Meshfree Automation of Engineering Analysis

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Abstract: Meshing has become one of the main bottlenecks in integrating geometric design and engineering analysis activities. We propose a meshfree approach for modelling and analysis based on RFM (the R-function method) that has the potential to achieve unprecedented levels of ease and automation.

Key words: Meshfree method, R-functions, geometric modelling, engineering analysis, parametric studies, moving domains and boundary conditions

1. MOTIVATION

Geometric design and engineering analysis have emerged as two separate activities that are only weakly connected, because they operate on distinct computer representations and require difficult representation conversions, such as finite element meshing and grid generation. This results in a slow and inefficient design-analysis cycle, inability to reflect the results of analysis in the original model, difficulty in integrating multiple analyses on a common design model, and severely restricted types of analyses available for applications with time-varying geometries.

Analysis involves solving a boundary/initial value problem of mathematical physics, where given a geometric domain and boundary/initial conditions, a time history of assumed physical quantities (such as forces, velocities, displacements, energy, etc.) must be computed. Most such models cannot generally be solved exactly, and require numerical approximations that are usually based on spatial discretization and approximation of the geometric domain. On the other hand, shape design activities are well supported by a geometric or solid modeling system, which today supports a variety of computations, spatial discretizations (for example, a finite element mesh) needed by various numerical methods to find approximate solutions to boundary/initial value problems.

The resulting situation is illustrated in Figure 1. As far as geometric design is concerned, most analyses are considered to be largely disjoint and external procedures that can be applied *after* geometry is fixed. Similarly, the majority of physical analysis and simulation codes treat geometric information as a given input, external to analysis activities. Let us briefly consider some consequences resulting from the lack of closer integration between geometric design and analysis.

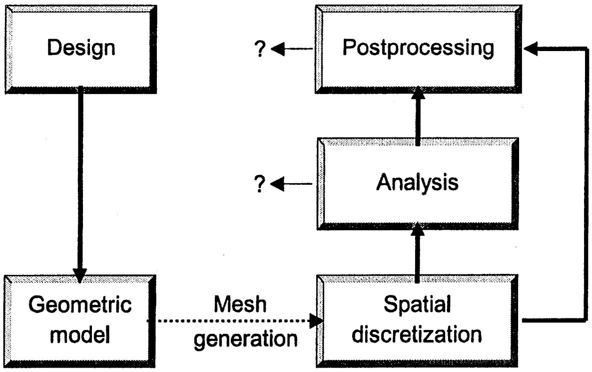

Figure 1. Lack of integration between geometric design and analysis leads to slow and inefficient design-analysis cycle

- *Meshing is a bottleneck*

Spatial discretization (meshing, grid generation) has emerged as a major research area and a bottleneck in integrating design and analysis. Semi-automatic meshing software is now commercially available, but complete automation is not possible, because good discretization has to take into account boundary conditions, singularities, changes in time, and perhaps other information about the nature of the exact solution to the problem.

- *Inefficient design-analysis cycle*

Spatial discretization is an inherently different representation scheme from those used in a geometric modelling system (Requicha, 1980). The latter usually relies on some combination of parametric, feature-based, boundary, constructive solid geometry representation to represent a solid model (Shapiro & Vossler, 1995). Thus, communicating results of the analysis back to the design model is often difficult, or impossible. This often result in a slow and inefficient design-analysis cycle in product development.

- *Lack of integrating framework*

The current state of the art in analysis and simulation of physical processes comprises painstakingly created special purpose tools. Since these representations and tools are usually created independently, and using distinct spatial discretizations, great difficulty arises in attempting to make them work together. For example, differences in type of models, and solution methods lead to significant difficulties in computer modeling and simulating problems involving multiple interacting physical phenomena. A common method of dealing with such problems is based on "coupling" the output of one program with input of another, which requires special-case handing and/or places significant restrictions on the solution techniques. Similar integration difficulties exist in combining analyses involving models at different levels of abstraction.

- *Difficulties with changing geometry*

The state of integration is inadequate even for problems with static geometry, but it is much worse in applications where geometry may be changing in time. Such applications include simulation of engine combustion, metal forming and removing operations, processes involving phase transitions (with moving boundaries). For such problems, the spatial discretization must be compatible with changes in geometry, which must be known *a priori*. It is widely acknowledged that spatial discretization dominates both the time and complexity of the many engineering analyses.

2. A MESHFREE APPROACH

We are developing a new technology for seamless integration of geometric design and analysis tools using a single hybrid computer representation. The high-level diagram explaining the proposed approach is shown in Figure 2. The key to integration is a hybrid computer representation that associates with the traditional geometric representation an *implicit real-valued function representation* and combines it with a piecewise analytic model of the analysis problem using a suitable choice of basis functions. Such representations can be constructed automatically from the standard geometric models at run time (Shapiro & Tsukanov, 1999b), and support *meshfree* analyses of spatially distributed problems, where the spatial discretization no longer needs to conform to the geometric domain or boundary conditions (Shapiro & Tsukanov, 1999a).

- *The term **meshfree** does not necessarily imply the absence of a spatial grid, but means that the spatial discretization (if any) does not need to conform to the geometry of the domain.*

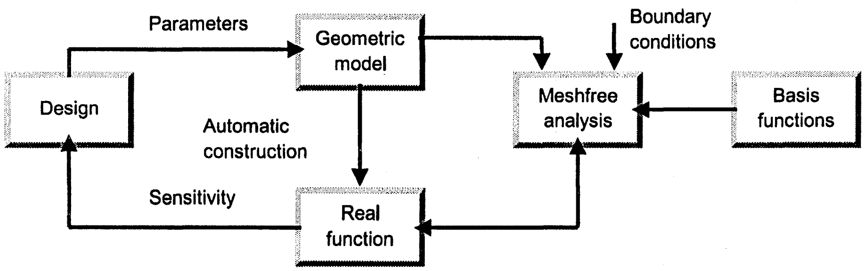

Figure 2. Proposed approach to integrating design and analysis

Our approach to solving engineering analysis problems was originally proposed by Kantorovich in 1950's (Kantorovich & Krylov, 1958) for restricted class of problems, and later generalized by Rvachev (1974) using the theory of R-functions. We will therefore refer to this method as RFM (R-Function Method); detailed description of the method is given by Rvachev & Sheyko (1995), and Rvachev et al (2000a).

It is important that all geometric computations required during the analysis (such as point membership classification) are performed using the authentic geometric model; the constructed functional representations are used only for computations of field values and derivatives. The associated implicit functions are automatically constructed using R-functions and provide the crucial link between the geometric and the field model. Detailed description of the constructions is provided by Shapiro and Tsukanov (1999) and Rvachev et al (2000b).

A particularly good choice for basis functions is a set of B-splines located on a uniform rectangular grid that spatially discretizes the whole space, but does not conform to the geometric model. The resulting method inherits all the usual advantages of B-splines including local support, multi-resolution, and other numerical properties. The computations are essentially meshfree, in the sense that representation of the engineering solution (on the regular mesh) is decoupled from the representation of the geometric model (by boundary representation), and the changes in geometry do not require changes in the mesh.

3. EXAMPLE APPLICATIONS

The proposed new approach to integrating CAD and CAE in a meshfree manner is illustrated on a simple torsion problem (see Figure 3). Given a standard geometric model of the cross-section (Figure 3(a)), associated implicit defining function is constructed automatically (Figure 3(b)); this function satisfies the homogeneous Dirichlet boundary conditions as required by the torsion problem. Combination of this function with a set of B-splines is shown in Figure 3(c), where the coefficients of B-splines are randomly assigned. The coefficients can also be chosen to minimize an appropriate functional of the problem, as shown in Figure 3(d). Figure 3(e) shows that computed results are in the complete agreement with the well known closed form approximation of the solution (Pilkey, 1994). If geometric model is parameterised, so are the automatically constructed functions; obtaining solutions for parametric variations as shown in Figure 3(f) is completely automatic, and requires no modification of the procedure or user intervention.

More generally, the method constructs solutions to engineering analysis problems in two stages: (1) first all prescribed boundary conditions (loads, displacements, temperatures, etc.) are interpolated directly from the given geometric model by a global function Rvachev et al (2000b); (2) this interpolating function is then combined with a set of additional basis functions (with unknown coefficients) to solve the specified analysis problem (Rvachev et al, 2000a). Implementation of the method requires automatic construction of the global interpolating functions, automatic differentiation, and computational integration over the geometric domains in a meshfree manner. Automatic differentiation and computational integration techniques are non-trivial but relatively standard procedures. Automatic construction of interpolating functions has been recently described in Rvachev et al (2000b) and is briefly illustrated below.

A common method for interpolating scattered data is based on inverse distance weighting (sometimes called a Shepard's method). The basic idea is that the influence of each known data sample should be inversely proportional to the distance to the point; the weights are also normalized to form a partition of unity and to satisfy the usual interpolation conditions (Hoschek, J. and D. Lasser, 1993). The method generalizes to transfinite interpolation over arbitrary sets that can be represented implicitly by smooth distance-like functions, such as those that are constructed using theory of R-functions. Figure 4 shows results of straightforward application of this technique in two dimensions. The functional value and normal derivative data are prescribed as shown in Figure 4(a) over the heterogeneous collection of points sets: two smaller discs, a circle, and a line segment. Application of inverse distance technique with R-functions yields the function shown in Figure 4(b). The same technique may be applied to a more general case with derivatives prescribed in any directions (Figure 4(c)). The constructed interpolating surfaces (functions) can be combined with a set of B-splines and smoothed to approximate solutions of desired boundary value problems (Figure 4(d), (e)). Detailed discussion of the method and additional examples are given by Rvachev et al, (2000b)

The technique has numerous applications and advantages, when compared to other methods of interpolation. In particular, because the method places no restrictions on topology, adjacency, and dimension of the interpolated sets, it eliminates the need for preprocessing or meshing and allows effortless modification and updates of the input geometry and data, which is particularly useful in modeling time-varying boundary conditions and geometry. Figure 5 shows application of the method to a typical problem of heat transfer in an internal (moving) combustion engine chamber that involves deforming domain and moving boundary conditions. Geometric model and boundary conditions (Figure 5(a)) are used to construct the associated functions (5(c)) that are combined with a uniform grid of B-splines (5(b)) to solve the heat transfer problem at multiple time steps and geometric configurations (5(d)). More detailed description of the methodology, specific constructions, and the solution technique is given by Shapiro & Tsukanov (1999a).

In summary, the proposed meshfree approach to integrating CAD/CAE activities is well suited for parametric studies, sensitivity computations, shape optimization, and multi-physics analysis, because the global interpolating function is expressed in terms of geometric parameters by construction, and the same functions may be used for different types of analysis. We will demonstrate SAGE, first fully implemented two-dimensional prototype of meshfree analysis based on RFM, and discuss progress towards development a three-dimensional system coupled with a solid modeler.

ACKNOWLEDGMENTS

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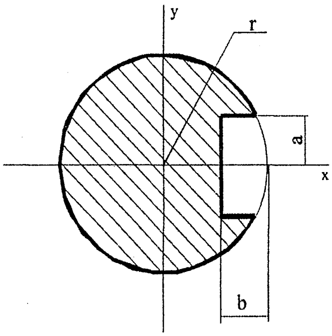
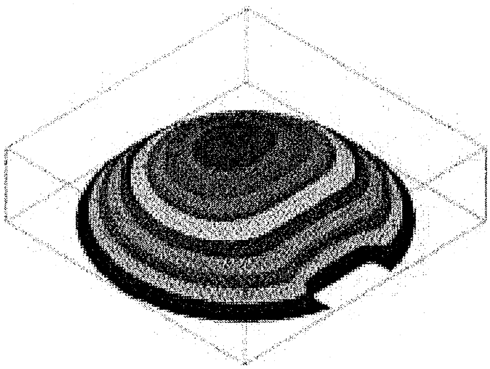
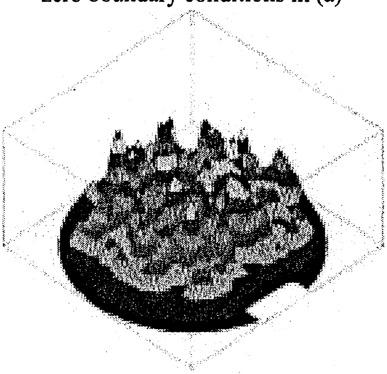


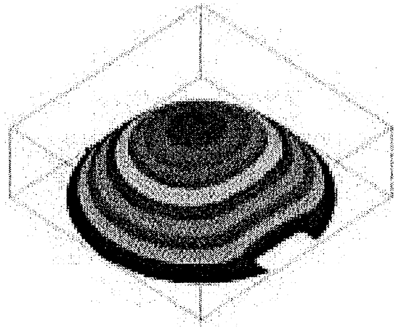
(a) Geometric model of parameteterized cross-section



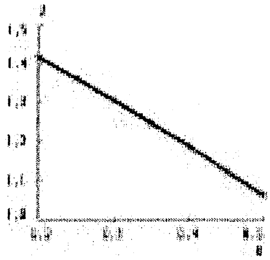
(b) Global function interpolating the zero boundary conditions in (a)



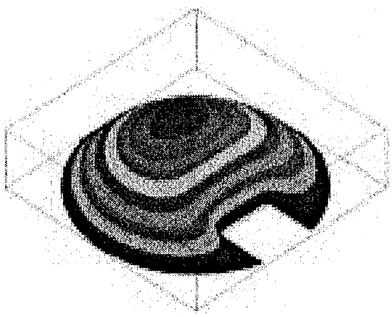
(c) Global function (b) combined with B-splines on a regular grid with random coefficients



(d) Function in (c) with B-spline coefficients minimizing the energy

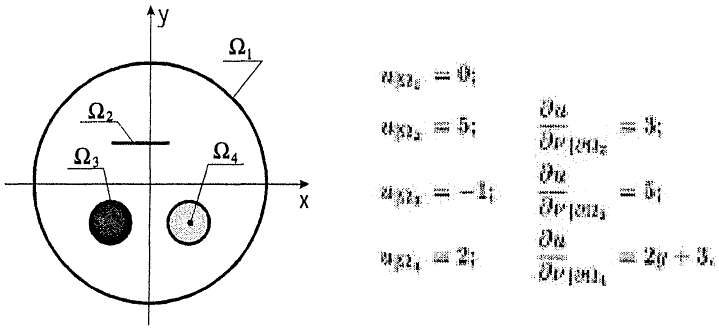


(e) Comparison with the analytically predicted solution

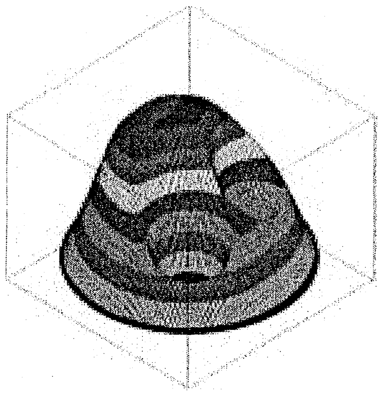


(f) Parametric variation of function in (d) automatically computed by varying geometry in (a)

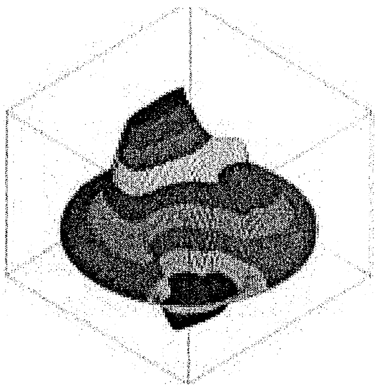
Figure 3. Illustration of the approach: torsion analysis using RFM



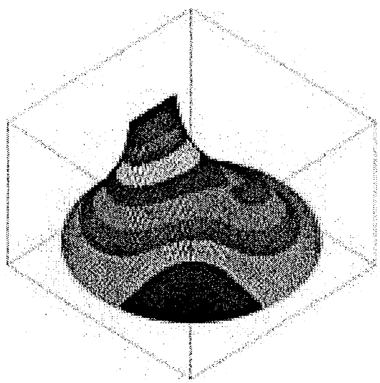
(a) Functional and normal derivative data are prescribed over heterogeneous collection of four point sets



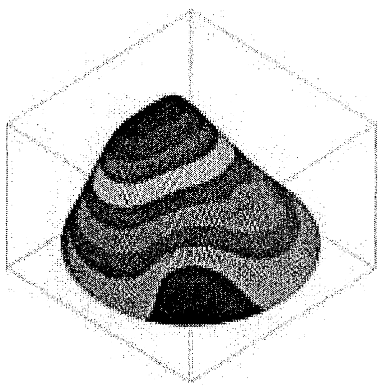
(b) Functional surface satisfying all prescribed boundary conditions in (a)



(c) Function interpolating other values and directional derivatives over heterogeneous in dimension objects in (a)

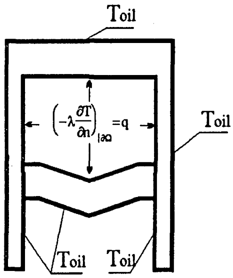


(d) Functional surface interpolating boundary conditions and minimizing tension energy

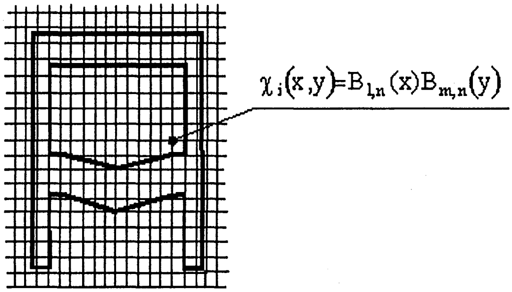


(e) Functional surface interpolating boundary conditions and minimizing bending energy

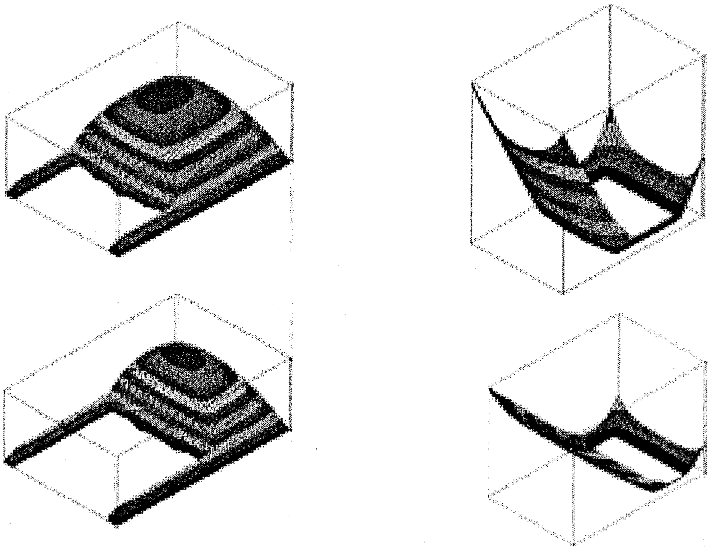
Figure 4. Transfinite interpolation over implicitly defined sets



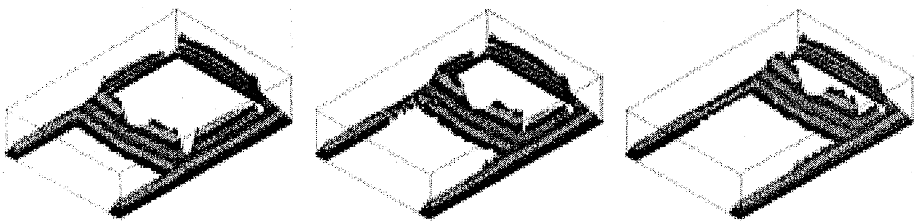
(a) Geometric two-dimensional model of a moving piston



(b) Non-conforming uniform grid of B-splines is used to approximate the solution at every step



(c) Functions representing the boundaries automatically change (move) with changes in geometry



(d) Temperature distributions computed for three different positions of the moving piston using the same interpolating functions shown in (c) and on the same non-conforming mesh shown in (b)

Figure 5. Transient Heat Transfer in Internal (Moving) Combustion Chamber