-10, 1-5, 10, 1-5, 6-1, 5-9, 1-5, 9-13, 10-15, 13-17, 15-18, 17-13, 18-22, 13-9, 5-14 \\ \end{aligned}$

Listings 4 and 5 show how a program with a GTV database on the strong side, searching to a depth of 31 plies for each move, will still allow repetition draws against a PPL database defending on the weak side. Appendix A contains the proper play for the strong side at each footnote, given in [1] below.

Listing 4. The PPL database defends the weak side of Figure 6 against a GTV database on the winning side, and a draw ensues via repetition.

27-24, 19-15, 24-20, 16-19, 21-25, 15-18, 20-24, ^[1] 19-16, 24-20, ^[2] 16-19, 25-30, 18-15, 07-02, 19-23, 20-24 ^[3] 15-10, 30-25, 10-14, 25-22, 14-10, 22-17, 23-18, 24-19, ^[4] 18-15, 19-23, ^[5] 15-11, 23-19, ^[6] 11-15, 19-16, 15-18, 16-20, ^[7] 18-15, 17-21, ^[8] 15-11, 21-25, 11-15, 20-16, 15-18, 16-19, ^[9] 18-15, 19-23, ^[10] 15-11, 23-19, 11-15, 19-24, ^[11] 15-18, 25-30, ^[12] 18-23, 24-27, ^[13] 23-18, 27-24, 18-23, 24-27, ^[14] 23-18, 27-24, 18-23, 24-27, ^[14] 23-18, 27-24, 18-23, 24-27, ^[16] 23-18, repetition draw.

Listing 5. The PPL database defends the weak side of Figure 7 against a GTV database on the winning side, and a different kind of draw is reached. The positions will repeat in a cycle every 66 plies.

 $\begin{array}{l} 28-24, \, 01-06, \, 24-19, \, 06-10, \, 19-23, \, 10-06, \, 03-07, \, 06-02, \, 07-11, \, 02-07, \, 11-15, \, 22-26, \, 15-19, \, 26-22, \, 19-24, \\ 22-26, \, 23-18, \, ^{[17]} \quad 26-31, \, 18-15, \, 32-28, \, 15-19, \, 28-32, \, 24-28, \, 07-11, \, 19-24, \, 32-27, \, 24-20, \, 27-32, \, 21-17, \\ 31-27, \, 30-25, \, 27-31, \, 25-22, \, ^{[18]} \, 31-27, \, 17-14, \, ^{[19]} \, 11-15, \, 22-17, \, ^{[20]} \, 27-23, \, 17-13, \, 23-27, \, 13-09, \, ^{[21]} \, 15-19, \\ 14-17, \, 19-23, \, 17-22, \, 23-19, \, 09-06, \, ^{[22]} \, 19-15, \, 22-17, \, ^{[23]} \, 15-19, \, 06-09, \, ^{[24]} \, 19-23, \, 09-13, \, 23-19, \, 17-21, \, 19-23, \, 13-17, \, 23-19, \, 17-14, \, ^{[25]} \, 19-15, \, 14-09, \, ^{[26]} \, 15-18, \, 21-17, \, 18-23, \, 09-13, \, ^{[27]} \, 23-19, \, 17-22, \, 19-15, \, 22-17, \, 15-18, \, 21-17, \, 18-23, \, 09-13, \, ^{[27]} \, 23-19, \, 17-22, \, 19-15, \, 22-17, \, 15-18, \, 21-17, \, 18-23, \, 09-13, \, ^{[27]} \, 23-19, \, 17-22, \, 19-15, \, 22-17, \, 15-18, \, 21-17, \, 18-23, \, 09-13, \, ^{[27]} \, 23-19, \, 17-22, \, 19-15, \, 22-17, \, 15-18, \, 21-17, \, 18-23, \, 09-13, \, 12-19, \, 17-22, \, 19-15, \, 22-17, \, 12-18, \, 21-17, \, 18-23, \, 09-13, \, 21-19, \, 17-22, \, 19-15, \, 22-17, \, 22-17, \, 12-19, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \, 12-12, \,$

25, ^[28] 15-11, 13-09, ^[29] 11-15, 20-16, 15-18, 16-11, 27-23, 09-13, 23-19, 13-17, 18-23, 25-22, ^[30] 19-24, 17-14, ^[31] 23-19, 22-17, ^[32] 19-23, 17-22, 23-19, 22-25, ^[33] 24-20, 11-08, 20-16, 08-12, 16-20, 14-18, ^[34] 20-24, 25-22, ^[35] 24-27, 22-17, ^[36] 27-24, 18-14, ^[37] 24-20, 14-10, ^[38] 19-23, 17-22, 20-24, 12-08, ^[39] 23-19, 08-11, ^[40] 24-20, 22-17, ^[41] 19-23, 17-22, 23-19, 10-06, 19-23, 06-09, ^[42] 23-19, 11-08, 19-23, 09-14, 20-16, 08-12, 16-20, 14-10, ^[43] 20-24, 12-08, ^[44] 23-19, 08-11, ^[45] 24-20, 22-17, ^[46] 19-23, 17-22, 23-19, 10-07, ^[47] 20-24, 11-08, 24-27, 07-02, 19-15, 02-06, 27-23, 08-12, 15-19, 06-10, 23-27, 10-14, 27-23, 22-18, ^[48] 23-27, 14-17, 27-24, 17-22, 24-27, 12-08, ^[49] 19-23, 18-14, 23-19, 08-11, ^[50] 27-23, 22-17, ^[51] 19-24, 17-22, 23-19, 11-08, 19-23, 17-14, ^[52] 20-16, 08-12, 16-20, 14-10, ^[53] 20-24, 12-08, ^[54] 23-19, 08-11, ^[55] 24-20, 22-17, ^[56] 19-23, 17-22, 23-19, 10-06, 19-23, 06-09, ^[57] 23-19, 11-08, ^[58] 19-23, 09-14, 20-16, 08-12, 16-20, 14-10, ^[59] 20-24, 12-08, ^[50] 23-19, 08-11, ^[61] 24-20, 22-17, ^[62] 19-23, 17-22, 23-19, 10-06, 19-23, 06-09, ^[57] 23-19, 11-08, ^[58] 19-23, 23-19, 10-06, 19-23, 06-09, ^[57] 23-19, 11-08, ^[58] 19-23, 23-19, 10-06, 19-23, 06-09, ^[57] 23-19, 11-08, ^[58] 19-23, 23-19, 10-06, 19-23, 06-09, ^[57] 23-19, 11-08, ^[58] 19-23, 23-19, 10-06, 19-23, 06-09, ^[57] 23-19, 11-08, ^[58] 19-23, 23-19, 10-06, 19-23, 06-09, ^[57] 23-19, 11-08, ^[58] 19-23, 23-19, 10-07, ^[61] 24-20, 22-17, ^[62] 19-23, 17-22, 23-19, 10-07, ^[61] 24-20, 22-17, ^[62] 19-23, 17-22, 23-19, 10-06, 19-23, 06-09, ^[57] 23-19, 11-08, ^[58] 19-23, 23-19, 10-07, ^[63] 20-24, 11-08, 24-27, infinite cycle of repetition.

It is interesting to note that after move 12 in Listing 5, which is from Figure 7, the same type of position as Figure 6 is created; i.e., one in which the Checker cannot crown since it is being blocked by an enemy King. Even with this common theme, two different types of draws result. In Listing 4, a "see-saw" draw occurs when the hash table saturates and moves leading to the win have all been played before. Recall one of the uses of the hash table is to score repeated moves of the same King as a draw, so that you do not shuffle the same piece back and forth twenty times and believe you have conducted a valid 40 ply search (20 for one side, 20 for the other). Likewise, arriving at the same position many times during the search via transposition without making progress should be discouraged. The program ends up in the undesirable situation where most or all of the winning lines are found in the hash table but none force the final simplifying win. They all appear to lead to "no progress" due to their high frequency of occurrence in the hash table, yet they are the only subset of moves that will win. In Listing 5, a lengthy cycle from moves 55 to 88 could theoretically repeat ad infinitum, starting at move 89. Without being able to search at least 67 plies into the future from move 55, this cycle cannot be avoided.

3.3 GTV Database Program vs. PPL Database Program

An experiment was performed to observe how two different Grandmaster-level programs (Waldteufel, 2002; Gilbert, 2002) would play against the PPL database from a "longest win" test position. In each case, the WORLD CHAMPIONSHIP CHECKERS (WCC) program (Dodgen and Trice, 2001) with the PPL database played the losing side of each ending, and both the WYLLIE program and KINGSROW program played the winning side. All of the programs had access to a GTV database probed in RAM during the search that contained at least all of the 19,055,258,760 7-piece positions featuring four against three. The WCC program consulted the PPL database when defending the weak side on every move. The starting position for each

game was the position shown in Figure 5. In this position, Black to move can win in 253 plies.

Listing 6. WYLLIE program vs. WCC, February 7, 2003. WCC was able to draw from the losing starting position shown in Figure 5.

08-11, 12-08, 01-05, 09-13, 10-14, 08-03, 11-16, 13-09, 14-18, 09-14, 18-23, 14-18, 23-27, 18-23, 27-32, 21-17, 32-28, 17-22, 28-24, 22-26, 24-20, 23-27, 16-19, 26-31, 19-24, 27-32, 20-16, 32-28, 16-19, 03-07, 04-08, 28-32, 19-16, 31-26, 05-09, 26-31, 16-20, 07-02, 08-11, 02-06, 09-13, 06-10, 20-16, 32-27, 24-28, 27-32, 16-19, 31-27, 11-08, 27-24, 19-23, 10-14, 08-11, 24-20, 11-15, 20-24, 23-26, 24-27, 26-30, 27-31, 15-11, 31-27, 30-25, 27-23, 25-21, 23-26, 13-17, 26-30, 11-15, 30-26, 15-19, 26-31, 17-22, 31-27, 22-25, 14-18, 21-17, 27-24, 19-16, 24-27, 25-29, 18-15, 29-25, 15-18, 25-30, 18-23, 17-22, 27-31, 16-12, 31-27, 22-26, 23-19, 26-31, 27-24, 30-26, 19-15, 26-22, 24-20, 31-26, 20-24, 22-17, 15-10, 12-16, 10-15, 17-14, 24-20, 16-12, 15-11, 14-17, 11-15, 17-13, 15-11, 26-31, 11-15, 13-09, 15-10, 12-08, 20-16, 31-26, 10-15, 08-12, 16-20, 09-06, 15-11, 26-22, 11-15, 06-02, 15-19, 22-26, 19-15, 02-07, 15-18, 07-10, 20-24, 10-06, 18-15, 26-31, 15-11, 06-09, 24-20, 09-05, 11-15, 31-26, 15-11, 26-30, 11-15, 05-09, 15-11, 09-14, 11-15, 30-25, 15-19, 14-17, 19-23, 17-22, 20-24, 12-08, 23-19, 22-17, 24-20, 17-13, 20-16, 08-12, 16-20, 25-22, 19-23, 13-17, 20-24, 17-21, 24-20, 22-25, 23-19, 25-30, 19-23, 21-25, 20-24, 12-08, 24-20, 08-11, 23-19, 11-08, 19-23, 25-22, 20-16, 08-12, 16-19, 22-17, 19-24, 17-14, 24-20, 14-09, 20-24, 12-08, 24-20, 08-11, 23-19, drawn by agreement.

Listing 7. KINGSROW program vs. WCC, March 3, 2003. WCC was able to draw from the losing starting position shown in Figure 5.

08-11, 12-08, 01-05, 09-13, 10-14, 08-03, 11-16, 13-09, 14-18, 09-14, 18-23, 14-18, 23-27, 18-23, 27-32, 21-17, 32-28, 17-22, 28-24, 22-26, 24-20, 23-27, 16-19, 26-31, 19-24, 27-32, 20-16, 32-28, 16-19, 03-07, 04-08, 28-32, 19-16, 31-26, 16-20, 07-10, 08-11, 26-31, 05-09, 10-06, 09-13, 06-10, 20-16, 32-27, 24-28, 27-32, 16-19, 31-27, 11-08, 27-24, 19-23, 10-14, 08-11, 24-20, 11-15, 20-24, 23-26, 24-27, 26-30, 27-31, 15-11, 31-27, 30-25, 27-23, 11-16, 14-18, 13-17, 18-14, 25-21, 23-26, 16-19, 26-31, 17-22, 31-27, 19-15, 27-31, 21-25, 32-27, 15-19, 27-32, 19-16, 14-10, 16-11, 31-27, 25-21, 10-14, 22-25, 14-18, 21-17, 27-24, 11-16, 24-27, 25-30, 18-23, 17-22, 27-31, 16-20, 23-19, 20-24, 19-23, 30-25, 32-27, 24-20, 27-32, 20-16, 31-27, 25-21, 27-24, 21-17, 24-20, 16-12, 20-24, 17-14, 24-20, 14-10, 20-24, 12-08, 23-19, 10-14, 19-23, 14-09, 24-20, 08-12, 20-24, 09-13, 24-20, 22-25, 23-19, 13-17, 19-23, 25-30, 20-24, 17-22, 24-19, 22-26, 23-27, 30-25, 27-23, 26-31, 23-18, 25-30, 18-15, 30-26, 19-24, 26-23, 15-11, 31-26, 11-15, 26-22, 24-27, 23-18, 15-19, 18-14, 27-23, 14-17, 19-24, 17-13, 24-20, 13-09, 20-24, 12-16, 24-20, 16-11, 23-19, drawn by agreement.

The WYLLIE program searched for 10 to 15 seconds per move, averaging about 620,000 moves per second during the search. The time duration was chosen for practical purposes. This ending was very long, and if we took a combined 30 seconds to type our moves to one another, the game would last over an hour if 250 plies were required to win.

The WYLLIE program was unable to win this ending, conceding the draw after 196 plies of play. Even after this lengthy engagement, the PPL database indicated that the win was 177 plies away for the WYLLIE program. In this respect, only 96 plies of progress were observed after 196 plies of actual play. The WYLLIE program got as close as 135 plies from the terminal position before it began to make non-optimal moves. Listing 6 shows the moves made by WYLLIE and WCC during this game.

The KINGSROW program also played with an average search time of 10-15 seconds per move, which was increased to 30 seconds per move (upon request of the KINGSROW programmer) once three Kings were on the board for the winning side. The program was able to get an average search depth of 33 plies during the course of play. The KINGSROW program got as close as 159 plies from the terminal position, but it too started to make non-optimal moves allowing WCC to push the win further away. At the end of 164 plies of play, the win was still 181 plies distant, so KINGSROW netted 72 plies of progress after 164 plies. Listing 7 shows the moves made by KINGSROW and WCC during this game.

4. Improving Play from the Fourth Position Ending

There is an arrangement on the checkerboard in which the weak side can have one less King than the strong side and still retain a draw. This study problem was designated *Fourth Position* by the checkers fraternity. With one slight modification to the arrangement of this position, or by altering the side to move, the strong side gains the ability to procure a win which requires precise timing of the disposal of one of its pieces.

Winning the textbook form of Fourth Position requires 81 plies according to the PPL database. The original published play from 1756 features a first move that would require a total of 85 plies to complete the win, but there was also a sub-optimal defensive move in this analysis. The sub-optimal line would surrender the game 12 plies more quickly than would the PPL database.

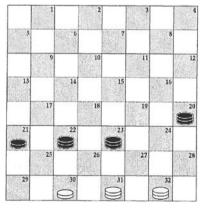


Figure 8. Payne's Fourth Position, 1756. Black to move wins in 81 plies. White to move draws.

The original solution entailed the wellmotivated retreat 22-18 (Payne, 1756) to start things off, but the PPL database offers 22-25! winning more quickly. Even more incredible is the fact that two moves later Black will play 25-29, a move that is usually strongly discouraged in almost every position in which White has a Checker on square 30. The improved PPL solution is presented in Listing 8. A subset of the classic solution to Fourth

Position is presented thereafter, with commentary correcting the play on the defensive side.

Listing 8. The PPL solution to Fourth Position from Figure 8.

22-25!, 31-27, 23-19, 32-28, 25-29!, 27-31, 20-24, 28-32, 24-28, 31-27, 29-25, 27-24, 19-16, 24-27, 16-20, 27-23, 25-22, 23-27, 22-26, 30x23, 28-24, 32-28, 24x31, 23-19, 20-24, 19-15, 24-19, 15-10, 19-15, 10-06, 31-26, 28-24, 26-22, 24-28, 22-18, 28-24, 21-25, 24-28, 18-14, 28-32, 25-30, 32-28, 30-26, 28-32, 14-10, 06-01, 26-23, 01-05, 10-06, 32-28, 23-19, 05-01, 06-10, 01-05, 19-24, 28x19, 15x24, 05-01, 10-14, 01-05, 24-27, 05-01, 27-23, 01-05, 23-18, 05-01, 14-09, 01-05, 18-14, 05-01, 09-05, 01-06, 05-01, 06-02, 14-18, 02-07, 18-15, 07-02, 15-11, 02-07, 11x02, Black wins.

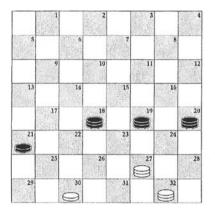


Figure 9. White to move after 22-18, 31-27, 23-19 from Figure 8.

The PPL database was able to identify some play on the weak side of Fourth Position that was not optimal. Figure 9 shows the position with White to move after: 22-18, 31-27, 23-19 which has traditionally been followed by the retreat of the King 27-31. The perfect play database announces that this move leads to a win in 69 plies for Black, but the optimal line will persist for 12 plies longer. The best defense from the position

shown in Figure 9 is 32-28, 18-22 (heading back to square 29, as suggested by the original improvement, is still the

fastest course of action from here) 27-31, 22-25, 31-27, 25-29, 27-31, 20-24, 28-32, 24-28, 31-27, 29-25, 27-24, 19-16, 24-27, 16-20, 27-23, 25-22, 23-27, 22-26, 30x23, 28-24, 32-28 and now 24x31 leads a 5-piece position that White will lose in 58 plies.

The purpose of the move 32-28 is to prevent the immediate 19-24 by Black. It should be noted that as long as White keeps the King on square 28, Black cannot play the strong 19-24 attack, which is instrumental in concluding the game more quickly. It is not the absence of 27-31 on move two for White that extends the life of the weak side, it is the presence of the move 32-28.

5. Conclusions

The game of checkers is deceptive in its apparent simplicity. Most strong contemporary checkers programs have large opening books capable of circumventing early losses, and are likewise capable of handling the tactics in the middle game beyond the ability of the strongest human players. But, as was demonstrated, the endgame domain is still sufficiently complex so as to prevent grandmaster-level programs from winning in positions that are known wins with as few as seven pieces on the board. This result was rather surprising and should underscore the complexity inherent in the game of checkers.

The Perfect Play databases of Dodgen and Trice are the only databases in existence that allow a software program to play the game of checkers perfectly in the endgame. The 7-piece perfect play lookup database allows the WORLD CHAMPIONSHIP CHECKERS program to announce a win from a distance of 253 plies.

We will continue to build larger PPL databases as time and personal computer resources will allow. The web site at WorldChampionshipCheckers.com will showcase the PPL database building progress and other items of interest to checkers and programming enthusiasts.

Acknowledgements

Without the pioneering effort of the CHINOOK programming team, and in particular Dr. Jonathan Schaeffer and Robert Lake, our work in the domain of endgame databases for the game of checkers would never have gotten off of the ground. The concept of applying an indexing function to create a sparsely populated matrix for the game of checkers is one of the critical components necessary to make run-time probable GTV databases possible. Without being able to create our own GTV databases, we certainly could not have made a PPL database. For showing us the way, we are ever thankful.

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Appendix A: Footnotes to Imperfect Moves Made

- 1. 25-30 wins in 201, but 20-24 allows White 4 additional plies. Cumulative slip = 4 plies.
- 2. A rare case where the only move to win is a reversal of the previous move. This indicates that the GTV database must be consulted at every node in the tree during the search, a very expensive computation. Usually when one side is ahead by once piece, only the root of the tree needs to consult the database and prune the moves leading to draws and losses. The reasoning behind this is that in many cases, just about every move wins, so probing the database does not prune any legal moves, but it does slow down the search a great deal, even if the entire database is RAM-resident.
- 3. This retreat again is correct, but shuffling back and forth is usually penalized by an evaluation function. Notice the position is changing ever-so-slightly as the weak side has not shuffled back and forth over the same moves as the strong side. This is a very difficult position to play properly!
- 4. It should be noted that 24-20, again a repeated move on the strong side, would also lead to an optimal win in 189.
- 5. 19-16 wins in 187, but 19-23 allows White 4 additional plies. Cumulative slip = 8 plies.
- 6. Another instance where a reversing of the previous move is the only move to win. In the principal variation, the program expects 15-18, a non-optimal defensive move, instead of 15-11, the PPL best defense.
- 7. At ply 31, the program chooses this over its previous best candidate, 16-12, which was the optimal move. 16-12 wins in 185, but 16-20 allows White 2 additional plies. Cumulative slip = 10 plies.
- 8. 20-16 wins in 187, but 17-21 allows White 4 additional plies. Cumulative slip = 14 plies.
- 9. 16-12 wins in 185, but 16-19 allows White 2 additional plies. Cumulative slip = 16 plies.
- 10. 19-16 wins in 187, but 19-23 allows White 4 additional plies. Cumulative slip = 20 plies.
- 11. 19-16 wins in 187, but 19-24 allows White 4 additional plies. Cumulative slip = 24 plies. The program searched 63 plies and moved instantly for the strong side here, reporting a draw. This is because the hash table was saturated with positions consisting of only one move to win. The program will not make a drawing or losing move, but all of the moves maintaining the win have already been tried. On the strong side, the program tries not to repeat moves, but the weak side has created a position that will cycle in the hash table. This is the beginning of some serious trouble.
- 12. 24-20 wins in 189, but 25-30 allows White 6 additional plies. Cumulative slip = 30 plies.
- 13. 30-25 wins in 195, but 24-27 allows White 2 additional plies. Cumulative slip = 32 plies.
- 14. 30-25 wins in 195, but 24-27 allows White 2 additional plies. Cumulative slip = 34 plies.
- 15. 30-25 wins in 195, but 24-27 allows White 2 additional plies. Cumulative slip = 36 plies.
- 16. 30-25 wins in 195, but 24-27 allows White 2 additional plies. Cumulative slip = 38 plies. At this point, the program has slipped into a position that will repeat and allow the weak side to draw.
- 17. 23-19 wins in 191, but 23-18 allows White 4 additional plies. Cumulative slip = 4 plies.
- 18. 17-14 wins in 179, but 31-27 allows White 4 additional plies. Cumulative slip = 8 plies.
- 22-25 wins in 181, but 17-14 allows White 4 additional plies. Cumulative slip = 12 plies.
 14-17 wins in 183, but 22-17 allows White 4 additional plies. Cumulative slip = 16 plies.
- 21. 20-16 wins in 183, but 13-09 allows White 8 additional plies. Cumulative slip = 24 plies.
- 22. 09-13 wins in 185, but 09-06 allows White 4 additional plies. Cumulative slip = 28 plies.
- 23. 06-09 wins in 187, but 22-17 allows White 4 additional plies. Cumulative slip = 32 plies.

- 24. The program makes the correct move after searching 63 plies then moving instantly, reporting a score of draw for the strong side. This is the same phenomenon that was observed in the other position, and the repetition spiral is about to begin.
- 25. 21-25 wins in 181, but 17-14 allows White 4 additional plies. Cumulative slip = 36 plies.
- 26. 21-25 wins in 183, but 14-09 allows White 8 additional plies. Cumulative slip = 44 plies.
- 27. Another instance of hitting the limit of 63 plies of search due to hash table saturation. This is the correct move to win optimally but the program is reporting a draw from the repetitions.
- 28. Although this is the correct move for the optimal win, the program searched for over four times as long to reach ply 31 on this move than for the average of the previous moves.
- 29. 13-17 wins in 181, but 13-09 allows White 4 additional plies. Cumulative slip = 48 plies.
- 30. 17-22 wins in 175, but 19-24 allows White 4 additional plies. Cumulative slip = 52 plies.
- 31. 17-21 wins in 177, but 17-14 allows White 4 additional plies. Cumulative slip = 56 plies.
- 32. 14-17 wins in 179, but 19-23 allows White 4 additional plies. Cumulative slip = 60 plies.
- 33. The classic problem of "wandering Kings" is present here. Now 45 moves into the game, with 31 moves of regression, the program has only advanced 14 full moves towards the goal. This was, by far, the longest time required to complete a 31 ply, search at 27 minutes 16 seconds.
- 34. Making the correct move, and spending only 47 seconds to complete 31 plies of search in this instance.
- 35. 18-22 wins in 171, but 24-27 allows White 4 additional plies. Cumulative slip = 64 plies.
- 36. 22-25 wins in 173, but 22-17 allows White 4 additional plies. Cumulative slip = 68 plies.
- 37. 18-22 wins in 175, but 18-14 allows White 4 additional plies. Cumulative slip = 72 plies.
- 38. 14-18 wins in 177, but 19-23 allows White 4 additional plies. Cumulative slip = 76 plies.
- 39. 10-14 wins in 177, but 23-19 allows White 4 additional plies. Cumulative slip = 80 plies.
- 40. Another long search, indicative of opportunities to give up the draw appearing in the anticipated line of play. 10-14 wins in 179, but 08-11 allows white 8 additional plies. Cumulative slip = 88 plies.
- 41. 10-06 wins in 185, but 22-17 allows White 4 additional plies. Cumulative slip = 92 plies.
- 42. Making the correct move after a research on ply 31, replacing the "wandering" 06-02 move.
- 43. 14-17 wins in 175, but 14-10 allows White 4 additional plies. Cumulative slip = 96 plies.
- 44. 10-14 wins in 177, but 12-08 allows White 4 additional plies. Cumulative slip = 100 plies.
- 45. 10-14 wins in 179, but 08-11 allows White 8 additional plies. Cumulative slip = 108 plies.
- 46. 10-06 wins in 185, but 22-17 allows White 4 additional plies. Cumulative slip = 112 plies.
- 47. 10-06 wins in 185, but 10-07 allows White 4 additional plies. Cumulative slip = 116 plies.
- 48. 14-17 wins in 175, but 23-27 allows White 4 additional plies. Cumulative slip = 120 plies.
- 49. 22-25 wins in 173, but 12-08 allows White 8 additional plies. Cumulative slip = 128 plies.
- 50. 08-12 wins in 177, but 08-11 allows White 4 additional plies. Cumulative slip = 132 plies.
- 51. 14-17 wins in 179, but 22-17 allows White 4 additional plies. Cumulative slip = 136 plies.
- 52. 08-12 wins in 175, but 17-14 allows White 4 additional plies. Cumulative slip = 140 plies.
- 53. 14-17 wins in 175, but 14-10 allows White 4 additional plies. Cumulative slip = 144 plies.
- 54. 10-14 wins in 177, but 12-08 allows White 4 additional plies. Cumulative slip = 148 plies.
- 55. 10-14 wins in 179, but 08-11 allows White 8 additional plies. Cumulative slip = 156 plies.
- 56. 10-06 wins in 185, but 22-17 allows White 4 additional plies. Cumulative slip = 160 plies.
- 57. As was seen in the position at [42], here too the correct move was made after a research on ply 31, replacing the "wandering" 06-02 move.
- 58. Another correct move, played here at move 94, leads to the same position at move 60, which was 68 plies ago.
- 59. 14-17 wins in 175, but 14-10 allows White 4 additional plies. Cumulative slip = 164 plies. The position here at move 97 is the same as was seen at move 63, playing in the cycle from 68 plies ago. See note [43].
- 60. 10-14 wins in 177, but 12-08 allows White 4 additional plies. Cumulative slip = 168 plies. The position here at move 98 is the same as was seen at move 64, playing in the cycle from 68 plies ago. See note [44].
- 61. 10-14 wins in 179, but 08-11 allows White 8 additional plies. Cumulative slip = 176 plies. The position here at move 99 is the same as was seen at move 65, playing in the cycle from 68 plies ago. See note [45].
- 62. 10-06 wins in 185, but 22-17 allows White 4 additional plies. Cumulative slip = 180 plies. The position here at move 100 is the same as was seen at move 66, playing in the cycle from 68 plies ago. See note [46].
- 63. 10-06 wins in 185, but 10-07 allows White 4 additional plies. Cumulative slip = 184 plies. The position here at move 102 is the same as was seen at move 68, playing in the cycle from 68 plies ago. See note [47].