Second-Order Statistics of Closed-Loop Power Controlled Signals in Multi-Path Rayleigh Fading Channels

Hafez Hadinejad-Mahram
and Xiaolong Jiang
Institute of Communications Engineering
Aachen University of Technology
hafez@ient.rwth-aachen.de

Abstract The second-order statistics of power controlled signals in multi-path Rayleigh fading channels are considered. These statistics can be used e.g. in design and evaluation of channel estimation and channel coding schemes. The simulation results presented here provide a comprehensive comparison between the figures obtained with and without power control. The measures considered include the autocorrelation function and the power spectrum of the received signal, the autocovariance function of the signal power, and finally the mean and variance of the received signal power. The results are based on single-user link level simulations with a fixed-step power control scheme.

Keywords: Closed-loop power control, signal statistics

1. Introduction

Power control (PC) is a key ingredient of direct-sequence code division multiple access (DS-CDMA) systems [Gilhousen et al., 1991]. Its major task is to prevent the so called near-far effect, i.e. the situation in which the strong signal of one or several users overwhelms the signals of the other users resulting in a significant degradation of the system performance. Power imbalance among the users of a DS-CDMA system is due to several phenomena including path loss, shadowing (slow or long-term fading), and multi-path (fast or short-term) fading. The former two exhibit reciprocity in the forward and reverse links and thus can be combated by means of open-loop PC [Lee and Steele, 1996, Tam and Lau, 1999]. For example in the uplink, each mobile will measure

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the averaged power from the base station over a long period of time and adjust its transmitted power inversely proportional to the averaged received power. The multi-path fading, on the other hand, varies very fast and is, in general, different for the forward and reverse link due to the large distance between the carrier frequencies of the two links and the frequency-selectivity of the channel. Thus, some form of closed-loop (adaptive) PC (CLPC) is needed to mitigate the effect of multi-path fading. However, it is intuitive and has also been shown by several researchers [Ariyavisitakul and Chang, 1993, Chockalingam et al., 1998, Sim et al., 1999] that CLPC can only be effective if it is fast enough to track the variations of the channel.

The conventional fixed-step CLPC algorithm [Lee and Steele, 1996] is quite simple: the receiver measures the averaged received signal power or the averaged signal-to-interference power ratio (SIR) over the current PC period and compares it with a preset target value. A command is then sent to the transmitter over a feedback channel requesting it to reduce or increase its transmit power depending on the outcome of the comparison. Both fixed-step and adaptive step-size algorithms have been considered in the literature [Lee and Steele, 1996, Park and Nam, 1999]. Besides the conventional approach several predictive approaches have been proposed [Lau and Tam, 2001a, Lau and Tam, 2001b, Freris et al., 2001].

Most of the recent publications dealing with the statistics of the signal and/or interference in DS-CDMA systems employing fast CLPC aim at estimating the system capacity based either on SIR cumulative distribution functions [Ariyavisitakul and Chang, 1993, Ariyavisitakul, 1994, Hashem and Sousa, 1999] or on the uncoded channel bit error rate (BER) [Chockalingam et al., 1998]. In [Chockalingam et al., 1998], in addition to the BER calculation, the autocovariance function of the power in dB is given which quantifies the ability of the CLPC to compensate for the time-varying channel. The given power correlation statistics are, however, restricted to the case of a flat fading channel. Pirinen [Pirinen, 2001] investigates, among others, the impact of mobility and CLPC on the autocorrelation function of the received signal in flat fading. However, the simulation results seem to be partly flawed.

In this paper, we study the second-order statistics of power controlled signals in multi-path Rayleigh fading channels. The importance of these statistics lies with their application in the design and evaluation of channel estimation and channel coding schemes. Extensive simulation results provide a comprehensive comparison between the figures obtained with and without power control parameterized by mobile speed and power control step size. The measures of interest are the autocorrelation func-

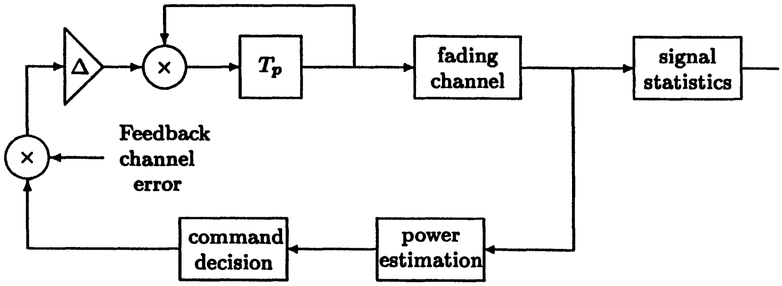


Figure 1. Closed-loop power control model

tion and the power spectral density (PSD) of the received signal, the autocovariance function of the signal power, and finally the mean and variance of the received signal power. The results are based on single-user link level simulations with a fixed-step CLPC scheme. Furthermore, path loss and shadowing are not considered in our model. The fixed parameters like carrier frequency, symbol (or chip) rate, and power control command rate are chosen so as to match those of WCDMA.

2. System Model

The CLPC model used in this study is shown in Fig. 1. The transmitted signal traverses the fading channel and arrives at the receiver. There, the received power is estimated and based on this estimate a power control decision is made which is then sent back to the transmitter. The transmitter can now adjust its transmission power by executing the power control command, i.e. by reducing or increasing its transmission power by a fixed factor Δ^2 (dividing or multiplying the transmitted signal by Δ). This procedure recurs every T_p seconds, where T_p is the power control command rate. Since the power command bit is assumed to be unprotected it is sensible to allow for a certain percentage of feedback channel errors which is also included in the model.

The fading channel is modeled as a multi-path Rayleigh fading channel including the case of a single propagation path (flat fading). Each tap of the channel is assumed to have the classical (Jakes) Doppler spectrum [Jakes, 1974]. We assume that we can perfectly separate the multi-path components, i.e. there is no loss in diversity due to the correlation among the diversity branches. In practical systems this loss is present because of the imperfect autocorrelation properties of the waveforms resulting in the multi-path components to be correlated after demodulation. We believe, that the simplified setting of the ideal diversity system – which has been adopted by many researchers before – captures the essence of the material and provides useful insight into the main characteristics of a power controlled DS-CDMA channel.

Power estimation is accomplished by an integrate and dump device which averages the instantaneous received power over the duration of a PC period T_p. The power command decision is made by comparing this power estimate with a preset threshold.

In the following we describe the statistics considered here in detail. First, we define the normalized sample autocorrelation function of the received signal of k-th tap as

$$\varphi_k(m) = \mathrm{Re}\left\{ \frac{\sum_{n=1}^{N} r_k^*(n)\, r_k(n+m)}{\sum_{n=1}^{N} |r_k(n)|^2} \right\}$$

where N is the number of samples and $r_k(n)$ is the n-th sample of the k-th tap. The PSD estimate of the received signal is calculated for each tap by applying a 1024-point FFT to the above sample autocorrelation function [Oppenheim and Schafer, 1989]. The length of the autocorrelation function is 199 (i.e. 2·100-1) samples where the sampling rate is chosen to be the Nyquist rate, i.e. $2f_D$ with f_D being the maximum Doppler frequency. Finally, the normalized sample autocovariance function of the (total) received signal power in dB is defined as

$$C(m) = \frac{\sum_{n=1}^{N} R(n)\, R(n+m) - \frac{1}{N} | \sum_{n=1}^{N} R(n)|^2}{\sum_{n=1}^{N} |R(n)|^2 - \frac{1}{N} | \sum_{n=1}^{N} R(n)|^2}$$

where $R(n)$ is the total received power in dB at time sample n.

Table 1 summarizes the fixed simulation parameters.

Carrier frequency	2 GHz
Chip rate	3.84 Mchip/s
PC command rate $\frac{1}{T_p}$	1.5 KHz
Spreading factor	256
PC loop delay	$T_p = \frac{1}{1500}$ s
Power threshold	0 dB

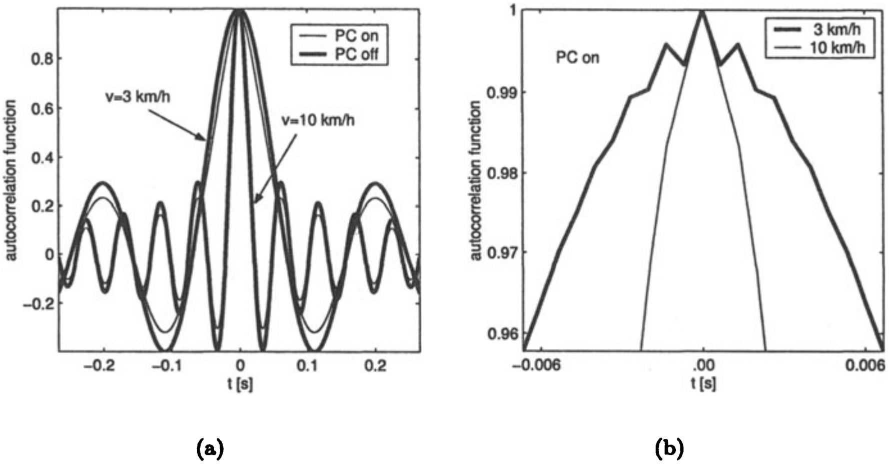
Table 1. Fixed simulation parameters

3. Numerical Results

In this section we present and discuss our simulation results. First we consider the case of a flat fading channel which already reveals the most important characteristics of power controlled signals. Then, we look at some results obtained for the multi-path case.

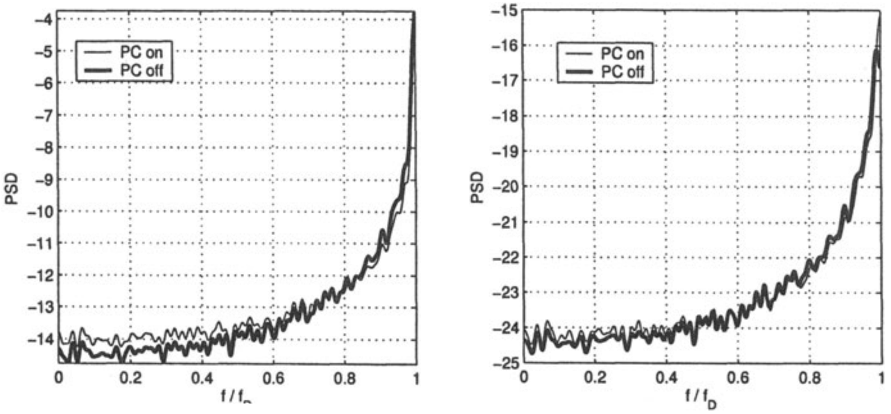
3.1. Flat Fading

The normalized sample autocorrelation function for different step sizes and different velocities was computed. Fig. 2(a) shows the results for 3 and 10 km/h and a power step size of 1 dB (if not otherwise stated, throughout the paper a step size of 1 dB is used). The results for other step sizes were quite similar. It is seen that the autocorrelation function slightly decreases with power control. This decrease becomes smaller for higher velocities and almost vanishes at velocities above 30 km/h (not shown in the figure). Fig. 2(b) shows the autocorrelation function in a narrow interval around zero. The zigzag shape of the curve for 3 km/h can be explained as follows. When the channel changes very slowly a correct PC command can partly compensate the variations of the channel during the previous PC periods which leads to an instant increase of the correlation function at the transition between two PC periods. Note that the extent of this compensation depends on the Doppler rate, thus, this effect can be observed only at the very low velocities.



(a) (b)

Figure 2. Autocorrelation function of the received signal for different velocities.

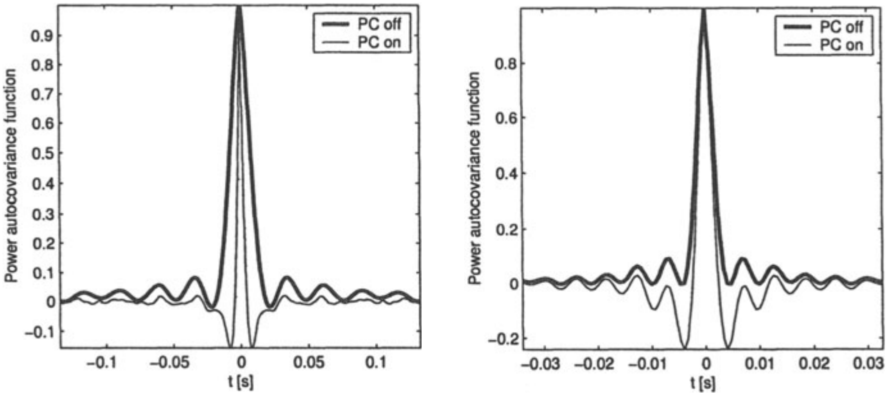


(a) 3 km/h

(b) 50 km/h

Figure 3. Power spectral density with and without PC.

Fig. 3(a) and 3(b) show the PSD for 3 and 50 km/h. In both cases the difference between the PSDs with and without PC is rather small as could be expected from the observation of the autocorrelation functions. However, the differences for 3 km/h is larger than for 50 km/h. In



(a) 10 km/h

(b) 50 km/h

Figure 4. Power autocovariance function with and without PC.

the former case PC is capable of tracking the fading channel thus a noticeable difference in the spectrum, even though small, is expected.

Fig. 4(a) and 4(b) show the normalized autocovariance function of the received signal power in dB. Similar results were reported in [Chockalingam et al., 1998] based on an analytical log-linear model. As can be seen from the comparison of the curves for 10 and 50 km/h, for low velocities, the difference between the curves with and without PC is larger. This is due to the random zigzag shape of the power controlled signal (in dB) at low velocities. The better the PC loop can track the channel variations the more random the curves will look and the less correlated the samples will be. Moreover, the main peak of the autocovariance function is observed to be broader for lower speeds, whereas the sidelobes decrease faster.

step size	0 dB		0.5 dB		1 dB		2 dB	
Velocity	mean	var	mean	var	mean	var	mean	var
3 km/h	0.996	0.992	1.027	0.063	1.016	0.037	1.039	0.085
10 km/h	0.997	0.993	1.192	0.727	1.080	0.241	1.063	0.171
50 km/h	0.997	0.993	1.040	2.123	1.399	2.194	1.352	1.797

Table 2. Mean and variance of the received power for different parameters.

Table 2 shows the mean and variance of the received signal power parameterized by mobile speed and PC step size. Obviously, the optimum step size, i.e. the one which yields a mean close to 1 and a variance close to 0 are different for different velocities.

3.2. Frequency-Selective Fading

Similar simulations as with the flat fading case were conducted for multi-path channels with 2 and 4 taps with different tap weight settings. The results were quite similar to those discussed in the previous section. The PSDs obtained for each tap had a shape like those shown in Fig. 3. However, the anyway small difference between the curves with and without PC was even smaller. As for the power autocovariance function, it could be observed that its sidelobes go to zero faster in the multi-path case and otherwise the shape of the curves are quite similar to those obtained with flat fading.

4. Conclusions

We conducted extensive simulations to asses the changes of the correlation properties of power controlled signals as compared to the case without power control. We looked at several measures including the autocorrelation function, the power spectral density, and the autocovariance function of the total received power. Our simulation results show that the changes of the correlation properties of the received signal are rather slight and can probabily be neglected for most applications even for low mobility.

References

[Ariyavisitakul, 1994] Ariyavisitakul, S. (1994). Signal and interference statistics of a CDMA system with feedback power control - part II. *IEEE Trans. Comm.*, 42(2/3/4):597–605.

[Ariyavisitakul and Chang, 1993] Ariyavisitakul, S. and Chang, L. (1993). Signal and interference statistics of a CDMA system with feedback power control. *IEEE Trans. Comm.*, 41(11):1626–1634.

[Chockalingam et al., 1998] Chockalingam, A., Dietrich, P., Milstein, L., and Rao, R. (1998). Performance of closed-loop power control in DS-CDMA cellular systems. *IEEE Trans. Vehicular Technology*, 47(3):774–788.

[Freris et al., 2001] Freris, N., Jeans, T., and Taaghol, P. (2001). Adaptive SIR estimation in DS-CDMA cellular systems using Kalman filtering. *Electronic Letters*, 37(5):315–317.

[Gilhousen et al., 1991] Gilhousen, K., Jacobs, I., Padovani, R., Viterbi, A., and Weaver, L. (1991). On the capacity of a cellular CDMA system. *IEEE Trans. Vehicular Technology*, 40:303–312.

[Hashem and Sousa, 1999] Hashem, B. and Sousa, E. (1999). Reverse link capacity and interference statistics of a fixed-step power-controlled DS/CDMA system under slow multipath fading. *IEEE Trans. Comm.*, 47(12):1905–1912.

[Jakes, 1974] Jakes, W. (1974). *Microwave Mobile Communication.* Wiley, New York.

[Lau and Tam, 2001a] Lau, F. and Tam, W. (2001a). Novel SIR-estimation-based power control in a CDMA mobile radio system under multipath environment. *IEEE Trans. Vehicular Technology*, 50(1):314–320.

[Lau and Tam, 2001b] Lau, F. and Tam, W. (2001b). Predictive closed-loop power control in CDMA mobile systems. *Electronic Letters*, 37(1):52–54.

[Lee and Steele, 1996] Lee, C.-C. and Steele, R. (1996). Closed-loop power control in CDMA systems. *IEE Proc. Comm.*, 143(4):231–239.

[Oppenheim and Schafer, 1989] Oppenheim, A. and Schafer, R. (1989). *Discrete-Time Signal Processing.* Prentice-Hall, Englewood Cliffs, NJ.

[Park and Nam, 1999] Park, S. and Nam, H. (1999). DS/CDMA closed-loop power control with adaptive algorithm. *Electronic Letters*, 35(17):1425–1427.

[Pirinen, 2001] Pirinen, P. (2001). Impact of mobility and closed-loop power control to received signal statistics in Rayleigh fading channels. In *Proc. IEEE Vehicular Technology Conf. VTC'01-Spring*, pages 2859–2863, Rhodes, Greece.

[Sim et al., 1999] Sim, M., Gunawan, E., Soong, B.-H., and Soh, C.-B. (1999). Performance study of close-loop power control algorithms for a cellular CDMA system. *IEEE Trans. Vehicular Technology*, 48(3):911–921.

[Tam and Lau, 1999] Tam, W. and Lau, F. (1999). Analysis of power control and its imperfections in CDMA cellular systems. *IEEE Trans. Vehicular Technology*, 48:1706–1717.