AN H^∞ DESIGN METHOD FOR FAULT DETECTION AND IDENTIFICATION PROBLEMS

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Abstract In the present paper an H^∞ type approach is developed in order to solve fault detection and identification problems for linear dynamic systems. It is shown that the design specifications lead to a multi–objectives optimization problem. Necessary and sufficient solvability conditions for these problems are given together with an illustrative numerical example.

Keywords: Fault detection and identification, H^∞ control, matrix inequalities.

1. Introduction

The fault detection and identification (FDI) problem has been intensively investigated over the last three decades. Roughly speaking, the problem consists in designing a system able to detect, based on the measured outputs of a monitored plant, any failure of the actuators or of the sensors that can occur in the plant functioning. This system is often called in the literature *residual generator*. The problem becomes more difficult when the influence of certain disturbances is taken into account. In this case the residual generator must "distinguish" between the

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failure occurrence and the disturbance influence in order to avoid false alarm signals. A more refined statement requires not only to detect a failure but to indicate what failure occurred. This design specification is known as *fault identification* requirement. Some early statements of the problem and corresponding results are given for instance in [1], [8]. Since then, a large number of methods have been proposed to solve FDI problems. Some of them use model–based techniques which include observer–based methods, factorization methods, fuzzy logic and neural control (e.g. [4], [5], [9], [10]). Other methods as are the eigen–structure assignment approaches are based on the properties of some structural invariant subspaces associated with the linear model of the monitored plant (e.g. [2], [3], [11], [12]). The H^2 and H^∞ norms have been also used in the optimization problems arising in the design of the residual generator ([6], [10], [13] and their references).

The present paper describes an H^∞ type method to solve FDI problems. The case when all faults can simultaneously occur at any moment is considered. It is shown that the specific design objectives can be expressed as γ–attenuation conditions. In contrast with the well–known H^∞ control problems, these γ–attenuation conditions cannot be expressed in function of lower linear fractional transformation and therefore the design methodology of the residual generator is different. The paper is organized as follows: in Section 2 the FDI problem is stated and the specific design objectives are transformed in γ–attenuation conditions. Section 3 includes the main result providing necessary and sufficient conditions for the solvability of the FDI problem and the design algorithm of the residual generator. Finally, in Section 4 an illustrative numerical example is presented.

2. Problem statement

Consider the following linear dynamic model of the monitored plant:

$$\begin{aligned} \dot{x} &= Ax + B_1 f + B_2 d \\ y &= Cx + D_1 f + D_2 d \end{aligned} \tag{1}$$

where $x \in \mathrm{R}^n$ is the state variable, $y \in \mathrm{R}^p$ denotes the measured output vector, $f \in \mathrm{R}^{m_f}$ is the fault vector and $d \in \mathrm{R}^{m_d}$ stands for the disturbance signal vector. It is assumed that $f \in L^2([0,\infty), \mathrm{R}^{m_f})$ and $d \in L^2([0,\infty), \mathrm{R}^{m_d})$, where $L^2(\cdot,\cdot)$ denotes the Lebesgue space of square integrable functions. It is also assumed that the system (1) is stable. This is a natural assumption indicating that the monitored plant has been previously stabilized by a corresponding control system which dynamics is included in the state space representation (1). The design problem consists in determining an H^∞ residual controller of n_k order with the state equations:

$$\begin{aligned} \dot{x}_k &= A_k x_k + B_k y \\ r &= C_k x_k + D_k y \end{aligned} \tag{2}$$

such that the following conditions are accomplished:

- *A_k is stable*
- *Fault detection:*

$$\inf_{\omega \in \mathrm{R}} \underline{\sigma}(G_{rf}(j\omega)) > \alpha \tag{3}$$

- *Disturbance attenuation:*

$$\|G_{rd}(j\omega)\|_\infty < \beta, \tag{4}$$

for some $\alpha > \beta > 0$, where $G_{rf} := KG_{yf}$, $G_{rd} := G_{yd}$ are the transfer functions from f and d to r , respectively, $\underline{\sigma}(\cdot)$ denotes the minimal singular value, and $\|\cdot\|_\infty$ stands for the H^∞ norm.

Another important design objective in the residual generator design is the *fault identification.* This condition requires to take the residual vector r with the same dimension m_f as the fault vector, that is the matrix $D_k D_1$ is square. Moreover, $D_k D_1$ must be invertible; indeed, if $D_k D_1$ is not invertible then for $\omega \to \infty$ the fault detection condition (3) cannot be fulfilled. The invertibility condition of $D_k D_1$ requires D_1 to be a full rank column matrix.

Remark 1 In applications where D_1 is not left invertible one can consider condition (3) on some specified finite frequency interval $\left[\underline{\omega}_f, \overline{\omega}_f\right]$. Introducing an anticausal filter $Q(s)$ such that the output $z(s)=Q(s)y(s)$ satisfies the condition:

$$\begin{aligned} &\sigma\left(G_{zf}(j\omega)\right) = \sigma\left(G_{yf}(j\omega)\right), \quad \forall\, \omega \in \left[\underline{\omega}_f, \overline{\omega}_f\right] \\ &\sigma\left(G_{zf}(j\omega)\right) \geq \inf_{\underline{\omega}_f \leq \omega \leq \overline{\omega}_f} \underline{\sigma}\left(G_{yf}(j\omega)\right), \ \forall\, \omega \notin \left[\underline{\omega}_f, \overline{\omega}_f\right] \end{aligned}$$

one can obtain an equivalent optimization problem in which $G_{zf}(j\omega)$ is invertible on the frequency domain of interest.

Taking into account the above remark, for the further developments the case when D_1 is left invertible will be considered. Then, based on the invertibility assumption of $G_{rf}(j\omega)$, the fault detection condition (3) can be rewritten as:

$$\inf_{\omega \in \mathrm{R}} \underline{\sigma}[(G_{rf}(j\omega)) = \frac{1}{\sup\limits_{\omega \in \mathrm{R}} \overline{\sigma}[G_{rf}^{-1}(j\omega)]} > \alpha,$$

or equivalently,

$$\left\|G_{rf}^{-1}(j\omega)\right\|_\infty < \frac{1}{\alpha}. \tag{5}$$

3. Main result

In this section necessary and sufficient conditions for the existence of a stable residual generator satisfying the (3) and (4) are derived. Condition (4) is an H^∞ norm optimization problem but it differs from the usual H^∞ control problems since $G_{rf}^{-1} = (KG_{yf})^{-1}$ cannot be expressed as a lower linear fractional transformation. Therefore condition (5) requires a specific analysis. Thus, since $G_{rf} = KG_{yf}$, based on the realizations (1) and (2) of G_{yf} and K respectively, direct computations give that $G_{rf}^{-1}(s)$ has the realization (A_i, B_i, C_i, D_i), where:

$$A_i = \begin{bmatrix} A - B_1 (D_k D_1)^{-1} D_k C & -B_1 (D_k D_1)^{-1} C_k \\ B_k \left(I - D_1 (D_k D_1)^{-1} D_k\right) C & A_k - B_k D_1 (D_k D_1)^{-1} C_k \end{bmatrix},$$
$$B_i = \begin{bmatrix} B_1 \\ B_k D_1 \end{bmatrix} (D_k D_1)^{-1},$$
$$C_i = -(D_k D_1)^{-1} [D_k C \quad C_k]$$
$$D_i = (D_k D_1)^{-1}.$$

Then using the Bounded Real Lemma, in linear matrix inequality (LMI) form, it follows that the equivalent fault detection condition (5) is accomplished if and only if it exists a symmetric matrix $X > 0$ with dimensions $(n + n_k) \times (n + n_k)$ such that:

$$\begin{bmatrix} A_i^T X + X A_i & X B_i & C_i^T \\ B_i^T X & -\frac{1}{\alpha} I & D_i^T \\ C_i & D_i & -\frac{1}{\alpha} I \end{bmatrix} < 0 \quad .$$

Considering the partition of X:

$$X = \begin{bmatrix} R & M \\ M^T & \tilde{R} \end{bmatrix},$$

with $R \in \mathrm{R}^{n \times n}$, for $D_k = U D_1^+$ with U invertible, the above inequality becomes:

$$\begin{bmatrix} \mathcal{L}_{11} & \mathcal{L}_{12} & \mathcal{L}_{13} & \mathcal{L}_{14} \\ \mathcal{L}_{12}^T & \mathcal{L}_{22} & \mathcal{L}_{23} & \mathcal{L}_{24} \\ \mathcal{L}_{13}^T & \mathcal{L}_{23}^T & -\frac{1}{\alpha} I_{m_f} & \mathcal{L}_{34} \\ \mathcal{L}_{14}^T & \mathcal{L}_{24}^T & \mathcal{L}_{34}^T & -\frac{1}{\alpha} I_{m_f} \end{bmatrix} < 0 \qquad (6)$$

with

$$\mathcal{L}_{11} = \left(A - B_1 D_1^+ C\right)^T R + R\left(A - B_1 D_1^+ C\right) + M B_k \left(I - D_1 D_1^+\right) C$$
$$+ C^T \left(I - D_1 D_1^+\right)^T B_k^T M^T$$
$$\mathcal{L}_{12} = \left(A - B_1 D_1^+ C\right)^T M + C^T \left(I - D_1 D_1^+\right)^T B_k^T \tilde{R}$$
$$- (R B_1 + M B_k D_1) (D_k D_1)^{-1} C_k$$

$$\begin{aligned}
\mathcal{L}_{13} &= (RB_1 + MB_kD_1)(D_kD_1)^{-1} \\
\mathcal{L}_{14} &= -C^TD_1^{+T} \\
\mathcal{L}_{22} &= A_k^T\tilde{R} + \tilde{R}A_k - \left(MB_1 + \tilde{R}B_kD_1\right)(D_kD_1)^{-1}C_k \\
&\quad -C_k^T(D_kD_1)^{-T}\left(MB_1 + \tilde{R}B_kD_1\right)^T \\
\mathcal{L}_{23} &= \left(M^TB_1 + \tilde{R}B_kD_1\right)(D_kD_1)^{-1} \\
\mathcal{L}_{24} &= -C_k^T(D_kD_1)^{-T} \\
\mathcal{L}_{34} &= (D_kD_1)^{-T}
\end{aligned}$$

where $(\cdot)^{-T}$denotes the transpose of the inverse and $(\cdot)^+$ is the pseudo-inverse of $(\cdot)$. Further one can set

$$B_k = \begin{cases} \begin{bmatrix} I_p \\ 0 \end{bmatrix} \; if \;\; \geq p, \;\; or \\ [I_{n_k} \;\; 0] \; if \;\; n_k < p. \end{cases} \tag{7}$$

This setting of B_k does not limit the generality of the problem. Indeed, since B_k is not full rank, it exists a small enough perturbation making it full rank and still satisfying the inequality (6). Taking into account the fault identification design objective with simultaneous occurrence of faults, a natural additional condition is

$$D_kD_1 = \delta I_{m_f} \tag{8}$$

with a specified scalar $\delta > 0$. Then direct algebraic computations show that condition (6) is equivalent with:

$$Z_1 + P_1^T\Omega Q_1 + Q_1^T\Omega^TP_1 < 0 \tag{9}$$

with

$$\begin{aligned}
Z_1 &= \begin{bmatrix} Z_{11} & Z_{12} \end{bmatrix} \\
Z_{11} &= \begin{bmatrix} \tilde{A}^TR + R\tilde{A} + \tilde{C}^TM^T + M\tilde{C} & \tilde{A}^TM + \tilde{C}^T\tilde{R} \\ M^T\tilde{A} + \tilde{R}\tilde{C} & 0 \\ B_1^TR + D_1^TB_k^TM^T & B_1^TM + D_1^TB_k^T\tilde{R} \\ -D_1^+C & 0 \end{bmatrix}, \\
Z_{12} &= \begin{bmatrix} RB_1 + MB_kD_1 & -C^TD_1^{+T} \\ M^TB_1 + \tilde{R}B_kD_1 & 0 \\ -\frac{\delta^2}{\alpha}I_{m_f} & I_{m_f} \\ I_{m_f} & -\frac{1}{\alpha}I_{m_f} \end{bmatrix}, \\
P_1 &= \begin{bmatrix} M^T & \tilde{R} & 0 & 0 \\ -\left(B_1^TR + D_1^TB_k^TM^T\right) & -\left(B_1^TM + D_1^TB_k^T\tilde{R}\right) & 0 & -I_{m_f} \end{bmatrix}, \\
Q_1 &= \begin{bmatrix} 0 & I_{n_k} & 0 & 0 \end{bmatrix}, \\
\Omega &= \begin{bmatrix} A_k \\ \tilde{C}_k \end{bmatrix}
\end{aligned} \tag{10}$$

where the following notations have been used:

$$\begin{aligned}\tilde{A} &:= A - B_1 D_1^+ C \\ \tilde{C} &:= B_k \left(I - D_1 D_1^+\right) C \\ \tilde{C}_k &:= \left(D_k D_1\right)^{-1} C_k\end{aligned} \tag{11}$$

In the virtue of the Projection Lemma ([11]), there exists Ω satisfying (9) if and only if:

$$W_{P_1}^T Z_1 W_{P_1} < 0 \tag{12}$$

$$W_{Q_1}^T Z_1 W_{Q_1} < 0 \tag{13}$$

where W_{P_1} and W_{Q_1} are bases of the null spaces of P_1 and Q_1, respectively. Considering the partition of X^{-1}:

$$X^{-1} = \begin{bmatrix} S & N \\ N^T & \tilde{S} \end{bmatrix} > 0$$

with $S \in \mathrm{R}^{n \times n}$, direct algebraic computations show that the above conditions are equivalent with:

$$AS + SA^T - \gamma_1 B_1 B_1^T < 0 \tag{14}$$

and:

$$\begin{bmatrix} \tilde{A}^T R + R\tilde{A} + \tilde{C}^T M^T + M\tilde{C} & RB_1 + MB_k D_1 & -C^T D_1^{+T} \\ B_1^T R + D_1^T B_k^T M^T & -\frac{\delta^2}{\alpha} I_{m_f} & I_{m_f} \\ -D_1^+ C & I_{m_f} & -\frac{1}{\alpha} I_{m_f} \end{bmatrix} < 0, \tag{15}$$

respectively.

Remark 2 Inspecting the 2×2 block matrix in the right lower corner of the matrix in (15) it follows that $\alpha < \delta$.

Recall that $X > 0$ and thus:

$$\begin{bmatrix} R & M \\ M^T & \tilde{R} \end{bmatrix} > 0. \tag{16}$$

Moreover, R and S are related through the condition $XX^{-1} = I$, which leads to the following additional conditions:

$$\begin{bmatrix} R & I_n \\ I_n & S \end{bmatrix} \geq 0 \tag{17}$$

and

$$rank\left(\begin{bmatrix} R & I_n \\ I_n & S \end{bmatrix}\right) \leq n + n_k. \tag{18}$$

Therefore the fault detection condition (5) is fulfilled if and only if there exist the matrices $R, M, \tilde{R}$ and S satisfying the conditions (14)–(18). In [7] it is proved that if $P_1P_1^T > 0$ and $Q_1W_{P_1}W_{P_1}^TQ_1^T > 0$ then the set of all Ω satisfying the basic LMI (9) has the parametrisation:

$$\Omega = \Phi_1 + \Phi_2 L \Phi_3, \tag{19}$$

where the matrix $L \in \mathrm{R}^{(n_k+m_f)\times n_k}$ is any matrix with $\|L\| < 1$ and Φ_1, Φ_2, Φ_3 depend on P_1, Z_1 and Q_1. The condition $P_1P_1^T > 0$ directly follows from the expression of P_1 in (10). On the other hand $Q_1W_{P_1}W_{P_1}^TQ_1^T = \tilde{R}^{-1}M^TM\tilde{R}^{-1}$. Then, for M full rank column matrix Ω has the parameterisation (19). Further, applying again the Bounded Real Lemma in LMI form, one obtains that the disturbance attenuation condition (4) is fulfilled if and only if it exists a symmetric matrix $X_R \in \mathrm{R}^{(n+n_k)\times(n+n_k)} > 0$ such that:

$$\begin{bmatrix} A_R^TX_R + X_RA_R & X_RB_R & C_R^T \\ B_R^TX_R & -\beta I_{m_d} & D_R^T \\ C_R & D_R & -\beta I_{m_d} \end{bmatrix} < 0 \tag{20}$$

where (A_R, B_R, C_R, D_R) is a realization of the system KG_{yd}. Performing the partition

$$X_R = \begin{bmatrix} X_1 & X_2 \\ X_2{}^T & X_3 \end{bmatrix} > 0, \tag{21}$$

with $X_1 \in \mathrm{R}^{(n\times n)}$, direct algebraic computations show that condition (20) can be rewritten in the equivalent form:

$$Z_2 + P_2^T\Omega Q_2 + Q_2^T\Omega^T P_2 < 0 \tag{22}$$

which, based on the parameterisation (19) of L leads to:

$$\mathcal{N}(X, L) = \begin{bmatrix} \mathcal{N}_{11} & \mathcal{N}_{12} & \mathcal{N}_{13} & \mathcal{N}_{14} \\ \mathcal{N}_{12}^T & \mathcal{N}_{22} & \mathcal{N}_{23} & \mathcal{N}_{24} \\ \mathcal{N}_{13}^T & \mathcal{N}_{23}^T & -\beta I_{m_d} & \mathcal{N}_{34} \\ \mathcal{N}_{14}^T & \mathcal{N}_{24}^T & \mathcal{N}_{34}^T & \mathcal{N}_{44} \end{bmatrix} < 0 \tag{23}$$

with

$$
\begin{array}{l}
\mathcal{N}_{11}(X) = A^T X_1 + X_1 A + C^T B_k^T X_2^T + X_2 B_k C \\
\mathcal{N}_{12}(X, L) = A^T X_2 + C^T B_k^T X_3 + X_2 (\Phi_{11} + \Phi_{21} L \Phi_3) \\
\mathcal{N}_{13}(X) = X_1 B_2 + X_2 B_k D_2 \\
\mathcal{N}_{14} = C^T D_1^{+T} \\
N_{22}(X, L) = X_3 (\Phi_{11} + \Phi_{21} L \Phi_3) + (\Phi_{11} + \Phi_{21} L \Phi_3)^T X_3 \\
\mathcal{N}_{23}(X) = X_2^T B_2 + X_3 B_k D_2 \\
\mathcal{N}_{24}(L) = \Phi_{12}^T + \Phi_3^T L^T \Phi_{22}^T \\
\mathcal{N}_{34} = D_2^T D_1^{+T} \\
\mathcal{N}_{44} = -\frac{\beta}{\delta^2} I_{m_f}
\end{array}
\tag{24}
$$

Φ_{ij}, $i, j = 1, 2$ *being given by the partition*

$$
[\Phi_1 \;\; \Phi_2] = \left[\begin{array}{cc} \Phi_{11} & \Phi_{21} \\ \Phi_{12} & \Phi_{22} \end{array} \right], \quad \Phi_{11} \in \mathrm{R}^{n_k \times n_k}, \Phi_{21} \in \mathrm{R}^{n_k \times (n_k + m_f)}.
$$

Based on the developments above, one proves the following result:

Theorem *Given* $\alpha > \beta > 0$, *the following are equivalent:*
(i) *There exists an* n_k*-order stable residual generator with* $D_k = \delta D_1^+$ *satisfying the conditions* (3) *and* (4);
(ii) *There exist the symmetric matrices* $X_1 \in \mathrm{R}^{(n \times n)}, X_3 \in \mathrm{R}^{(n_k \times n_k)}$ *and the rectangular matrices* $X_2 \in \mathrm{R}^{(n \times n_k)}, L \in \mathrm{R}^{(n_k + m_f) \times n_k}$ *satisfying* (21), (23) *and*

$$
\left[\begin{array}{cc} I_{n_k + m_f} & L \\ L^T & I_{n_k} \end{array} \right] > 0. \tag{25}
$$

If these conditions are satisfied then the matrices A_k *and* C_k *are given by:*

$$
\left[\begin{array}{c} A_k \\ C_k \end{array} \right] = \left[\begin{array}{cc} I & 0 \\ 0 & \delta I_{m_f} \end{array} \right] (\Phi_1 + \Phi_2 L \Phi_3). \tag{26}
$$

The algorithm to determine an n_k order residual generator solving the fault detection problem (3), (4) is the following:

Step1 Solve the system of matrix inequalities (14)–(18) with respect to $R, S, \tilde{R}$ and Δ;

Step 2 Determine the parameterisation (19) of the set of all Ω with the free parameter L, $\|L\| < 1$;

Step 3 Solve the system of matrix inequalities (21), (23), (25) with respect to X_1, X_2, X_3 and L;

Step 4 Introduce the computed L into (26) to obtain A_k and C_k.

4. A numerical example

Consider the linearized longitudinal dynamics of an aircraft subjected to wind disturbances:

$$\begin{aligned} \dot{x} &= Ax + B_{\omega}\omega + B_{\delta}\delta \\ y &= Cx, \end{aligned}$$

where $x \in \mathrm{R}^5$ the state vector including the longitudinal body axis velocity u, the normal body axis velocity w, the pitch rate q, the pitch angle θ and the wind gust w_g. The control variable δ is the elevon deflection angle and ω is the white noise input with unit spectral density generating the wind gust disturbance w_g. The measured outputs vector y includes: the pitch rate q, the angle of attack α, the normal acceleration a_z and the longitudinal acceleration a_x. For the F16XL fighter at the altitude $h = 10000ft$ and the speed Mach=0.9 the matrices of the state–space realization are ([6]):

$$A = \begin{bmatrix} -0.0674 & 0.0430 & -0.8886 & -0.5587 & 0.0430 \\ 0.0205 & -1.4666 & 16.5800 & -0.0299 & -1.4666 \\ 0.1377 & -1.6788 & -0.6819 & 0 & -1.6788 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1.1948 \end{bmatrix},$$

$$B_{\omega} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1.57 \end{bmatrix}, \quad B_{\delta} = \begin{bmatrix} -0.1672 \\ -1.5172 \\ -9.7842 \\ 0 \\ 0 \end{bmatrix},$$

$$C = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0.0591 & 0 & 0 & 0.0591 \\ 0.0139 & 1.0517 & 0.1495 & -0.0299 & 0 \\ -0.0677 & 0.0431 & 0.0171 & 0 & 0 \end{bmatrix}.$$

The two faults are considered in this example, namely the elevon and the normal acceleration accelerometer failures, respectively. Then $m_f = 2$ and, corresponding to representation (1),

$$B_1 = [B_{\delta} \;\; 0_{5\times 1}], \quad B_2 = B_{\omega}, D_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad D_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Since D_1 is not full rank the fault detection condition (3) cannot be accomplished for high frequency failure signals. Then G_{yf} have been scaled according with Remark 1. Applying the theoretical results presented in Section 3 an H^∞ residual generator of order $n_k = 5$ was obtained. The nonlinear matrix inequality (23) has been solved by an iterative *centering algorithm.* The time responses corresponding to individual and simultaneous failures have been determined. The numerical simulations are performed for unit fault signals occurring at $t = 3$ seconds and they are presented in Figures 1, 2 and 3, respectively.

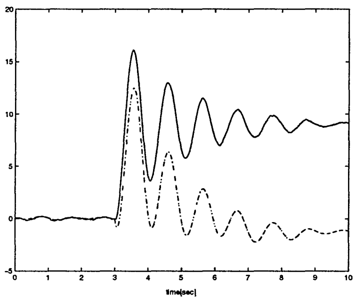

Figure 1. The step responses of the residuals (r_1-, $r_2 - \cdot$): the first fault occurs

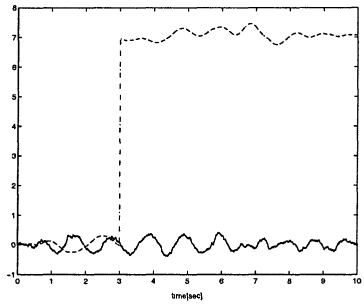

Figure 2. The step responses of the residuals (r_1-, $r_2 - \cdot$): the second fault occurs

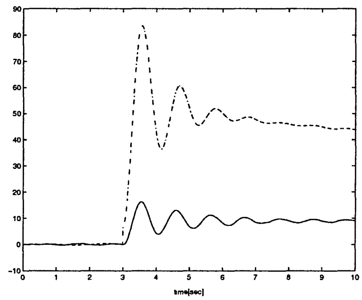

Figure 3. The step responses of the residuals (r_1-, $r_2 - \cdot$): the case when the two faults simultaneously occur

5. Conclusions

An H^∞ type method to solve fault detection and identification problems has been considered. Necessary and sufficient solvability conditions have been derived in terms of some systems of matrix inequalities. The proposed approach allows to develop similar design methods in the discrete–time, stochastic and hybrid representations of the monitored plants.

References

[1] Beard, R.U. "Failure accommodation in linear-systems through self reorganization", Ph.D. Thesis, Department of Aeronautics and Astronautics, MIT, Cambridge, MA, 1971.

[2] Chen, R.H., Speyer J.L. "Optimal Stochastic Multiple-Fault Detection Filter" Proceedings of the 38^{th} CDC, Phoenix, Arizona, USA, 1999.

[3] Choi J.W. "A simultaneous assignment methodology of right/left eigenstructures", *IEEE Transactions on Aerospace and Electronic Systems*, 1998, **34**(2), 625-634;

[4] Frank P.M., Ding X. "Frequency domain approach to optimally robust generation and evaluation for model-based fault diagnosis", *Automatica*, 1994, **30**, 789-904.

[5] Frank P.M., Ding X. "Survey of robust residual generation and evaluation methods in observer-based fault detection systems", *Journal of Process Control* , 1997, **7**, 403-424.

[6] Douglas R.K., Speyer J.L. "H_∞Bounded Fault Detection Filter", *Journal of Guidance, Control and Dynamics*, **22**(1), 129-139, 1999.

[7] Iwasaki, T., Skelton, R.E., " All controllers for the general H_∞ control problem: LMI existence conditions and state space formulas", *Automatica,* 1994, **30**, pp. 1307-1317.

[8] Jones H.L. "Failure detection in linear systems", Ph.D. thesis, Department of Aeronautics and Astronautics, MIT, Cambridge, MA, 1973.

[9] Patton R.J., Chen J. "Parity space approach to model-based fault diagnosis. A tutorial survey and some new results", *Proceedings of IFAC/IMACS Symposium of SAFEPROCESS'91*, Baden-Baden, 1991.

[10] Patton R.J., Hou M. "H_∞estimation and robust fault detection", *Proceedings of European Control Conference, Karlsruhe, Germany, 1999.*

[11] Patton R.J., Chen J. "On eigenstructure assignment for robust fault diagnosis", *International Journal of Robust and Nonlinear Control,* 2000, **10**:1193-1208.

[12] Saberi A., Stoorvogel A., Sannuti P., Niemann H. "Fundamental problems in fault detection and identification", *International Journal of Robust and Nonlinear Control,* 2000, **10**, 1209-1236.

[13] Sauter D., Rambaux F., Hamelin F. "Robust fault diagnosis in an H_∞setting", *Proceedings of SAFAPROCESS'97*, 1997, 879-884.

[14] Skelton R.E., Iwasaki T., Grigoriadis K. *A Unified Algebraic Approach to Linear Control Design*, Taylor & Francis, 1998.