

Satisfiability-Based Functional Delay Fault Testing

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Functional delay fault testing is concerned with propagating a transition from a primary input to a primary output of a combinational circuit. Since it does not consider individual paths in the circuit, it can overcome the biggest limitation of path delay fault testing: the explosion in the size of fault lists. Functional delay fault testing can also be used to derive test sets for IP (Intellectual Property) circuits whose implementation details are not provided. Boolean Satisfiability (SAT) and BDDs have been widely used for a variety of EDA (Electronic Design Automation) applications. Even though there have been few experimental studies to conclude the superiority of one to the other, they have been compared for a number of specific tasks in the EDA field. In this paper we show that SAT-based functional delay fault testing can yield very competitive results with careful construction of the CNF formulas for the target faults. In particular, using simple structural analysis of the circuit formulas of minimum size can be easily generated. CNF formula construction based on the circuit consistency function is presented and experimental results for ISCAS 85 and 89 circuits are reported.

1. INTRODUCTION

Delay fault testing, which addresses manufacturing defects that affect temporal behavior, is usually performed after a fabricated circuit has been tested for stuck-at faults. There are two fault models that have been widely used in delay fault testing: the *gate delay fault model* [3] which ascribes faulty behavior to individual gates having excessive delay, and the *path delay fault model* [16] which attempts to capture the distributed effects of defects over entire circuit paths leading to excessive path delays. In either case, the existence of a delay fault causes the circuit to fail to operate at the expected clock frequency.

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There have been numerous research efforts on delay fault testing based on the path delay fault model [5, 10]. The major drawback of this model is that the size of the fault list (number of paths) for circuits with a large amount of reconvergence can become exponentially large. Attempts to overcome this limitation included the use of incremental path sensitization [6, 8] and targeting a group of paths using the primitive path delay fault model [7]. Such attempts, however, still fail to handle circuits with billions of paths (e.g., C6288 of the ISCAS 85 benchmark suites [2]).

An alternative to these two fault models, called the *functional delay fault model*, was first proposed in [12] and also investigated in [17]. In [13] two approaches for functional delay fault test generation, one based on a Binary Decision Diagram (BDDs) formulation and the other on a Boolean Satisfiability (SAT) formulation, were described and compared.¹ The experimental results indicated that several hours of run time for the ISCAS 85 benchmarks circuits were required in both approaches, with the BDD method resulting in a slightly better performance.

The Boolean satisfiability problem (SAT) has received a lot of attention recently, resulting in a number of robust and efficient heuristics and implementations [1, 14, 18]. It has also been used in many CAD applications such as test pattern generation for both stuck-at and delay fault models, logic verification and timing analysis [4, 8, 9, 11, 15]. In this paper, we present a SAT-based test pattern generation method for the functional delay fault model. We show that with careful construction of the target CNF formulas, this method can yield very competitive results. Promising results for both ISCAS 85 and 89 circuits are presented.

The remainder of the paper is organized as follows. In the next section, definitions that are used throughout the paper are presented. In Section 3, a method to generate the target CNF formula for functional delay fault testing based on the circuit consistency functions is presented with an example. In Section 4, experimental results for ISCAS 85 and 89 circuits are presented and Section 5 concludes the paper.

2. DEFINITIONS

Most of the definitions in Section 2.1 and Section 2.2 are taken directly from [15]. They are repeated here for completeness.

¹ The details of CNF formula generation for the SAT approach were not described.

2.1 Combinational Circuits

A combinational circuit C is represented as a directed acyclic graph $C = (V, E)$ where V denotes the circuit nodes and $E \subseteq V \times V$ denotes the connections between nodes. The following definitions also apply:

- $O(x)$ denotes the **fanout** nodes of node x , i.e., $\{y \in V \mid (x, y) \in E\}$.
- $O^*(x)$ denotes the **transitive fanout** nodes of node x , i.e., the set of all nodes y such that there is a path from x to y .
- $I(x)$ denotes the **fanin** nodes of node x , i.e., $\{y \in V \mid (y, x) \in E\}$.
- $I^*(x)$ denotes the **transitive fanin** nodes of node x , i.e., the set of all nodes y such that there is a path from y to x .
- $SI(\Psi)$ denotes the **side inputs** of a set of nodes $\Psi \subseteq V$ and is defined as follows:

$$SI(\Psi) = \{x \in V \mid I(x) \cap \Psi \neq \emptyset\}$$

The set of primary input nodes are referred to as PI, and the set of primary output nodes as PO. Figure 1 illustrates the definitions given in this section for a small benchmark circuit from [2].

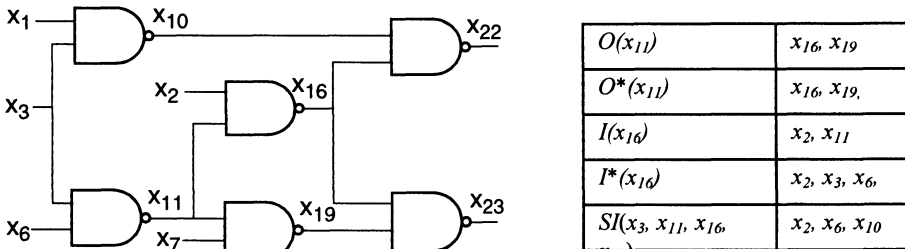


Figure 1. Example ISCAS 85 circuit C17

2.2 Boolean Satisfiability

We consider Boolean functions represented in Conjunctive Normal Form (CNF). A *literal* is an occurrence of a variable x or its complement x' . A *clause* is the disjunction (OR) of literals. A CNF formula φ on n binary variables x_1, x_2, \dots, x_n is the conjunction (AND) of m clauses w_1, w_2, \dots, w_m . A CNF formula is said to be *satisfiable* when there is at least one truth

assignment to its variables that makes all clauses equal to 1. A CNF formula is said to be *unsatisfiable* when no such assignment exists.

For each gate g in a circuit, the *gate consistency function* φ_g is a Boolean function that denotes the valid input-output assignments admissible by the gate's logic function. A detailed description of the gate consistency function and its CNF representation for primitive gates is given in [15]. The *circuit consistency function* φ is defined as the conjunction of the gate consistency functions for each node in the circuit. If we view a CNF formula as a set of clauses, the circuit consistency function can be defined using a set union operator as follows:

$$\varphi = \bigcup_{x \in V} \varphi_x$$

2.3 Functional Delay Fault Testing

The objective of functional delay testing is to propagate a transition $tI \in \{\text{rising}, \text{falling}\}$ from a primary input node to a primary output node as a transition $tO \in \{\text{rising}, \text{falling}\}$. By analogy to the path delay fault model, robust propagation for functional delay fault testing can be defined as follows:

Definition 1: For a given fault (I, O, tI, tO) , a two-pattern input combination (v_1, v_2) is said to function-robustly propagate a transition tI from input I to a transition tO on output O if the value on O changes if and only if the value on I changes.

In generating a test vector pair, two modes can be considered. In the single-input-transition mode, only one primary input is allowed to change, all other inputs being set to fixed values. In the multi-input-transition mode, any number of primary inputs are allowed to change. In this paper, we consider only the single-input-transition mode; it must be noted, though, that the proposed test pattern generation procedure can be readily extended to the multi-input-transition mode.

Definition 2: A two-pattern input combination (v_1, v_2) is a function robust test for a given fault (I, O, tI, tO) under the single-input-transition mode if and only if I is the only input that changes its values between v_1 and v_2 as a transition tI and the transition tO is observed at the output O .

Consider a primary output O of a circuit that implements the function f and a primary input I of the circuit. The detection of the faults $(I, O, \text{rising}, \text{rising})$ and $(I, O, \text{falling}, \text{falling})$ can be accomplished by checking the function $f_I \wedge (f_I)'$ for satisfiability. To detect the faults $(I, O, \text{rising}, \text{falling})$ and $(I, O, \text{falling}, \text{rising})$ the satisfiability of the function $f_I \wedge (f_I)'$ must be checked. Note that these two formulas represent the two terms of df/dI , the Boolean derivative of f with respect to I .

3. FUNCTIONAL DELAY FAULT GENERATION

3.1 Fault List Construction

Before the test pattern generation procedure can begin, a fault list needs to be created. The method used in [12] is used in constructing fault lists. The upper bound on the size of the fault list for a circuit with $|PI|$ inputs and $|PO|$ outputs is $4|PI||PO|$ since for each input/output pair, there are four (I, O, tI, tO) tuples.

There are two techniques to reduce the size of the fault list. First, for a given output, the fan-in cone for the output node can be identified and only the primary input nodes that appear in the fan-in cone are considered when creating the fault list. Second, a check can be performed for each input/output pair to determine whether there is at least one path between them with an even or an odd number of inversions. If there are no paths with an even number of inversions (I, O, tI, tO) tuples with identical (tI, tO) values can be dropped from the fault list. Similarly, (I, O, tI, tO) tuples with opposite (tI, tO) values can be dropped if there are no path between I and O with an odd number of inversions.

Figure 2 shows the fault list created for the C17 circuit. E denotes that there is at least one path between the given primary input and primary output nodes with an even number of inversions and O denotes that there is at least one path with an odd number of inversion. “-” denotes that there are no paths between the two given nodes.

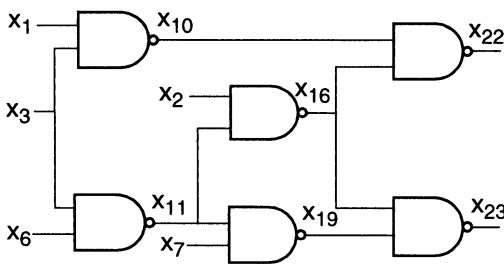


Figure 2. Fault list for C17

PI	PO	Parity
x_1	x_{22}	E
x_2	x_{22}	E
x_3	x_{22}	E, O
x_6	x_{22}	O
x_7	x_{22}	-
x_1	x_{23}	-
x_2	x_{23}	E
x_3	x_{23}	O
x_6	x_{23}	O
x_7	x_{23}	E

3.2 CNF Formula Construction

Once a fault list is set up, test generation is performed for each fault in the fault list. The overall flow of test pattern generation for delay fault testing is shown in Figure 3.

Since the formulas that must be checked involve both the positive and negative cofactors of each output with respect to each input in its transitive fan-in, the circuit is duplicated and consistency CNF formulas for both “co-factor” circuits are generated. A set of clauses from the CNF formulas thus created are then selectively chosen to create a target SAT instance for a specific functional delay fault. The clauses from the duplicate circuit are added to the target formula only when necessary. After generating the circuit consistency function, outputs are chosen one at a time. For each *PO*, the fan-in cone is identified. Then for each *PI* which has at least one path to the chosen output node, the fanout cone of the node is identified. For each (*PI*, *PO*) pair (*i*, *o*), the set of nodes that can potentially propagate a transition can be identified by ($I^*(o) \mid o$) ($O^*(i) \mid i$). The function `duplicate_nodes()` will extract the gate consistency functions from the duplicated circuit for those nodes.

Figure 4 presents an example of the test pattern generation procedure with a target fault ($x_6, x_{22}, \text{rising}, \text{rising}$). The fan-in cone for the target output node x_{22} is shown in the upper half. Since ($I^*(x_{22}) \mid x_{22}$) ($O^*(x_6) \mid x_6$) = { $x_6, x_{11}, x_{16}, x_{22}$ }, the duplicated nodes for these four nodes are identified and their gate consistency functions are selected by the function `duplicate_nodes()`. The function `connect_side_inputs()` identifies the side inputs of the given nodes in the duplicated circuit and connects them to the corresponding nodes in the original circuit. In this example, $SI(x_6, x_{11}, x_{16}, x_{22}) = \{x_2, x_3, x_{10}\}$, and the clauses that enforce the condition that each node in this set has the same value as its duplicate equivalent are added, which has the same effect as connecting a node in the original circuit to its duplicate node. Finally, for each fault between the target primary input and output pair, constraints are added based on the direction of the input and output transitions. For the example in Figure 4, the fault ($x_6, x_{22}, \text{rising}, \text{rising}$) is detected if the function

$$f_{x_6} \text{ ? } \overline{f_{x_6}}$$

is satisfiable, where f is the logic function at output node x_{22} . This is accomplished by setting x_6 and x_{22} to logic 1 and x'_6 and x'_{22} to logic value 0.

```

begin functional delay fault generation
foreach  $o$  of  $PO$ 
   $\varphi_m = \text{Identify } I^*(o)$ ;
  foreach  $i$  of  $PI$  // consider only  $i \in I^*(o)$ 
    Identify  $O^*(i)$ ;
     $\varphi_{dup} = \text{duplicate\_nodes}((I^*(o) \mid o) (O^*(i) \mid i))$ ;
     $\varphi_{conn} = \text{connect\_side\_inputs}(SI((I^*(o) \mid o) (O^*(i) \mid i)))$ ;
    foreach  $(tI, tO) \in \{(r, r), (r, f)\}$ 
       $\varphi_{cons} = \text{generate\_constraints}((tI, tO))$ ;
       $\varphi = \varphi_m \mid \varphi_{dup} \mid \varphi_{conn} \mid \varphi_{cons}$ ;
      solve( $\varphi$ );
end
  
```

Figure 3. Overall flow of functional delay fault test generation

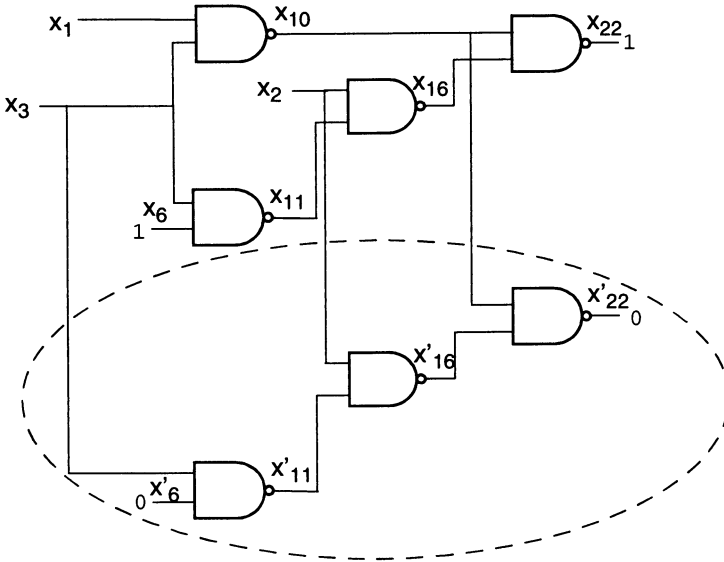


Figure 4. Target circuit for a fault $(x_6, x_{22}, \text{rising}, \text{rising})$

4. EXPERIMENTAL RESULTS

The functional delay fault test pattern generator described in this paper was implemented in C++ and integrated with the GRASP SAT solver [14]. The experiment was run on a PC running Linux with a Pentium II 300 MHz CPU and 256 Mbyte of memory. The results for ISCAS 85 circuits are presented in Table 1.

The first two columns list the name and number of faults of each circuit. From column 3 to column 5, D denotes detectable faults, R redundant faults and A aborted faults. %D in column 6 denotes the percentage of detectable faults with respect to the total number of faults. Columns 7 and 8 give the average size of the generated CNF formulas for each circuit. Although the complexity of SAT problems is not necessarily proportional to the size of the target SAT formula, it is observed that large CNF formulas usually take longer to solve. Column 7 gives average number of variables and column 8 gives the average number of clauses in each formula. Finally column 9 shows the run time in seconds. The GRASP SAT solver is called with the “+dLCS” switch, one of the decision making heuristics that turned out to be most effective for this application. The time limit for each fault was set to 100 seconds.

Functional delay test pattern can be generated for all the circuits in Table 1 with no aborted faults except for C6288. For this circuit, we applied an approximate method introduced in [13] (see Table 2). With this approximation method, a certain percentage of primary inputs are assigned random fixed values (0 or 1) before the target SAT instance is solved. The first column in Table 2 denotes the percentage of primary inputs that are assigned fixed values. This method can cause a detectable fault to be declared as redundant, but will not make a redundant fault detectable; it provides a lower bound on the number of detectable faults.

The same experiment was performed with the combinational portions of the ISCAS 89 benchmark circuits. The results are reported in Table 3. None of the faults were aborted in this case, with very reasonable run times.

Table 1. Results for ISCAS 85 circuits

Circuit	# faults	#D	#R	#A	%D	Avg # vars	Avg # cls	Time(s)
C432	794	538	256	0	67.8	249.0	667.6	84.1
C499	5248	5184	64	0	98.8	190.5	518.7	525.8
C880	1004	1004	0	0	100.0	162.2	385.4	79.8
C1355	5248	5184	64	0	98.8	442.6	1210.3	2628.4
C1908	1870	1758	112	0	94.0	619.4	1522.4	322.7
C2670	2440	2160	280	0	88.5	456.2	1125.5	487.8
C3540	2284	2242	42	0	98.2	1033.4	2817.4	5308.2
C5315	7440	7328	112	0	98.5	368.5	906.3	901.3
C6288	3036	2862	44	130	94.3	2299.3	6826.7	30490.0

Circuit	# faults	#D	#R	#A	%D	Avg # vars	Avg # cls	Time(s)
C7552	7144	6228	916	0	87.2	581.0	1374.5	1215.3

Table 2. Results for C6288 using approximation

Approximation factor	#D	#R	#A	%D	time (s)
10%	2950	46	40	97.2	35950.9
25%	2972	54	10	97.9	32456.7
50%	2972	62	2	97.9	31162.7
75%	2964	72	0	97.6	30876.7

Table 3. Results for ISCAS 89 circuits

Circuit	# faults	#D	#R	#A	%D	Avg # vars	Avg # cls	Time(s)
s208.1	152	152	0	0	100.0	48.8	108.2	6.8
s298	230	218	12	0	94.8	39.3	96.7	9.9
s344	282	278	4	0	98.6	47.4	108.2	12.8
s349	284	280	4	0	98.6	47.6	108.8	12.3
s382	408	400	8	0	98.0	41.0	91.4	17.5
s386	308	308	0	0	100.0	49.7	111.7	13.5
s400	388	380	8	0	97.9	41.3	93.1	16.8
s420	390	390	0	0	100.0	72.5	157.0	19.0
s420.1	424	424	0	0	100.0	72.3	156.8	20.9
s444	544	528	16	0	97.1	46.5	105.9	24.0
s510	310	310	0	0	100.0	77.1	189.8	16.0
s526	492	480	12	0	97.6	47.4	113.6	21.8
s526n	492	480	12	0	97.6	47.4	113.6	22.0
s641	1064	1010	54	0	94.9	160.1	349.6	71.4
s713	1148	994	154	0	86.6	165.3	370.4	78.5
s820	514	514	0	0	100.0	64.3	165.3	25.8
s832	514	514	0	0	100.0	64.4	167.8	25.7
s838	790	790	0	0	100.0	112.4	242.7	45.3
s838.1	1352	1352	0	0	100.0	107.2	222.8	75.0
s953	934	934	0	0	100.0	99.1	232.7	52.3
s1196	1208	1158	50	0	95.9	183.3	469.8	94.5
s1238	1224	1160	64	0	94.8	188.2	497.8	98.4
s1423	5836	5458	378	0	93.5	209.7	477.0	469.4
s1488	794	792	2	0	99.7	95.9	240.5	44.6
s1494	794	792	2	0	99.7	95.7	241.3	45.1
s5378	5760	5404	356	0	93.8	197.1	436.4	442.6
s9234.1	9474	8070	1404	0	85.2	327.0	738.7	999.9
s13207.1	11448	10286	1162	0	89.8	445.8	1135.4	1676.9
s15850.1	57160	44326	12834	0	77.5	973.6	2369.7	15440.1
s38417	87394	83236	4158	0	95.2	403.4	922.6	11117.4
s38584.1	47272	41420	5852	0	87.6	194.1	449.5	3697.8
s35932	22274	20674	1600	0	92.8	92.5	223.6	1228.4

5. CONCLUSIONS

In this paper, a satisfiability-based functional delay fault test generation method which uses the circuit consistency function in generating the target CNF formula was presented. Promising results for ISCAS benchmark circuits are reported.

[12] suggests generating more than one test per each fault in the fault list in order to achieve high path delay fault coverage. Incremental Satisfiability (ISAT) has been proven successful in solving multiple instances of closely related SAT problems [8]. We are currently looking at using ISAT to generate multiple tests per each functional delay fault.

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