RISK CONTROL AND OPTIMIZATION FOR STRUCTURAL FACILITIES

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Abstract Optimization techniques are essential ingredients of reliability-oriented optimal designs of technical facilities. Although many technical aspects are not yet solved and the available spectrum of models and methods in structural reliability is still limited many practical problems can be solved. A special one-level optimization is proposed for general cost-benefit analysis and some technical aspects are discussed. However, focus is on some more critical issues, for example, "what is a reasonable replacement strategy for structural facilities?", "how safe is safe enough?" and "how to discount losses of material, opportunity and human lives?". An attempt has been made to give at least partial answers.

Keywords: Structural reliabiltiy, optimization, risk acceptability, discount rates

1. Introduction

The theory of structural reliability has been developed to fair maturity within the last 30 years. The inverse problem, i.e. how to determine certain parameters in the function describing the boundary between safe and failure states for given reliability, has been addressed only recently. It is a typical optimization problem. Designing, erecting and maintaining structural facilities may be viewed as a decision problem where maximum benefit and least cost are sought and the reliability requirements are fulfilled simultaneously. In what follows the basic formulations of the various aspects of the decision problem are outlined making use of some more recent results in the engineering literature. The structure of a suitable objective function is first discussed. A renewal model proposed as early as 1971 by Rosenblueth/Mendoza [42], further developed in [17], [40] and extended in [36], [18] is presented in some detail. Theory and methods of structural reliability are reviewed next where it is pointed

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out that the calculation of suitable reliability measures is essentially an optimization problem. Focus is on the concepts of modern first- and second reliability methods [20]. The problem of the value of human life is then discussed in the context of modern health-related economic theories. Some remarks are made about appropriate discount rates. Finally, details of a special version of modern reliability-oriented optimization techniques based on work in [26] are outlined followed by an illustrative example.

2. Optimal Structures

A structure is optimal if the following objective is maximized:

$$Z(\mathbf{p}) = B(\mathbf{p}) - C(\mathbf{p}) - D(\mathbf{p}) \tag{1}$$

Without loss of generality it is assumed that all quantities in eq. (1) can be measured in monetary units. $B(\mathbf{p})$ is the benefit derived from the existence of the structure, $C(\mathbf{p})$ is the cost of design and construction and $D(\mathbf{p})$ is the cost in case of failure. $\mathbf{p}$ is the vector of all safety relevant parameters. Statistical decision theory dictates that expected values are to be taken. In the following it is assumed that $B(\mathbf{p}), C(\mathbf{p})$ and $D(\mathbf{p})$ are differentiable in each component of $\mathbf{p}$. The cost may differ for the different parties involved, e.g. the owner, the builder, the user and society. A structural facility makes sense only if $Z(\mathbf{p})$ is positive for all parties involved within certain parameter ranges. The intersection of these ranges defines reasonable structures.

The structure which eventually will fail after a long time will have to be optimized at the decision point, i.e. at time $t = 0$. Therefore, all cost need to be discounted. We assume a continuous discounting function $\delta(t) = \exp[-\gamma t]$ which is accurate enough for all practical purposes and where γ is the interest rate.

It is useful to distinguish between two replacement strategies, one where **the facility is given up after failure** and one where **the facility is systematically replaced after failure**. Further we distinguish between structures which **fail upon completion or never** and structures which **fail at a random point in time** much later due to service loads, extreme external disturbances or deterioration. The first option implies that loads on the structure are time invariant. Reconstruction times are assumed to be negligibly short. At first sight there is no particular preference for either of the replacement strategies. For infrastructure facilities the second strategy is a natural strategy. Structures only used once, e.g. special auxiliary construction structures or boosters for space transport vehicles fall into the first category.

3. The Renewal Model

3.1 Failure upon completion due to time-invariant loads

The objective function for a **structure given up after failure at completion** due to time-invariant loads (essentially dead weight) is

$$Z(\mathbf{p}) = B^* R_f(\mathbf{p}) - C(\mathbf{p}) - H P_f(\mathbf{p}) = B^* - C(\mathbf{p}) - (B^* + H) P_f(\mathbf{p}) \quad (2)$$

$R_f(\mathbf{p})$ is the reliability and $P_f(\mathbf{p}) = 1 - R_f(\mathbf{p})$ the failure probability, respectively. H is the direct cost of failure including demolition and debris removal cost. For **failure at completion and systematic reconstruction** we have

$$\begin{aligned} Z(\mathbf{p}) &= B^* - C(\mathbf{p}) - (C(\mathbf{p}) + H) \sum_{i=1}^{\infty} i P_f(\mathbf{p})^i R_f(\mathbf{p}) \\ &= B^* - C(\mathbf{p}) - (C(\mathbf{p}) + H) \frac{P_f(\mathbf{p})}{1 - P_f(\mathbf{p}))} \end{aligned} \quad (3)$$

After failure one, of course, investigates its causes and redesigns the structure. However, we will assume that the first design was already optimal so that there is no reason to change the design rules leading to the same $P_f(\mathbf{p})$. If each structural realization is independent of each other formula (3) holds.

A certain ambiguity exists when assessing the benefit B^* taken here and in the following as independent of $\mathbf{p}$. If the intended time of use of the facility is t_s it is simply

$$B^* = B(t_s) = \int_0^{t_s} b(t)\delta(t)dt \quad (4)$$

For constant benefit per time unit $b(t) = b$ one determines

$$B^* = B(t_s) = \frac{b}{\gamma}\left[1 - \exp\left[-\gamma t_s\right]\right] \underset{t_s \to \infty}{=} \frac{b}{\gamma} \quad (5)$$

3.2 Random Failure in Time

Assume now random failure events in time. The time to the first event has distribution function $F_1(t, \mathbf{p})$ with probability density $f_1(t, \mathbf{p})$. If the **structure is given up after failure** it is obviously

$$B(t_s) = \int_0^{t_s} b(t)\delta(t)R_1(t, \mathbf{p})dt \quad (6)$$

$$D(t_s) = \int_0^{t_s} f_1(t,\mathbf{p})\delta(t)H dt \tag{7}$$

and therefore

$$Z(\mathbf{p}) = \int_0^{t_s} b(t)\delta(t)R_1(t,\mathbf{p})dt - C(\mathbf{p}) - \int_0^{t_s} \delta(t)f_1(t,\mathbf{p})H dt \tag{8}$$

For $t_s \to \infty$ and $f_1^*(\gamma,\mathbf{p}) = \int_0^\infty e^{-\gamma t} f_1(t,\mathbf{p})dt$ the Laplace transform of $f_1(t,\mathbf{p})$ it is instead

$$Z(\mathbf{p}) = \frac{b}{\gamma}[1 - f_1^*(\gamma,\mathbf{p})] - C(\mathbf{p}) - Hf_1^*(\gamma,\mathbf{p}) \tag{9}$$

For the more important case of **systematic reconstruction** we generalize our model slightly. Assume that the time to first failure has density $f_1(t)$ while all other times between failure are independent of each other and have density $f(t)$, i.e. failures and subsequent renewals follow a modified renewal process [11]. This makes sense because extreme loading events usually are not controllable, i.e. the time origin lies somewhere between the zeroth and first event. The independence assumption is more critical. It implies that the structures are realized with independent resistances at each renewal according to the same design rules and the loads on the structures are independent, at least asymptotically. For constant benefit per time unit $b(t) = b$ we now derive by making use of the convolution theorem for Laplace transforms

$$\begin{aligned} Z(\mathbf{p}) &= \int_0^\infty be^{-\gamma t}dt - C(\mathbf{p}) - (C(\mathbf{p}) + H)\sum_{n=1}^{\infty}\int_0^\infty e^{-\gamma t} f_n(t,\mathbf{p})dt \\ &= \frac{b}{\gamma} - C(\mathbf{p}) - (C(\mathbf{p}) + H)\frac{f_1^*(\gamma,\mathbf{p})}{1 - f^*(\gamma,\mathbf{p})} \\ &= \frac{b}{\gamma} - C(\mathbf{p}) - (C(\mathbf{p}) + H)h_1^*(\gamma,\mathbf{p}) \end{aligned} \tag{10}$$

where $h_1^*(\gamma,\mathbf{p})$ is the Laplace transform of the renewal intensity $h_1(t,\mathbf{p})$. For regular renewal processes one replaces $f_1^*(\gamma,\mathbf{p})$ by $f^*(\gamma,\mathbf{p})$. For the renewal intensity and its Laplace transform there is an important asymptotic result [11]:

$$\lim_{t\to\infty} h(t,\mathbf{p}) = \lim_{\gamma\to 0}\gamma h^*(\gamma,\mathbf{p}) = \frac{1}{m(\mathbf{p})} \tag{11}$$

where $m(\mathbf{p})$ is the mean of the renewal times.

If, in particular, the events follow a stationary Poisson process with intensity λ we have

$$f_1^*(\gamma) = f^*(\gamma) = \int_0^\infty \exp[-\gamma t] \lambda \exp[-\lambda t] \, dt = \frac{\lambda}{\gamma + \lambda} \tag{12}$$

and

$$h^*(\gamma) = \frac{\lambda}{\gamma} \tag{13}$$

This result is of great importance because structural failures should, in fact, be rare, independent events. Then, the Poisson intensity λ can be replaced by the so-called outcrossing rate ν^+ to be described below - even in the locally non-stationary case. Finally, if at an extreme loading event (e.g. flood, wind storm, earthquake, explosion) failure occurs with probability $P_f(\mathbf{p})$ and $f_1(t)$ and $f(t)$, respectively, denote the densities of the times between the loading events one obtains by similar considerations

$$g_1^*(\gamma, \mathbf{p}) = \sum_{n=1}^{\infty} f_1^*(\gamma) f_{n-1}^*(\gamma) P_f(\mathbf{p}) R_f(\mathbf{p}))^{n-1} = \frac{P_f(\mathbf{p}) f_1^*(\gamma)}{1 - R_f(\mathbf{p}) f^*(\gamma)} \tag{14}$$

For the case treated in eq. (13) we have for stationary Poissonian load occurrences:

$$h^*(\gamma, \mathbf{p}) = \frac{g_1^*(\gamma, \mathbf{p})}{1 - g^*(\gamma, \mathbf{p})} = \frac{P_f(\mathbf{p}) \lambda}{\gamma} \tag{15}$$

Unfortunately, Laplace transforms are rarely analytic. Taking Laplace transforms numerically requires some effort but taking the inverse Laplace transform must simply be considered as an numerically ill-posed problem. Then, however, one always can resort to the asymptotic result which can be shown to be accurate enough for all practical purposes.

The foregoing results can be generalized to cover multiple mode failure, loss of serviceability, obsolescence of the facility and inspection and maintenance. Also, the case of non-constant benefit, a case of obsolescence, or non-constant damage has been addressed. Further developments are under way.

4. Computation of Failure Probabilities and Failure Rates

4.1 Time-invariant Reliabilities

The simplest problem of computing failure probabilities is given as a volume integral

$$P_f(\mathbf{p}) = P(F) = \int_{F(\mathbf{p})} dF_{\mathbf{X}}(\mathbf{x}, \mathbf{p}) = \int_{F(\mathbf{p})} f_{\mathbf{X}}(\mathbf{x}, \mathbf{p}) d\mathbf{x} \tag{16}$$

where the failure event is $F(\mathbf{p}) = \{h(\mathbf{x},\mathbf{p}) \leq 0\}$ and the random vector $\mathbf{X} = (X_1, X_2, ..., X_n)^T$ has joint distribution function $F_{\mathbf{X}}(\mathbf{x})$. Since n usually is large and $P_f(\mathbf{p})$ is small serious numerical difficulties occur if standard methods of numerical integration are applied. However, if it is assumed that the density $f_{\mathbf{X}}(\mathbf{x},\mathbf{p})$ of $F_{\mathbf{X}}(\mathbf{x})$ exists everywhere and $h(\mathbf{x},\mathbf{p})$ is twice differentiable, then, the problem of computing failure probabilities can be converted into a problem of optimization and some simple algebra. For convenience, a probability preserving distribution transformation $\mathbf{U} = T^{-1}(\mathbf{X})$ is first applied [19]. Making use of Laplace integration methods [4] one can then show that with $h(\mathbf{x},\mathbf{p}) = h(T(\mathbf{u}),p) = g(\mathbf{u},\mathbf{p})$ [5], [20]

$$P_f(\mathbf{p}) = \int_{h(\mathbf{x},\mathbf{p})<0} f_{\mathbf{X}}(\mathbf{x},\mathbf{p})d\mathbf{x} = \int_{g(\mathbf{u},\mathbf{p})<0} \varphi_{\mathbf{U}}(\mathbf{u},\mathbf{p})d\mathbf{u}$$
$$\sim \Phi(-\beta)\prod_{i=1}^{n-1}(1-\beta\kappa_i)^{-1/2} \approx \Phi(-\beta) \quad (17)$$

for $1 < \beta \to \infty$ with

$$\beta = \|\mathbf{u}^*\| = \min\{\mathbf{u}\} \text{ for } \{\mathbf{u} : g(\mathbf{u},\mathbf{p}) \leq 0\}, \quad (18)$$

$\varphi_{\mathbf{U}}(\mathbf{u})$ the multinormal density, $\Phi(.)$ the one-dimensional normal integral, $g(\mathbf{0},\mathbf{p}) > 0$ and κ_i the main curvatures of the failure surface $\partial F = \{g(\mathbf{u},\mathbf{p}) = 0\}$. Of course, it is assumed that a unique "critical" point $\mathbf{u}^*$ exists but methods have been devised to also locate and consider appropriately multiple critical points. In line two the asymptotic result is given denoted by second order since the Hessian of $g(\mathbf{u},\mathbf{p}) = 0$ is involved. The last result represents a first-order result corresponding to a linearization of $g(\mathbf{u},\mathbf{p})$ in $\mathbf{u}^*$ already pointed out by [16]. Very frequently this is sufficiently accurate in practical applications.

4.2 Time-variant Reliabilities

Much more difficult is the computation of time-variant reliabilities. Here, the question is not that the system is in an adverse state at some arbitrary point in time but that it enters it for the first time given that it was initially at time $t = 0$ in a safe state. The problem is denoted by first passage problem in the engineering literature. But exact results for distributions of first passage times are almost inexistent. However, good approximations can be obtained by the so-called outcrossing approach [13]. The outcrossing rate is defined by

$$\nu^+(\tau) = \lim_{\Delta\to 0}\frac{1}{\Delta}P(N(\tau,\tau+\Delta) = 1) \quad (19)$$

or for the original vector process

$$\nu^+(\tau) = \lim_{\Delta \to 0} \frac{1}{\Delta} P_1(\{\mathbf{X}(\tau) \in \bar{F}\} \cap \{\mathbf{X}(\tau + \Delta) \in F\}) \tag{20}$$

One easily sees that the definition of the outcrossing rate coincides formally with the definition of the renewal intensity. The counting process $N(.)$ of outcrossings must be a regular process [12] so that the mean value of outcrossings in $[0, t]$ is given by

$$E[N(t)] = \int_0^t \nu^+(\tau) d\tau \tag{21}$$

One can derive an important upper bound. Failure occurs either if $\mathbf{X}(0) \in V$ or $N(t) > 0$. Therefore [28]

$$\begin{aligned} P_f(t) &= 1 - P(\mathbf{X}(\tau) \in \overline{F}) \text{ for all } \tau \in [0, t] \\ &= P(\{\mathbf{X}(0) \in F\} \cup \{N(t) > 0\}) \\ &= P(\mathbf{X}(0) \in F) + P(N(t) > 0) - P(\{\mathbf{X}(0) \in F\} \cap \{N(t) > 0\}) \\ &\leq P(\mathbf{X}(0) \in F) + P(N(t) > 0) \\ &\leq P(\mathbf{X}(0) \in V) + E[N(t)] \end{aligned} \tag{22}$$

If the original process is sufficiently mixing one can derive the asymptotic result [13]:

$$P_f(t) \sim 1 - \exp\left[-E[N(t)]\right] \tag{23}$$

justifying the remarks below eq. (13). A lower bound can also be given. It is less useful.

Consider a stationary vectorial rectangular wave renewal process each component having renewal rate λ_i and amplitude distribution function $F_i(x)$. The amplitudes X_i are independent. Regularity assures that only one component has a renewal in a small time interval with probability $\lambda_i\ \Delta$. Then [9]

$$\begin{aligned} \nu^+(F)\Delta &= P(\bigcup_{i=1}^{n} \{\text{renewal in } [0, \Delta]\} \cap \{\mathbf{X}_i \in \overline{F}\} \cap \{\mathbf{X}_i^+ \in F\}) \\ &= \sum_{i=1}^{n} \Delta\ \lambda_i P(\{\mathbf{X}_i \in \overline{F}\} \cap \{\mathbf{X}_i^+ \in F\}) \\ &= \sum_{i=1}^{n} \Delta\ \lambda_i [P(\mathbf{X}_i^+ \in F) - P(\{\mathbf{X}_i \in F)\} \cap \{\mathbf{X}_i^+ \in F\})] \end{aligned} \tag{24}$$

$\mathbf{X}_i$ denotes the process $\mathbf{X}$ before and $\mathbf{X}_i^+$the process after a jump of the $i-$th component. If the components are standard normally distributed and the failure domain is a half-space $F = \left\{\alpha^T \mathbf{u} + \beta \leq 0\right\}$ one

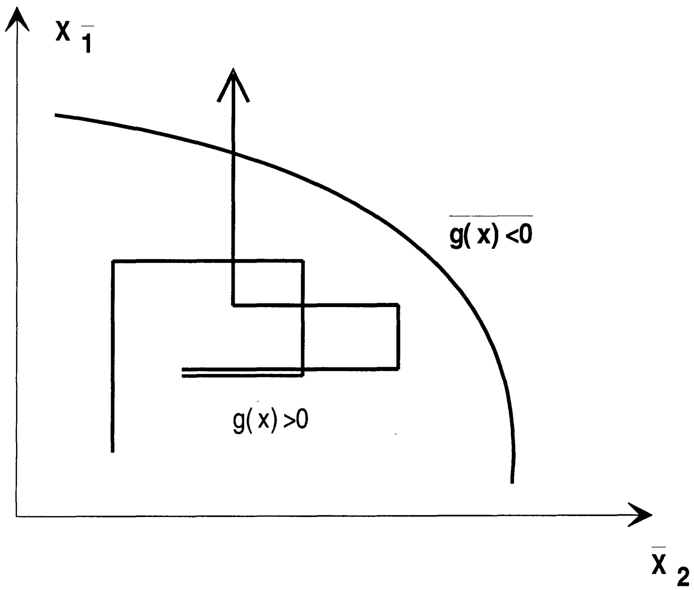

Figure 1. Outcrossings of a vectorial rectangular wave renewal process

determines

$$\nu^+(V) = \sum_{i=1}^{n} \lambda_i \left[P\left(\{\alpha^T \mathbf{U}_i > -\beta\} \cap \{\alpha^T \mathbf{U}_i^+ \leq -\beta\}\right)\right]$$
$$= \sum_{i=1}^{n} \lambda_i \left[\Phi_2(\beta, -\beta; \rho_i)\right] \leq \sum_{i=1}^{n} \lambda_i \Phi(-\beta) \tag{25}$$

where $\rho_i = 1 - \alpha_i^2$ is the correlation coefficient of the process before and after a jump and $\Phi_2(.,.;.)$ the bivariate normal integral. For general non-linear failure surfaces one can show that asymptotically [8]

$$\nu^+(F) = \sum_{i=1}^{n} \lambda_i \Phi(-\beta) \prod_{i=1}^{n-1} (1 - \beta\ \kappa_i)^{-1/2}; 1 < \beta \to \infty \tag{26}$$

with $\beta = \|\mathbf{u}^*\| = \min\{\|\mathbf{u}\|\}$ for $g(\mathbf{u}) \leq \mathbf{0}$ and κ_i the main curvatures in the solution point $\mathbf{u}^*$. This corresponds to the result in eq. (17). The same optimization problem as in the time-invariant case has to be solved. Rectangular wave renewal processes are used to model life loads, sea states, traffic loads, etc..

For stationary vector processes with differentiable sample paths it is useful to standardize the original process $\mathbf{X}(t)$ and its derivative (in mean square) process $\dot{\mathbf{X}}(t) = \frac{d}{dt}\mathbf{X}(t)$ such that $E\left[\mathbf{U}(t)\right] = E\left[\dot{\mathbf{U}}(t)\right] = 0, \mathbf{R}(0) = \mathbf{I}$ where $\mathbf{R}(\tau) = \mathbf{E}\left[\mathbf{U}(0)\mathbf{U}(\tau)^T\right]$ is the matrix of correlation functions and $\tau = |t_1 - t_2|$. A matrix of cross correlation functions between $\mathbf{U}(t)$ and $\dot{\mathbf{U}}(t)$, $\dot{\mathbf{R}}(\tau) = \mathbf{E}\left[\mathbf{U}(0)\dot{\mathbf{U}}(\tau)^T\right]$, as well as of the derivative process $\ddot{\mathbf{R}}(\tau) = \mathbf{E}\left[\dot{\mathbf{U}}(0)\dot{\mathbf{U}}(\tau)^T\right]$ also exists. The general outcrossing rate is defined by [38], [3]

$$\nu^+(t) = \lim_{\Delta\tau\to 0} \frac{P(\{\mathbf{U}(t)\in \boldsymbol{\Delta}(\partial F(t))\} \bigcap \left\{\dot{U}_N(t) > \partial\dot{F}(t)\right\} \text{ in } [\tau \le t \le \tau + \Delta\tau])}{\Delta\tau} \tag{27}$$

where $\dot{U}_N(t) = \mathbf{n}^T(\mathbf{u},t)\dot{\mathbf{U}}(t)$ the projection of $\dot{\mathbf{U}}(t)$ on the normal $\mathbf{n}(\mathbf{u},t) = -\alpha(\mathbf{u},t)$ of $\partial F(t)$ in $(\mathbf{u},t)$. $\boldsymbol{\Delta}(\partial F(t))$ is a thin layer around $\partial F(t)$ with thickness $(\dot{u}_N(t) - \partial\dot{F}(t))\Delta\tau$. Hence, it is:

$$\begin{aligned} &P(\{\mathbf{U}(t) \in \boldsymbol{\Delta}(\partial F(t))\} \bigcap \left\{\dot{U}_N(t) > \partial\dot{F}(t)\right\} \text{ in } [\tau \le t \le \tau + \Delta\tau]) \\ &= \int_{\boldsymbol{\Delta}(\partial F(t))} \int_{\dot{U}_N(t) > \partial\dot{F}(t)} \varphi_{n+1}(\mathbf{u},\dot{u}_N,t)d\mathbf{u}d\dot{u}_N \\ &= \Delta\tau \int_{\partial F(t)} \int_{\dot{U}_N(t) > \partial\dot{F}(t)} (\dot{u}_N - \partial\dot{F}(t))\varphi_{n+1}(\mathbf{u},\dot{u}_N,t)ds(\mathbf{u})d\dot{u}_N \end{aligned} \tag{28}$$

In the stationary case one finds with $\partial F \equiv g(\mathbf{u}) = 0$

$$\begin{aligned} \nu^+(\partial F) &= \int_{\partial F} \int_0^\infty \dot{u}_N \varphi_{n+1}(\mathbf{u}, \dot{u}_N) d\dot{u}_N ds(\mathbf{u}) \\ &= \int_{\partial F} \int_0^\infty \dot{u}_N \varphi_1(\dot{u}_N | \mathbf{U} = \mathbf{u}) \varphi_n(\mathbf{u}) d\dot{u}_N ds(\mathbf{u}) \\ &= \int_{\partial F} E_0^\infty \left[\dot{U}_N | \mathbf{U} = \mathbf{u}\right] \varphi_n(\mathbf{u}) ds(\mathbf{u}) \\ &= \int_{\mathrm{R}^{n-1}} E_0^\infty \left[\dot{U}_N | \mathbf{U} = \mathbf{u}\right] \varphi_{n-1}(\tilde{\mathbf{u}}, p(\tilde{\mathbf{u}})) T(\tilde{\mathbf{u}}) d\tilde{\mathbf{u}} \end{aligned} \tag{29}$$

where $u_n = p(\tilde{\mathbf{u}}) = g^{-1}(u_1, u_2, ..., u_{n-1})$ a parameterization of the surface and $T(\tilde{\mathbf{u}})$ the corresponding transformation determinant.

Explicit results are available only for special forms of the failure surface. For example, if it is a hyperplane

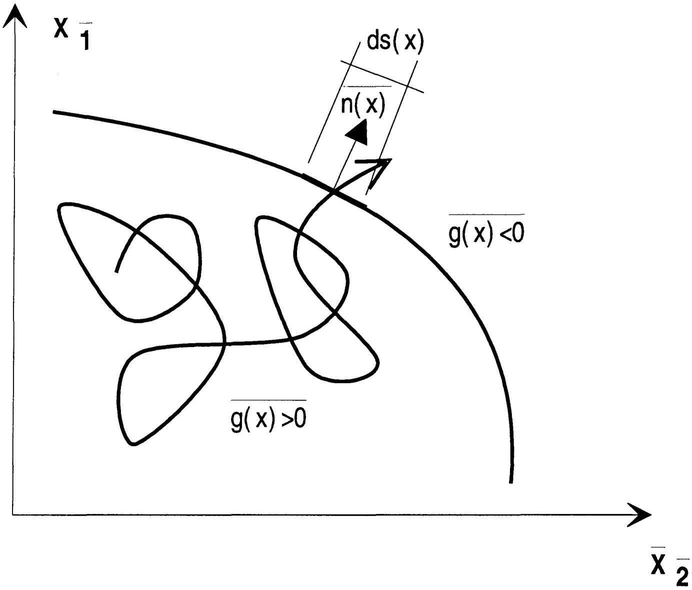

Figure 2. Outcrossing of a vectorial differentiable process

$$\partial V = \left\{ \sum_{i=1}^{n} \alpha_i u_i + \beta = 0 \right\} \tag{30}$$

the outcrossing rate of a stationary standardized Gaussian process is [51]:

$$\nu^+(\partial F) = E\left[\dot{U}_N\right] f(\partial F) = \frac{\kappa_N}{\sqrt{2\pi}} \varphi(\beta) \tag{31}$$

with $\kappa_N^2 = \alpha^T \ddot{\mathbf{R}}(\tau)\alpha$.. An asymptotic result for general non-linear surfaces has been derived in [7]:

$$\nu^+(\partial F) = \omega_0 \frac{\varphi(\beta)}{\sqrt{2\pi}} \prod_{i=1}^{n-1} (1 - \beta\kappa_i)^{-1/2} \tag{32}$$

with

$$\omega_0^2 = \mathbf{n}(\mathbf{u}^*)^T \left[\ddot{\mathbf{R}}(0) + \dot{\mathbf{R}}(0)^T \mathbf{G}(\mathbf{u}^*)\dot{\mathbf{R}}(0)\right] \mathbf{n}(\mathbf{u}^*)$$

provided that $g(\mathbf{0}) > 0$ and with $\dot{\mathbf{R}}(0) = \mathbf{E}\left[\mathbf{U}(0)\dot{\mathbf{U}}(0)^T\right]$ and

$$\mathbf{G}(\mathbf{u}^*) = \left\{ \nabla g(\mathbf{u}^*)^{-1} \frac{\partial^2 g(\mathbf{u}^*)}{\partial u_i \partial u_j}; i, j = 1, \ldots, n \right\}$$

Here again we have $\beta = \|\mathbf{u}^*\| = \min\{\|\mathbf{u}\|\}$ for $g(\mathbf{u}) \leq \mathbf{0}$ and κ_i are the main curvatures of ∂F in the solution point $\mathbf{u}^*$. Differentiable processes are used to model the turbulent natural wind, wind waves and earthquake excitations but also the output of dynamical systems.

Exact or approximate results have also been obtained for non-gaussian rectangular wave processes with or without correlated components [34], certain non-gaussian differentiable processes [14] and a variety of non-stationarities of the processes or the failure surfaces [35]. If one is not satisfied with the (asymptotic) approximations one can apply importance sampling methods in order to arrive at an arbitrarily exact result. Due to regularity of the crossings one can combine rectangular wave and differentiable processes. The processes can be intermittent [46], [22]. This allows the modelling of disturbances of short to very short duration (earthquakes, explosions). Such models have also been extended to deal with occurrence clustering [55], [45].

It is remarkable that the "critical" point $\mathbf{u}^*$, i.e. the magnitude of β, plays an important role in all cases as in the time-invariant case. It must be found by a suitable algorithm. Sequential quadratic programming algorithms tuned to the special problem of interest turned out to solve the optimization problem reliably and efficiently in practical applications [1].

However, it must be mentioned that in time-variant reliability more general models, e.g. renewal models with non-rectangular wave shapes, filtered Poisson process models, etc. can be easily formulated but hardly made practical from a computational point of view.

5. The Value of Human Life and Limb in the Public Interest

Two questions remain: a. Is it admissible to optimize benefits and cost if human lives are endangered and b. can we discount the "cost of human lives"? First of all, modern approaches to these questions do not speak of a monetary value of the human life but rather speak of the cost to save lives.. Secondly, any further argumentation must be within the framework of our moral and ethical principles as laid down in our constitutions and elsewhere. We quote as an example a few articles from the BASIC LAW of the Federal Republic of Germany:

- *Article 2: (1) Everyone has the right to the free development of his personality ...(2) Everyone has the right to life and to inviolability of his person*

- *Article 3: (1) All persons are equal before the law. (2) Men and women have equal rights. (3) No one may be prejudiced or favored because of his sex, his parentage, his race, his language, his homeland and origin, his faith or his religious or political opinions.*

Similar principles are found in all modern, democratic constitutions. But H. D. Thoreau (1817-1862 p.Chr.) realistically says about the value of human life: "*The cost of a thing is the amount of what I will call life which is required to be exchanged for it, immediately or in the long run.* ... [29].

Can these value fixings be transferred to engineering acceptability criteria? This is possible when starting from certain social indicators such as life expectancy, gross national product (GNP), state of health care, etc.. Life expectancy e is the area under the survivor curve $S(a)$ as a function of age a, i.e. $e = \int_0^\infty S(a)da$. A suitable measure for the quality of life is the GNP per capita, despite of some moral indignation at first sight. The GNP is created by labor and capital (stored labor). It provides the infrastructure of a country, its social structure, its cultural and educational offers, its ecological conditions among others but also the means for the individual enjoyment of life by consumption. Most importantly in our context, it creates the possibilities to "buy" additional life years through better medical care, improved safety in road traffic, more safety in or around building facilities or from hazardous technical activities, etc.. Safety of buildings via building codes is an investment into saving lives. The investments into structural safety must be efficient, however. Otherwise investments into other life saving activities are preferable. In all further considerations only about 60% of the GNP, i.e. g $\approx$ 0.6 GNP which is the part available for private use, are taken into account.

Denote by $c(\tau) > 0$ the consumption rate at age τ and by $u(c(\tau))$ the utility derived from consumption. Individuals tend to undervalue a prospect of future consumption as compared to that of present consumption. This is taken into account by some discounting. The life time utility for a person at age a until she/he attains age $t > a$ then is

$$
\begin{aligned}
U(a,t) &= \int_a^t u\left[c(\tau)\right] \exp\left[-\int_a^\tau \rho(\theta)d\theta\right] d\tau \\
&= \int_a^t u\left[c(\tau)\right] \exp\left[-\rho(\tau - a)\right] d\tau
\end{aligned}
\tag{33}
$$

for $\rho(\theta) = \rho$. It is assumed that consumption is not delayed, i.e. incomes are not transformed into bequests. ρ should be conceptually distinguished from a financial interest rate and is referred to as rate of time preference of consumption. A rate $\rho > 0$ has been interpreted as the effect of human impatience, myopia, egoism, lack of telescopic faculty, etc.. Exponential population growth with rate n can be considered by replacing ρ by $\rho - n$ taking into account that families are by a factor $exp[nt]$ larger at a later time $t > 0$. The correction $\rho > n$ appears always necessary, simply because future generations are expected to be larger and wealthier. ρ is reported to be between 1 and 3% for health related investments, with tendency to lower values [53]. Empirical estimates reflecting pure consumption behavior vary considerably but are in part significantly larger [25].

The expected remaining present value life time utility at age a (conditional on having survived until a) then is (see [2] [43] [39] [15])

$$\begin{aligned} L(a) = E[U(a)] &= \int_a^{a_u} \frac{f(t)}{\ell(a)} U(a,t)dt \\ &= \int_a^{a_u} \frac{f(t)}{\ell(a)} \int_a^t u[c(\tau)] \exp[-(\rho - n)(\tau - a)]\, d\tau dt \\ &= \frac{1}{\ell(a)} \int_a^{a_u} u[c(t)] \exp[-(\rho - n)(t - a)]\, \ell(t) dt \\ &= u[c]\, e_d(a, \rho, n) \end{aligned} \tag{34}$$

where $f(t)dt = \left(\mu(\tau) \exp\left[-\int_0^t \mu(\tau)d\tau\right]\right) dt$ is the probability of dying between age t and $t + dt$ computed from life tables. The expression in the third line is obtained upon integration by parts. Also, a constant consumption rate c independent of t has been introduced which can be shown to be optimal under perfect market conditions [43]. The "discounted" life expectancy $e_d(a, \rho, n)$ at age a can be computed from

$$e_d(a, \rho, n) = \frac{\exp((\rho - n)a)}{\ell(a)} \int_a^{a_u} \exp\left[-\int_0^t (\mu(\tau) + (\rho - n))d\tau\right] dt \tag{35}$$

"Discounting" affects $e_d(a, \rho, n)$ primarily when $\mu(\tau)$ is small (i.e. at young age) while it has little effect for larger $\mu(\tau)$ at higher ages. It is important to recognize that "discounting" by ρ is initially with respect to $u[c(\tau)]$ but is formally included in the life expectancy term.

For $u[c]$ we select a power function

$$u[c] = \frac{c^q - 1}{q} \tag{36}$$

with $0 \leq q \leq 1$, implying constant relative risk aversion according to Arrow-Pratt. The form of eq. (36) reflects the reasonable assumption that marginal utility $\frac{du[c]}{dc} = c^{q-1}$ decays with consumption c. $u[c]$ is a concave function since $\frac{du[c]}{dc} > 0$ for $q \geq 0$ and $\frac{d^2u[c]}{dc^2} < 0$ for $q < 1$. The numerical value has been chosen to be about 0.2 (see [43] [15] and elsewhere as well as table 2 below). It may also be derived from from the work-leisure optimization principle as outlined in [29] where $q = \frac{w}{1-w}$ and w the average fraction of e devoted to (paid) work (see [37] for estimates derived from this principle). This magnitude has also been verified empirically (see, for example, [25]). For simplicity, we also take $c = g \gg 1$.

Shepard/Zeckhauser [43] now define the "value of a statistical life" at age a by converting eq. (34) into monetary units in dividing it by the marginal utility $\frac{du(c(t))}{dc(t)} = u'[c(t)]$:

$$\begin{aligned} VSL(a) &= \int_a^{a_u} \frac{u[c(t)]}{u'[c(t)]} \exp\left[-(\rho - n)(t-a)t\right] \frac{\ell(t)}{\ell(a)} dt \\ &= \frac{u[c]}{u'[c]} \frac{1}{\ell(a)} \int_a^{a_u} \exp\left[-(\rho - n)(t-a)\right] \ell(t) dt \\ &= \frac{g}{q} \frac{1}{\ell(a)} \int_a^{a_u} \exp\left[-(\rho - n)(t-a)\right] \ell(t) dt \\ &= \frac{g}{q} e_d(a, \rho, n) \end{aligned} \tag{37}$$

because $\frac{u[c(t)]}{u'[c(t)]} = \frac{g}{q}$. The "willingness-to-pay" has been defined as

$$WTP(a) = VSL(a)\, dm \tag{38}$$

In analogy to Pandey/Nathwani [31], and here we differ from the related economics literature, these quantities are averaged over the age distribution $h(a, n)$ in a stable population in order to take proper account of the composition of the population exposed to hazards in and from technical objects. One obtains the "societal value of a statistical life"

$$\overline{SVSL} = \frac{g}{q} \bar{E} \tag{39}$$

with

$$\bar{E} = \int_0^{a_u} e_d(a, \rho, n) h(a, n) da \tag{40}$$

and the "societal willingness-to-pay" as:

$$\overline{SWTP} = \overline{SVSL}\; dm \tag{41}$$

For $\rho = 0$ the averaged "discounted" life expectancy $\bar{E}$ is a quantity which is about 60% of e and considerably less than that for larger ρ. In this purely economic consideration it appears appropriate to define also the undiscounted average lost earnings in case of death, i.e. the so-called "human capital":

$$HC = \int_0^{a_u} g(e-a)h(a,n)da \tag{42}$$

Table 1 shows the $\overline{SVSL}$ for some selected countries as a function of ρ indicating the importance of a realistic assessment of ρ.

		France	Germany	Japan	Russia	USA
e		78	78	80	66	77
n		0.37%	0.27%	0.17%	-0.35	0.90%
g		14660	14460	15960	5440	22030
q		0.174	0.167	0.208	0.188	0.222
	0%	4.05	3.96	3.46	0.93	5.83
	1%	3.05	3.00	2.62	0.74	4.28
ρ	2%	2.38	2.36	2.06	0.61	3.27
	3%	1.92	1.92	1.67	0.51	2.59
	4%	1.59	1.59	1.39	0.54	2.11

Table 1: $\overline{SVSL}$ 10^6 in PPP US$ for some countries for various ρ (from recent complete life tables provided by national statistical offices)

It can reasonably be assumed that the life risk in and from technical facilities is uniformly distributed over the age and sex of those affected. Also, it is assumed that everybody uses such facilities and, therefore, is exposed to possible fatal accidents. The total cost of a safety related regulation per member of the group and year is $\overline{SWTP} = -dC_Y(p) = -\frac{1}{N}\sum_{i=1}^{r} dC_{Y,i}(p)$ where r is the total number of objects under discussion, each with incremental cost $dC_{Y,i}$ and N is the group size. For simplicity, the design parameter is temporarily assumed to be a scalar. This gives:

$$-dC_Y(p) + \overline{SVSL}\ dm = 0 \tag{43}$$

Let dm be proportional to the mean failure rate $dh(p)$, i.e. it is assumed that the process of failures and renewals is already in a stationary state that is for $t \to \infty$. Rearrangement yields

$$\frac{dC_Y(p)}{dh(p)} = -k\overline{SVSL} \tag{44}$$

where

$$dm = kdh(p), 0 < k \leq 1 \tag{45}$$

the proportionality constant k relating the changes in mortality to changes in the failure rate. Note that for any reasonable risk reducing intervention there is necessarily $dh(p) < 0$.

The criterion eq. (44) is derived for safety-related regulations for a larger group in a society or the entire society. Can it also be applied to individual technical projects? $\overline{SVSL}$ as well as HC were related to one anonymous person. For a specific project it makes sense to apply criterion (44) to the whole group exposed. Therefore, the "life saving cost" of a technical project with N_F potential fatalities is:

$$H_F = HC\ kN_F \tag{46}$$

The monetary losses in case of failure are decomposed into $H = H_M + H_F$ in formulations of the type eq. (10) with H_M the losses not related to human life and limb.

Criterion (44) changes accordingly into:

$$\frac{dC_Y(p)}{dh(p)} = -\overline{SVSL}kN_F \tag{47}$$

All quantities in eq. (47) are related to one year. For a particular technical project all design and construction cost, denoted by $dC(p)$, must be raised at the decision point $t = 0$. The yearly cost must be replaced by the erection cost $dC(p)$ at $t = 0$ on the left hand side of eq. (47) and discounting is necessary. The method of discounting is the same as for discharging an annuity. If the public is involved $dC_Y(p)$ may be interpreted as cost of societal financing of $dC(p)$. The interest rate to be used must then be a societal interest rate to be discussed below. Otherwise the interest rate is the market rate. g in $\overline{SVSL}$ also grows approximately exponentially with rate ζ, the rate of economic growth in a country. It can be taken into account by discounting. The acceptability criterion for individual technical projects then is (discount factor for discounted erection cost moved to the right hand side):

$$\begin{aligned} \frac{dC(p)}{dh(p)} &= -\frac{\exp[\gamma t] - 1}{\gamma \exp[\gamma t]} \overline{SVSL} kN_F \frac{\zeta \exp[\zeta t]}{\exp[\zeta t] - 1} \\ &\underset{t\to\infty}{\to} -\overline{SVSL}kN_F \frac{\zeta}{\gamma} \end{aligned} \tag{48}$$

It must be mentioned that a similar very convincing consideration about the necessary effort to reduce the risk for human life from technical objects has been given by Nathwani et al. [29] and in [31] producing estimates for the equivalent of the constant $\overline{SVSL}$ very close to those given in table 1. The estimates for $\overline{SVSL}$ are in good agreement with several other estimates in the literature (see, for example, [49], [43]; [52]; [24] and many others) which are between 1000000 and 10000000 PPP US$ with a clustering around 5000000 PPP US$.

6. Remarks about Interest Rates

A cost-benefit optimization must use interest rates. Considering the time horizon of some 20 to more than 100 years for most structural facilities but also for many risky industrial installations it is clear that average interest rates net of in/deflation must be chosen. If the option with systematic reconstruction is chosen one immediately sees from eq. (14) that the interest rate must be non-zero. For the same equation we see that there is a maximum interest rate $\gamma_{\max}$ for which $Z(\mathbf{p})$ becomes negative for any $\mathbf{p}$

$$\gamma_{\max} = \frac{m(\mathbf{p})b - (C(\mathbf{p}) + H)}{m(\mathbf{p})C(\mathbf{p})} \tag{49}$$

and, therefore, $0 < \gamma \leq \gamma_{\max}$. Also $m(\mathbf{p})b > C(\mathbf{p}) + H$ must be valid for any reasonable project which further implies that $b/\gamma > 1$. Very small interest rates, on the other hand, cause benefit and damage cost to dominate over the erection cost. Then, in the limit

$$Z(\mathbf{p}) = b - \frac{(C(\mathbf{p}) + H)}{m(\mathbf{p})} \tag{50}$$

where the interest rate vanishes. Erection cost are normally weakly increasing in the components of $\mathbf{p}$ but $m(\mathbf{p})$ grows significantly with $\mathbf{p}$. Consequently, the optimum is reached for $m(\mathbf{p}) \to \infty$, that is for perfect safety which is not attainable in practice. In other words the interest rate must be distinctly different from zero. Otherwise, the different parties involved in the project may use interest rates taken from the financial market at the decision point $t = 0$.

The cost for saving life years also enters into the objective function and with it the question of discounting those cost also arises. At first sight this is not in agreement with our moral value system. However, a number of studies summarized in [32] and [23] express a rather clear opinion based on ethical and economical arguments. The cost for saving life years must be discounted at the same rate as other investments, especially in view of the fact that our present value system should be maintained also for future generations. Otherwise serious inconsistencies cannot be avoided.

What should then the discount rate for public investments into life saving projects be? A first estimate could be based on the long term growth rate of the GNP. In most developed, industrial countries this was a little more than 2% over the last 50 years. The United Nations Human Development Report 2000 gives values between 1.2 and 1.9 % for industrialized countries during 1975-1998. If one extends the consideration to the last 120 years one finds an average growth rate ζ of about 1.8% (see table 1). Using data in [47], [27] and the UN Human Development Report 2000 [50] the following table has been compiled from a more detailed table.

	1850	1998								
Ctry.	GNP	GNP	g	e	$n\%$	q	$\rho\%$	$\zeta\%$	$\gamma\%$	$\overline{SVSL}$
UK	3109	23500	15140	77	0.23	0.19	0.5	1.3	1.3	$3.1 \cdot 10^6$
US	1886	34260	22030	78	0.90	0.22	1.3	1.8	2.3	$3.9 \cdot 10^6$
F	1840	24470	14660	78	0.37	0.17	0.7	1.9	1.9	$3.3 \cdot 10^6$
S	1394	23770	12620	79	0.02	0.18	0.3	1.9	1.6	$2.7 \cdot 10^6$
D	1400	25010	14460	77	0.27	0.17	0.6	1.9	1.9	$3.3 \cdot 10^6$
AUS	4027	25370	15750	80	0.99	0.21	0.7	1.2	1.9	$3.3 \cdot 10^6$
J	969	26460	15960	80	0.17	0.20	1.2	2.7	2.3	$2.8 \cdot 10^6$

Table 1: Social indices for some developed industrial countries (all monetary values are in US$, 1998)

It is noted that economic growth the first half of the last century was substantially below average while the second half was well above average. The above considerations can at least define the range of interest rates to be used in long term public investments into life saving operations. For the discount rates to be used in long term public investments the growth theory established by Solow [48] is applied, i.e.

$$n + \zeta(1 - \epsilon) < \rho < \gamma \leq \gamma_{\max} < n + \epsilon\zeta \tag{51}$$

where $\epsilon = 1 - q$ the so-called elasticity of marginal consumption (income). There is much debate about interest rates for long term public investments, especially if sustainability aspects are concerned. But there is an important mathematical result which may guide our choice. Weitzman [54] and others showed that the far-distant future should be discounted at the lowest possible rate > 0 if there are different possible scenarios each with a given probability of being true.

7. A One-Level Optimization for Structural Components

Let us now turn to the technical aspects of optimization. Cost-benefit optimization according to eq. (3) or (10) in principle requires two levels of optimization, one to minimize cost and the other to solve the reliability of optimization, one to minimize cost and the other to solve the reliability problem. However, it is possible to reduce it to one level by adding the Kuhn-Tucker condition of the reliability problem to the cost optimization task provided that the reliability task is formulated in the transformed standard space. For the task in eq. (3) we have

$$\begin{array}{ll} \textbf{Maximize:} & Z(\mathbf{p}) = B^* - C(\mathbf{p}) - (C(\mathbf{p}) + H_M + H_F) \cdot \frac{P_f(\mathbf{p})}{1-P_f(\mathbf{p}))} \\ \textbf{Subject to:} & \\ & g(\mathbf{u},\mathbf{p}) = 0 \\ & u_i \|\nabla_{\mathbf{u}} g(\mathbf{u},\mathbf{p})\| + \nabla_{\mathbf{u}} g(\mathbf{u},\mathbf{p})_i \|\mathbf{u}\| = 0; \; i = 1,...,n-1 \\ & h_k(\mathbf{p}) \leq 0, k = 1,\ldots,q \\ & \nabla_p C(\mathbf{p}) \geq k\overline{SVSL}N_F \frac{\zeta}{\gamma} \nabla_p P_f(\mathbf{p}) \end{array} \tag{52}$$

where the first and second condition represent the Kuhn-Tucker condition for a valid "critical" point, the third condition some restrictions on the parameter vector $\mathbf{p}$ and the forth condition the human life criterion in eq. (48). Frequently, the term $\frac{P_f(\mathbf{p})}{1-P_f(\mathbf{p}))}$ in the objective can be replaced by $P_f(\mathbf{p})$. The failure probability is

$$P_f(\mathbf{p}) \approx \Phi(-\beta(\mathbf{p}))C_{SORM} \tag{53}$$

and we have to require that $\|\mathbf{u}\| \neq 0$ and $\|\nabla_{\mathbf{u}} g(\mathbf{u},\mathbf{p})\| \neq 0$. It is assumed that the second-order correction C_{SORM} is nearly independent of $\mathbf{p}$. In fact, at the expense of some more numerical effort, one can use any update of the first-order result $\Phi(-\beta(\mathbf{p}))$, for example an update by importance sampling provided that the result of importance sampling is formulated as a correction factor to the first-order result. $\nabla_p C(\mathbf{p})$ usually must be determined numerically.

For time-variant problems as in eq. (10) one finds the outcrossing rate for a combination of rectangular wave and differentiable processes as:

$$\nu^+(\mathbf{p}) = \left(\sum_{i=1}^{n_J} \lambda_i \Phi(-\beta) + \omega_0 \frac{\varphi(\beta)}{\sqrt{2\pi}} \right) C_{SORM} \tag{54}$$

The optimization task is

Minimize: $Z(\mathbf{p}) = \frac{b}{\gamma} - C(\mathbf{p}) - (C(\mathbf{p}) + H_M + H_F) \cdot \frac{\nu^+(\mathbf{p})}{\gamma}$

Subject to:

$$g(\mathbf{u}, \mathbf{p}) = 0$$
$$u_i \|\nabla_{\mathbf{u}} g(\mathbf{u}, \mathbf{p})\| + \nabla_{\mathbf{u}} g(\mathbf{u}, \mathbf{p})_i \|\mathbf{u}\| = 0; \; i = 1, ..., n-1$$
$$h_k(\mathbf{p}) \leq 0, k = 1, \ldots, q$$
$$\nabla_p C(\mathbf{p}) \geq k\overline{SVSL}N_F \tfrac{\zeta}{\gamma} \nabla_p \nu^+(\mathbf{p}) \tag{55a}$$

For the case in eq. (15) one replaces $\frac{\nu^+(\mathbf{p})}{\gamma}$ by $\frac{\lambda P_f(\mathbf{p})}{\gamma}$ and $\nabla_p \nu^+(\mathbf{p})$ by $\nabla_p \lambda P_f(\mathbf{p})$.

The optimization tasks in eq. (52) or in (55a) are conveniently performed by suitable SQP-algorithms (for example, [44], [33]). For both formulations eq. (52) and (55a), respectively, gradient-based optimizers require the gradients of the objective as well as the gradients of all constraints. This means that second derivatives are required in order to calculate the gradient of second condition as well as of the human value criterion, in particular, the entries into the Hessian of $g(\mathbf{u}, \mathbf{p})$. This is also the most serious objection against this form of a one level approach. One can, however, proceed iteratively for well-behaved failure surfaces. Initially, one assumes a linear or linearized failure surface and sets $C^{(0)}_{SORM} = 1$. Then, all entries $\frac{\partial^2 g(\mathbf{u}.\mathbf{p})}{\partial u_i \partial u_j}$ are zero. After a first solution of problem (52) or (55a) one determines the Hessian once in the solution point $(\mathbf{u}^{*(1)}, \mathbf{p}^{(1)})$ and with it also calculates $C^{(1)}_{SORM}$. Problems (52) or (55a) are then solved a second time with fixed Hessian $\mathbf{G}(\mathbf{u}^{*(1)}, \mathbf{p}^{(1)})$ and so forth. This schemes is repeated until convergence is reached which usually is after a few steps. From a practical point of view it is frequently sufficient to use first-order reliability results and no iteration is necessary.

In closing this section it is important to note that the optimization tasks as formulated in eq. (52) and (55a) are among the easiest one can think of. In practice safety related design decisions additionally include changes in the lay-out, in the structural system or in the maintenance strategy. Optimization is over discrete sets of design alternatives. Clearly, this is more difficult and very little is known how to do it formally except in a heuristic, empirical manner in small dimensions.

8. Example

As an example we take a rather simple case of a system where failure is defined if the random resistance or capacity is exceeded by the random demand, i.e. the failure event is defined as $F = \{R - S(t) \leq 0\}$. The demand is modelled as a one-dimensional, stationary marked Poissonian renewal process of disturbances (earthquakes, wind storms, explosions, etc.) with stationary renewal rate λ and random, independent sizes of the disturbances $S_i, i = 1, 2, ...$Random resistance is log-normally distributed with mean p and a coefficient of variation V_R. The disturbances are also independently log-normally distributed with mean equal to unity and coefficient of variation V_S. A disturbance causes failure with probability:

$$P_f(p) = \Phi\left(-\frac{\ln\left\{p\sqrt{\frac{1+V_S^2}{1+V_R^2}}\right\}}{\sqrt{\ln\left((1+V_R^2)(1+V_S^2)\right)}}\right) \tag{56}$$

Thus, the failure rate is $\lambda P_f(p)$ and the Laplace transform of the renewal density is:

$$h^*(\gamma, p) = \frac{\lambda P_f(p)}{\gamma} \tag{57}$$

An appropriate objective function given systematic reconstruction then is

$$\frac{Z(p)}{C_0} = \frac{b}{\gamma C_0} - \left(1 + \frac{C_1}{C_0}p^a\right) - \left(1 + \frac{C_1}{C_0}p^a + \frac{H_M}{C_0} + \frac{H_F}{C_0}\right)\frac{\lambda P_f(p)}{\gamma} \tag{58}$$

which is to be maximized. The criterion (62) has the form:

$$\frac{d}{dp}\left(1 + \frac{C_1}{C_0}p^a\right) \geq -k\overline{SVSL}N_F\frac{\zeta}{\gamma}\frac{d}{dp}\left(\lambda P_f(p)\right) \tag{59}$$

Some more or less realistic, typical parameter assumptions are: $C_0 = 10^6$, $C_1 = 10^4$, $a = 1.25$, $H_M = 3 \cdot C_0$, $V_R = 0.2$, $V_S = 0.3$, and $\lambda = 1$ $[1/year]$. The socio-economic demographic data are $e = 77$, $GDP = 25000$, $g = 15000$, $w = 0.15, N_F = 100, k = 0.1$ so that $H_F = HC\ kN_F = 5.8 \cdot 10^6$ and $\overline{SVSL}kN_F = 3.3 \cdot 10^7$. The value of N_F is chosen relatively large for demonstration purposes. Monetary values are in US\$. Optimization is performed for the public and for the owner separately.

For the public $b_S = 0.02C_0$ and $\gamma_S = 0.0185$ are chosen. Also, we take $\frac{\zeta}{\gamma_S} = 1$ for simplicity. In particular, benefit and discount rate are chosen such that the public does not make direct profit from an economic activity of its members. Optimization including the cost H_F gives $p_S^* = 4.35$, the corresponding failure rate is $1.2 \cdot 10^{-5}$. Criterion (48) is already fulfilled for $p_l = 3.48$ corresponding to a yearly failure rate of $1.6 \cdot 10^{-4}$ but $Z_S(p_l)/C_0$ being already negative. It is interesting to see that in this case the public can do better in adopting the optimal solution rather than just realizing the facility at its acceptability limit as pointed out already earlier.

The owner uses some typical values of $b_O = 0.07C_0$ and $\gamma_O = 0.05$ and does or does not include life saving cost. If he includes life saving cost the objective function is shifted to the right (dashed line). The calculations yield $p_O^* = 3.76$ and $p_O^* = 4.03$, respectively, and the corresponding failure rates are $7.1 \cdot 10^{-5}$ and $3.2 \cdot 10^{-5}$. The acceptability criterion limits the owner's region for reasonable designs. Inclusion of life saving cost has relatively little influence on the position of the optimum.

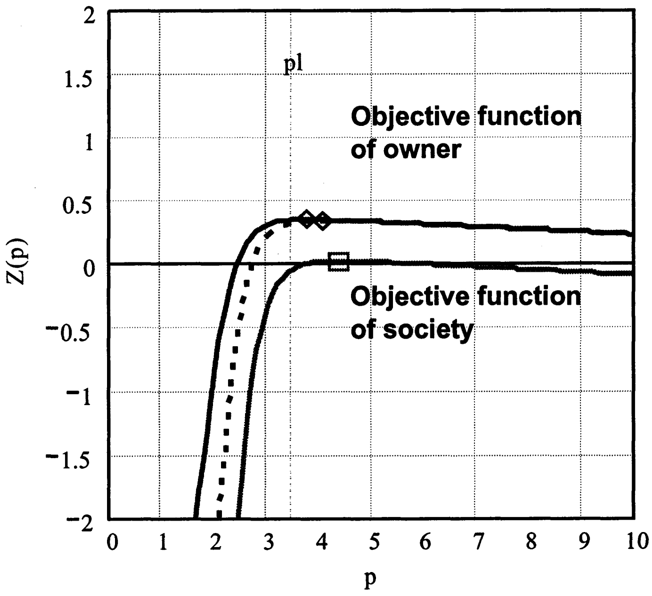

Figure 3. Objective function of owner and society

It is noted that the stochastic model and the variability of capacity and demand also play an important role for the magnitude and location of the optimum as well as the acceptability limit. The specific marginal cost (rate of change) of a safety measure and its effect on a reduction of the failure rate are equally important.

9. Conclusions

Optimization techniques are essential ingredients of reliability-oriented optimal designs of technical facilities. Although many technical aspects are not yet solved and the available spectrum of models and methods in structural reliability is still limited many practical problems can be solved. A special one-level optimization is proposed for general cost-benefit analysis. In this paper, however, focus is on some more critical issues, for example, "what is a reasonable replacement strategy for structural facilities?", "how safe is safe enough?" and "how to discount losses of material, opportunity and human lives?". An attempt has been made to give at least partial answers. Only if those issues have an answer overall optimization of technical facilities with respect to cost makes sense.

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